

## Research on the Toll-gate on Freeway Based on the Delay Time Model

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**Abstract.** For the sake of providing scientific the theoretical basis for establishing appropriate counts of toll booth on freeway, we build a delay time model which is established by hydromechanics theory and queuing theory. Using this model which can get the average total delay time required for a vehicle to pass the toll plaza. In the transition stage, traffic flow is compared to fluid. Based on assumptions, we can discuss how the traffic flows at the transition area. Based on queuing theory, average delay time and the number of toll booths can build a connection. In General, when the average total delay time is minimized, then the number of toll booths is optimized.

## The Establishment of the Model

The process of vehicles passing Toll Plaza can be divided into 3 stages [1], according to the regional division of Toll Plaza, There are the transition stage I, transition stage II and Waiting for service stage. This is showed in Figure 1:





Figure 1 shows the layout of the single direction toll plaza. When the vehicles entering the toll plaza, it will enter the first stage-the transition stage I. The road gradually widened as the vehicle approaches waiting for service stage, after entering Waiting for service stage, Vehicles are queued and charged. Finally, vehicles leave transition stage II. Vehicles have different behavior in the three different processes. We are divided into three sub models, in order to calculate time dividedly.

The total model can be divided into three sub-models. Each sub-model is interconnected by traffic volume and traffic speed. The volume of traffic flow is constant when vehicles pass the transition stage I and transition stage II. But when vehicles pass waiting for service stage, he volume of traffic flow is not constant. so; we concluded that the total time, which is:

$$T = T_1 + T_2 + T_3 \tag{1}$$

Where: T is the total average time of traffic flow passing through toll plaza;  $T_1$  is average time of traffic flow in the transition stage I;  $T_2$  is average time of traffic flow in waiting for service stage;  $T_3$  is average time of traffic flow in waiting for service stage II.



**The Sub Model I.** Highway traffic flow has fluid properties to a certain extent. The traffic flow of highway is regarded as continuous flow using the theory of fluid mechanics simulation. According to the fluid continuity equation [2], the continuity equation of traffic flow can be established. Assume that the traffic flow is free flow. This is showed in Figure 2. The traffic flow enters the toll plaza with the initial traffic volume  $q_0$  and the initial traffic speed  $V_0$ . According to traffic flow theory, we can get:

$$q_o = v_0 \cdot k_0 \tag{2}$$

When the traffic flows in a certain point in waiting for service stage, the relationship between traffic volume, traffic speed and traffic density is:

$$q_0 = v(x).k(x) \tag{3}$$

Where: v(x) is the traffic speed with respect to x; k(x) is the traffic density with respect to k.

Due to traffic volume is constant. The relationship between the traffic density at *X* and the width of the transition area is:

$$k_0 w_n = k(x) . w(x) \tag{4}$$

Where: w(x) is the width of the transition area at *X*, According to geometric knowledge, we can establish the equation:

$$\frac{y}{x} = \frac{w_f - w_n}{l_n} \tag{5}$$

$$w(x) = w_n + y \tag{6}$$

The relationship of transition area, the length of the transition section and the number of toll stations is

$$w_f = nd \tag{7}$$

$$l_n = n.a_l \tag{8}$$

Where: *d* is the coefficient for single direction lane number, used to compute *Z*, we take 0.02. According to the above equation can be calculated *X*, Then we can get  $T_1$ .

$$dT = \frac{dx}{v(x)} \tag{9}$$

$$T_1 = \int_0^{l_n} dt \tag{10}$$

According to the above equation, we can get the final expression of  $T_1$ .

$$T_1 = \frac{n.w_n.a_l}{v_0 \left(nd - w_n\right)} \tag{11}$$

It is worth mentioning that the process of vehicle pass the transition stage I is similar to the process of vehicle pass the transition stage II. This is showed in Figure 2. So we omit the intermediate derivation, the final result is given as Figure 3.



