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Study on the Sufficient Certification of Cauchy Convergence Criterion

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Abstract

Cauchy criterion is widely used in mathematical analysis, is the basic theory of mathematical analysis. We prove the Cauchy convergence criterion by using the interval proof theorem and the compactness theorem. In most of the research results, the basic theorem of the real number system is proved in a chain-like manner, and finally an argument ring is formed. The proof of the Cauchy criterion is the focus and the difficulty, especially its adequacy. This paper focuses on the Cauchy convergence criterion sufficient proof, the necessity is simple, this paper only gives a proof.

Keywords: Cauchy Convergence Criterion, Application study

1 Introduction

In the current mathematical analysis of textbooks, Cauchy convergence of the proof that most of the same, for some beginners difficult to understand, mainly reflected in the idea of some ring true. The author believes that the criterion is the Heine theorem, the proof of the adequacy of the use of Heine theorem from the thinking is more smooth, more clear, and the actual teaching effect is good.

2 Cauchy convergence principle and its sufficient proof

Cauchy convergence principle is one of the most important theorems in mathematical analysis. This principle provides a new way to study the limit of series and function limit.

$$\lim(fx) \Leftrightarrow \forall \varepsilon > 0, \exists \delta > 0, \forall x'x'' : |x' - a| < \delta$$
$$0 < |x'' - a| < \delta$$
$$|f(x') - f(x'')| < \varepsilon$$

The Necessity of Evidence Adequacy using Heine's theorem) is known

$$\forall \varepsilon > 0, \exists \delta > 0, \forall x'x''$$
$$0 < |x'' - a| < \delta$$
$$|f(x') - f(x'')| < \varepsilon$$
$$\forall \varepsilon > 0, \exists \delta > 0, \forall x'x''$$
$$0 < |x'' - a| < \delta$$
$$|f(x') - f(x'')| < \varepsilon$$

The above $|a_n|$

$$\delta > 0, \exists N > 0, \forall n_1, n_2 > N$$
$$0 < |a_1 - a| < \delta$$
$$0 < |a_2 - a| < \delta$$
$$|f(a_1 - a_2)| < \varepsilon$$

By the Cauchy convergence criterion of the series limit, $f(|a_n|)$ Convergent set

$$\lim_{n\to\infty}(f(|a_n|)) = b$$

Take any of this column

$$\{C_n\} C_n \neq a$$

And $\lim_{n\to\infty} C_n = a$

The same applies to the above

$$\delta > 0, \exists N > 0, \forall n > N_2$$
$$0 < |C_n - a| < \delta$$
$$N = \max\{N_1 N_2\}$$
$$\forall n > N$$

$0 < |a_n - a| < \delta, 0 < |C_n - a| < \delta$ is ok at the same time

Then

$$\{f(C_n) - f(a_n)\} < \varepsilon$$

$$f(C_n - b) = \left| f(C_n) - f(a_n) + f(a_n) - b \right|$$
$$\leq \left| f(a_n) - f(a_n) \right| + \left| f(a_n) - b \right|$$
$$< 2\varepsilon$$

Thus for any number of columns $\left|C_n\right| c_{\ n} \neq a \lim C_{\ n} = a$

Column $f(C_n)$ } are convergent to b, by the completeness of the Heine theorem $\lim_{x \to n} f(x) = b$

3 Another Cauchy Convergence Analysis

A necessary and sufficient condition for the convergence of the sequence is that for any given positive number ε there is always a positive integer N such that when m> N and n> N, $|x_n - x_m| < \varepsilon$

We call {xn} that satisfies this condition the Cauchy sequence, then the above theorem can be expressed as: Sequence {xn} converge if and only if it is a Cauchy sequence. The geometrical meanings of the criterion indicate that the necessary and sufficient condition for the convergence of the sequence {xn} is that the elements in the sequence become closer as the ordinal number increases, that is, any two terms are close enough to be infinitely close.

Since the Cauchy convergence criterion of sequence is one of the manifestations of real continuity, we prove that {xn} converges with the real axiom-Dedekind law.

