Research on the Optimal Disposal of Space Debris for Private Firms Cost Analysis and Risk Aversion

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Keywords: Space Debris, Private Firms, Processing method, Cost Analysis, Risk Aversion

Abstract. Space debris, dispersed in space densely, has become a hot global issue since collisions happened increasingly. How to develop a time-dependent model to remove debris and benefit from this project is a rare commercial opportunity for private firms. Firstly, cost analysis and planning is one of the most important components. It clearly shows the costs in different process. Also, relied on opportunity cost principle, accountant principle, science of financial management and analytical hierarchy process, the risks and benefits are analyzed in detail. It turns out the methods are stable. It can help private firms clean up space debris and enjoy profits.

Introduction

Space debris, also called orbital debris, has become a hot global issue since collisions in space happened increasingly. According to data reported by NASA, there are more than 50,000 pieces of space debris around the Earth[1].

The quantity of space debris around the earth reached the tipping point already in 2011. And risks of being struck increase rapidly threatening safety of artificial satellites [2]. To reduce the risk of collision between spacecraft and orbit debris, the U.S. Space Surveillance Network tracks all debris larger than 10 centimeters [3].

Nowadays, scientists have proposed a large number of methods to remove the debris. Nevertheless, it is still difficult for us to solve this problem. Despite all this, more and more private firms long for a model to assess the different solutions to space debris problems so that they can benefit from this commercial opportunity.

Cost Analysis and Planning

At first, we take the first part of the whole problem, rocket launch costs, into consideration. And the stick figure of multistage rocket is shown.

In our model, the launch cost that is concerned with the stage of rocket and tonnage of fuels can be worked out. Through this model, we can get optimal solution. According to basic physical principles[4], the derivational.

Due to our assumptions, the rocket is attracted by the Earth only. The running velocity of devices can be worked out by Newton's law of universal gravitation.

In the rocket ascent, the rocket structure mass and fuel mass decrease simultaneously in a proportion of λ and 1-λ.

According to the conservation laws of momentum and energy, we get an equation.

$$m(t)v(t) = m(t+\Delta t) - (1-\lambda)\frac{dm}{dt}(v(t)-u)\Delta t \tag{1}$$

Then, we plug into Equ.1[5], another equation can be got.

$$v_2 = 3\ln\left[\frac{\left(\frac{m_1}{m_2+m_p}+1\right)\left(\frac{m_2}{m_p}+1\right)}{\left(\frac{0.1m_1}{m_2+m_p}+1\right)\left(\frac{0.1m_2}{m_p}+1\right)}\right] = 6\ln\left(\frac{k+1}{0.1k+1}\right) \tag{2}$$

In the same way, we can calculate v3 and v4.

$$\begin{gathered} v_3 = 9\ln\left(\frac{k+1}{0.1k+1}\right) \\ v_4 = 12\ln\left(\frac{k+1}{0.1k+1}\right) \end{gathered} \tag{3}$$

When the final velocity of each stage rocket is equal to v_0, we achieve the minimum. Plug the final velocity, we get the results.

$$\begin{cases} S_2 = 47.2m_p \\ S_3 = 25m_p \\ S_4 = 19m_p \end{cases} \tag{4}$$

So, the cost is

$$\begin{cases} C_2 = c_1 + c_2 + 47.2m_p \\ C_3 = c_1 + c_2 + c_3 + 25m_p \\ C_4 = c_1 + c_2 + c_3 + c_4 + 19m_p \end{cases} \tag{5}$$

According to assumptions, it's known that devises orbit the earth to do uniform circular motion. Then the motion state equation of devices can be listed based on law of uniform circular motion.

$$\begin{cases} \frac{GMm}{r^2} = m\frac{v^2}{r} \\ T_1 = \frac{2\pi r}{v} \end{cases} \tag{6}$$

At near surface, we can get the equation.

$$\begin{cases} \frac{GMm}{R^2} = mg \\ T_1 = \frac{2\pi r}{v} \end{cases} \tag{7}$$

Then, we get the equation as follow

$$T_1 = 2\pi\sqrt{\frac{(r-d)^3}{gR^2}} \tag{8}$$

In the whole process, we can know the equation.

$$\theta = 2\pi t\left(\frac{1}{T_2} - \frac{1}{T_1}\right) \tag{9}$$

Then we plug T1 and T2 into Eq.9, the equation can be listed.