FIGURE 2 The layout sub mode





FIGURE 3 The lay in sub model (unilateral direction)

$$I_3 = -\frac{n^2 d a_l}{v_2 \left(nd - w_n\right)} I_n \left(\frac{w_n}{nd}\right)$$
(12)

**The Sub Model II.** Traffic volume  $q_1$  is evenly distributed to each toll intersection, after the vehicles are queued for payment, vehicles pass the transition stage II. The traffic volume through Phase 1 is constant, so  $q_1$  is equal to  $q_0$ . According to the hydrodynamic model, Each toll intersection traffic flow is . According to queuing theory, each toll intersection obeys the M / M / 1 system. This is showed in Figure 4. If we obtain average arrival rate  $\lambda$  and average service rate  $\tau$ , we can get the average consumption of time [3], which is:

$$T_2 = \frac{1}{\mu - \lambda} \tag{13}$$

Due to  $\lambda = q_1 / n$ , we can get:

$$T_2 = \frac{n}{n\mu - q_1} \tag{14}$$



FIGURE 4 Area of wait and serve

Taking into account the system state must be stable, we assume average arrival rate  $\lambda$  more than average service rate  $\tau$ , and we give the constraints on the number of toll booths,

$$n > q_0 / \tau$$

(15)

To simplify the problem. We suppose there is an initial speed  $v_2$  when the vehicle leaves the transition stage I and enters the transition stage II.

**Model Solving.** Through the above research and analysis, We integrate the objective function, which is:

$$T = -\frac{n^2 \cdot d \cdot a_l}{v_2 (nd - w_n)} \cdot l_n \left(\frac{w_n}{nd}\right) + \frac{n \cdot w_n \cdot a_l}{v_0 (nd - w_n)} + \frac{n}{n\mu - q_1}$$
(16)

In order to make the data closer to reality, we consult the New Jersey highway traffic data summary table, and we calculate the throughput by specific data. Average value was regarded as a calculated value. At the same time some variables should also arouse our attention, the highway is assumed to be a standard cross-section[4]. Single lane width is 3.55 M. The width of each toll station is 5 meters, Refer to the following chart for detailed parameters.

TABLE 1 Value of parameter								
throughput(c/h)	$w_{\rm d}$ (km)	$w_{\rm u}$ (km)	$E(\tau)$ (pcu/h)	$E(q_0)(\text{pcu/h})$	$v_{\rm o}({\rm km/h})$	v <sub>2</sub> (kn	n/h) d	
			(km)			·		
2598	0.0075	0.0075	820	3900	40	20	0.005	

There is only one main variable" n" in the whole model, The optimal solution can be obtained by changing N, That is, we can get the best number of toll booths. "n" from small to large in a certain range by integer, The purpose of this is to keep the system in a steady state.  $T_1$ ,  $T_2$ ,  $T_3$  shows different values for different N, we can see from table:

TABLE 2 Value of calculate						
n	$T_1$	$T_3$	$T_2$	Т		
7	0.0022	0.0480	0.0176	0.0681		
8	0.0024	0.0140	0.0219	0.0384		
9	0.0025	0.0091	0.0264	0.0379		
10	0.0025	0.0072	0.0308	0.0406		
11	0.0027	0.0060	0.0356	0.0445		
12	0.0027	0.0054	0.0406	0.0488		
13	0.0029	0.0051	0.0456	0.0536		
14	0.0028	0.0048	0.0508	0.0585		

Bring parameters into the equation [5], through the simulation; we can get the figure as follow. The lines in the picture are respectively represent  $T_1$ ,  $T_2$ ,  $T_3$  and T. We can see intuitively in the picture 5 that the average time of vehicles passing toll stations is the minimum when Toll stations are between 7 and 8, After calculating integer of toll station, We obtain the optimal solution: n=8.



## Conclusion

The fluid mechanics simulation theory describes the process of the vehicle passes through the toll station. Based on queuing theory, we calculate the number of optimal toll stations n=8. The average time spent in the queuing system d=21s. A delay time model which is established by hydro-



mechanics theory and queuing theory providing scientific the theoretical basis for establishing appropriate counts of toll booth on freeway.

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