It is first proved that the Cauchy sequence is bounded. According to the definition of Cauchy sequence, for any ε> 0, there exists positive integer N, when m, n> N, | xn-xm | Then take m = N +1, then when n> N, | xn-xN +1 | Xn + 1 + ε, that is, when n> N, {xn} has an upper bound and a lower bound, so it is bounded. The first N terms of {xn} are added to the above sequence to get {xn} itself. Since the first N terms are deterministic real numbers, the boundedness of {xn} will not be changed (even if {xn} Change). Therefore, for any positive integer n, {xn} is bounded. Suppose {xn} ⊆ [a, b]. Second, the Cauchy sequence is proved to converge. There is a set of real numbers A, and any element c in A satisfies that there are at most {xn} finite terms in the interval (-∞, c) (note the word "max", meaning there may be 0 items) The infinite term in {xn} falls in [c, + ∞). And the complement of A in R is set to B, then:

① by taking the law can be seen a ∈ A, and obviously b ∈ B. That is, A and B are non-empty sets.

②A∪B = R.

According to the definition of set A, B, any element in A is less than any element in B. From Dedekind theorem, there exists a unique real number η such that η is either the maximum in A orMinimum value.

4 Cauchy convergence applications

One of the applications of the Cauchy convergence criterion is that we can prove the exact bound principle of real numbers. Boundary Principle: There are upper (lower) bounds of nonempty, there must be upper (lower) bounds. Proof: There are bounds on the upper bounds of the primes. Let S be a nonempty set of numbers with upper bounds and b1 be an upper bound. We know that there exists a real number a1 such that a1 is less than an element of S, that is, a1 is not an upper bound of S, because of the nonempty property of S and the orderliness of real numbers.Let the closed interval [a1, b1] bisect, taking into account the midpoint of the closed interval, if S is the upper bound, then;Repeat this step, ie.

if the midpoint of a closed interval is the upper bound of S, let go. In this way, a series of closed intervals are obtained1 <bn + 1, Bn And by the construction of the closed interval, we know that for any natural number n, an are not the upper bound of S, and bn is the upper bound of S. The next card {an}, {bn} converges. From the definition of limit, we know that, when n> N, | bn-an | <ε.And for any positive integer n and p, according to

① know, an ≤ a + p <bn + p ≤ bn.Then, when n> N, 0 ≤ | an + p-an | & lt; | bn-an | & lt;Let n + p = m, we can get {an} is a Cauchy sequence, converge by the Cauchy convergence criterion {an}. Set, by

② get. R is the upper bound of S. Since bn is the upper bound of S, then for any element x in S there are x ≤ bnFrom the preservation of the limit of the theorem, we know that x ≤ r, that is, r is the upper bound of S. Also take any r '<r, byAnd the limit order of nature, there is a positive integer N, when n> N, there an> r '. ∀an is not the upper bound of S that is, the number smaller than r is no longer the upper bound of S. According to the definition of upper bound, r is the upper bound of S, that is, the set of upper bounds of nonempty space must have upper bound.

Secondly, we prove that there are bounds for the lower bounds of nonempty spaces. Let B be a nonempty set of lower bounds, and A be the set of all lower bounds of B. According to the definition of lower bound, there are a ≤ b. In other words, all elements in B are upper bounds of A, and A is a nonempty set of upper bounds. Since it is proved that there are bounded upper bounds on the nonempty space, then A has the upper bound, and the upper bound is r. The next card r is also the bound of B's. It is clear that r ∈ A, because if r ∉ A, then r must be the minimum in B (according to the definition of the upper bound), that is, for any element B, b ≤ r. According to the definition of lower bound, r is also a lower bound of B, so that r ∈ A contradicts the hypothesis. And r "> r, so r"∉A, that is greater than the number of r is no longer the lower bound of B. According to the definition of the lower bound, r is the lower bound of B, that is, the lower bound of the nonempty set must have a lower bound.

References

[1] Proof of the basic theorem . Optimization and Management Science. 1985 (02)

[2] Li Huan. Fahrenheit economic mathematical basic theorem of the promotion . Optimization and Management Science, 2(2), pp. 102–106, 1999.
[3] Hu Chaoyang. On the Ляпунов second method of the basic theorem of the extension . Journal of Jiangxi Normal University, 3(1), pp. 70–75, 2005.
[4] Li Jing. Discussion on Equivalence of Fundamental Theorem of Real Number .Chinese Journal of Scientific and Technical Innovation, 12(2), pp. 31–35, 2011.
[5] Lin Qun.T he basic theorem: the integral and partial conversion formula . Mathematics in Practice and Theory, 12(3), pp. 26–35,2008.
[6] Hua Luogeng. Mathematical Theory of Large-scale Optimization of Planned Economy - IX. Proof of Fundamental Theorem . Chinese Science Bulletin, 6(1), pp. 39–51, 1983.
[7] Tan Zhongtang. The basic theorem about the change of observer and space-time transform .Acta Solida Mechanica Sinica, 5(1), pp. 61–65, 2008.