$$t_0 = \frac{\theta}{R\left(\sqrt{\frac{g}{(r-d)^3}} - \sqrt{\frac{g}{r^3}}\right)} \tag{10}$$

It is easy to know that $Sum = \beta\frac{T}{t}$, so the equation of Sum can be calculated.

$$Sum = \beta \frac{TR}{\theta}\left(\sqrt{\frac{g}{(r-d)^3}} - \sqrt{\frac{g}{r^3}}\right)$$

Risk Aversion and Benefit Analysis:

In the process of collision, we set up Three-dimensional modeling to predict potential risks. At first, the covariance matrix can be constructed due to errors in velocity and position of *X*, *Y* and *Z* axis[6]. On the basis of assumption, equations can be listed[6].

$$p(\bar{\bar{\rho}}) = \frac{1}{(2\pi)^{3/2}(\det A)^{1/2}}\exp(-P) \tag{11}$$

$$P = \left(\bar{\bar{\rho}} - \bar{\bar{\rho}}_0\right)^T A^{-1}\left(\bar{\bar{\rho}} - \bar{\bar{\rho}}_0\right)/2 \tag{12}$$

After a series of coordinate transformation, *A* can be simplified to positively definite matrix.

$$A = \begin{pmatrix} \sigma_1^{\ 2} & 0 & 0 \\ 0 & \sigma_2^{\ 2} & 0 \\ 0 & 0 & \sigma_3^{\ 2} \end{pmatrix} \tag{13}$$

And its quadratic form Eq.12 means a surface of an ellipsoid whose three axes are σ_1, σ_2 and σ_3. We called this error ellipsoid representing the probability density distribution in collision

Through coordinates transform, extents of error to devices and space debris in their coordinates are projected to coordinates system for terrestrial equator inertia. At the same time, rendezvous reference systems will be built up every time they collide. Under this coordinate system, probability density function of devices and space debris positions errors can be got.

$$\begin{aligned} f_{x.y}(x,y) &= \frac{1}{2\pi\sigma_x\sigma_y}\exp\left\{-\frac{1}{2}\left[\frac{(x-x_0)^2}{\sigma_x^{\ 2}} + \frac{(y-y_0)^2}{\sigma_y^{\ 2}}\right]\right\} \\ x_0 &= R_{rel}\sin\phi \\ y_0 &= R_{rel}\cos\phi \end{aligned} \tag{14}$$

In Eq.13, $\sigma_{x,y}$ is the standard deviation of devices (x) and space debris (y) positions.

x_0 and y_0 show the relative positions relationship of devices and space debris. *Rrel* is the size of projection that relative position vector projected to rendezvous plane. ϕ, a relative vector, is the angle between the y-axis and the projection on rendezvous plane. In calculation, the device is regarded as a circle with a radius of R_S located at the origin of coordinates. So, collision probability is equal to the integral of probability density in a certain safety zone.

- Yellow warning value: $P_c \le 10^{-5}$. It's safe of devices in this rendezvous.
- Orange warning value: $10^{-5} \le P_c \le 10^{-4}$. It's slightly dangerous of devices in this rendezvous. In case that has no influence on devices' main tasks and doesn't damage the loan of devices, they should ignite to swing and avoid.

Red warning value: $P_c \ge 10^{-4}$. It's very dangerous of devices in this rendezvous, so they should ignite to swing and avoid immediately.

For private firm, only taking investment costs, returns and relevant risk into considerations to find a suitable sustainable method can clean up space debris, and in the meanwhile benefit [8]. Hence, the assessment of revenue risks for private firms is extremely significant if they want to clean up space debris.

According to Launch Cost Model we build, it is known that total mass can be determined once quantity and variety of devices are determined. By comparing with different launch costs of different stage rocket, the minimum launch cost can be determined. That is, C_1 means minimum total launch cost and we can get the equation.

$$C = C_1 + C_2 + mne \tag{15}$$

Then we consider the benefit problem. In assumptions, space debris distribute uniformly, so it can be predicted that space debris around devices also distribute uniformly. In this way, we can get following equation.

$$P_g = \lambda r P_0 D + \alpha t\eta + S_2 \cdot ne + Sum \cdot S_1 \cdot ne \tag{16}$$

And we know equations about *B*, ne and Sum (Eq.13).

$$B = P_g - C \tag{17}$$

$$ne = v_0 t \tag{18}$$

Combining Eq14, Eq.15, Eq.16 and Eq.17, we can get the equation.

$$B = \lambda r P_0 D + \alpha t\eta + v_0 t S_2 + S_1 v_0 t \cdot Sum - (C_1 + C_2 + m v_0 t) \tag{19}$$

- When $\alpha\eta + (S_2 + S_1 \cdot Sum - m)v_0 > 0$, the slope of figure is positive, and private firms can make a profit.
- When $t = \dfrac{2(C_l + C_c - \lambda r P_0 D)}{v_0(S_2 + S_1 \cdot Sum - m) + \alpha\eta}$, private firms will pay back their investment.

Results and Analysis

The Impact of Error Matrix A. The relative size of error matrix is limited from zero to five. Changing the component of devices' velocity in rendezvous (L), we draw a curve about the relationship of Pc and A when L=500,700,900.

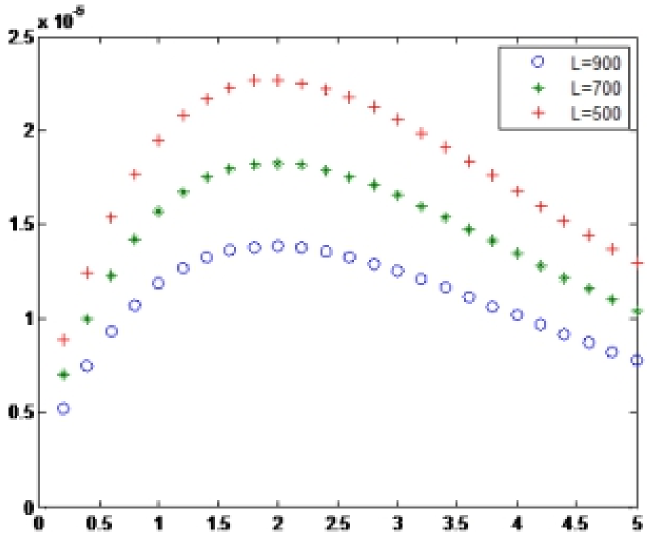

Fig.1 The curve about the relationship of P_C and ***A***

From Fig.1, when error value A is very small, the value of P_c is also very small. When error value increasing, and so do PC increase. And when error value increases to an extent, P_c is going to decrease. That is, P_c is increasing rapidly when it gets to the maximum with the change of error value, so it is important of this curve on early warning. All in all, results are not sensitive to different values of A in a large range[9].

The Impact of R_s. In different values of R_s, we draw the following figure. Each curve represents the variation of P_c with the change of error matrix in different values of R_s [10].

AS Fig.2 shows, when R_s increasing, P_c is also increasing, but it means that collision warning conservative. In conclusion, RS has little influence on P_c and we don't need to set size of the safety radius, namely, RS has little influence on collision.

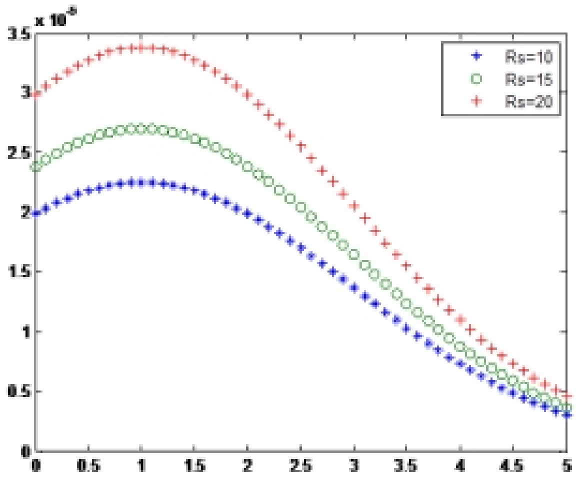

Fig.2 The curve about the relationship of P_C and R_S

Conclusion

What we have discussed above that can clean up space debris and make profits for private firms includes launch costs, handling methods of different sizes of space debris, assessment of revenue risks for private firms and comparison of different options. Taking these actual factors into consideration, we developed four typical models—Launch Cost Model, Cleaning Model, Collision Model and Revenue Risk Assessment Model to help private firms choose an optimal scheme. In addition, we make a comparison with three cleaning methods, that is, laser removing, satellite cleaning and suicide satellite eliminating. Through fuzzy evaluation method, we draw a conclusion that laser removing is more efficient in real life. All in all, it is recommended for private company adopt the combination of eliminating and avoid debris whereas the actual conditions. It is well established that building a laser removing base and Collision warning system can be the most valuable approach. Private firms can enjoy profits of removing space debris through our models.

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