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Set of Fuzzy Time Series Forecasting Models Based on the Difference Rate

Xiaojing Zhu, Hongxu Wang, Chengguo Yin and Xiaoli Lu
Hainan Tropical Ocean University, Hainan, Sanya, China, 572022

Abstract—**Song & Chissom introduced the concept of fuzzy time series in 1993[1], and many fuzzy time series methods have been proposed, however, the prediction accuracy is not high, among which, Jilani, Burney and Ardil (2007) proposed prediction model has achieved a high accuracy. This paper improves their predicted model, and proposed the set of fuzzy time series forecasting models Based on the difference rate, simplified as SFBDR, it contains the predicted model SFBDR (0.000001, 0.000003) and SFBDR (0.000003, 0.000001), in the historical enrollment of University of Alabama it can get the highest predicted accuracy of AFER=0% and MSE=0.**

Keywords-fuzzy time series forecasting method; SFBDR fuzzy number function; SFBDR inverse fuzzy number function; SFBDR Predicted function

I. INTRODUCTION

Song and Chissom[1] (1993) put forward the fuzzy time series forecasting model by using the fuzzy set theory [2], and first of all studied the predicted problem of the enrollments of the University of Alabama. Although many fuzzy time series methods have been proposed, but the predicted accuracy is not high. Literature [5-9, 11, 12, 14-17] improved the predicted model of Jilani, Burney and Ardil [10, 13] (2007), the predicted accuracy of the predicted model rose obviously, but still cannot reach the ideal state. This paper further improve their forecasting model and dig out the set of fuzzy time series forecasting models Based on the difference rate, simplified as SFBDR, when the discussion of the forecasting process for the historical enrollments of University of Alabama, it contains the predicted model SFBDR (0.000001, 0.000003) and SFBDR (0.000003, 0.000001), which can make the Average Forecasting Error Rate and the Mean Square Error to reach the ideal state: AFER=0% and MSE=0.

II. SET OF FUZZY TIME SERIES FORECASTING MODEL SFBDR

This paper extends the concepts used in Jilani, Burney and Ardil[10,13], especially applying it's the basic idea . For the time series forecasting problem, Let B be the historical universe of discourse, $B = \{B_1, B_2...B_n\}$. The Year- To- Year Percentage Change of historical data is calculated as $C_r = (B_r - B_{r-1}) /B_r$. Let C be the differential rate discourse universe of the historical data $C = \{C_2, C_3\ldots C_n\}$.

The literature Jilani, Burney and Ardil[10,13] "prediction formula" is generalized to fuzzy number function, inverse fuzzy number function and predicted function are as follows:

Definition 1: for the time series prediction problem, Let B be the universe of discourse of the historical data, $B = \{B_1, B_2\ldots B_n\}$, Let C be the difference rate for the universe of discoursed of the historical data ,$C = \{C_2, C_3\ldots C_n\}$, let fuzzy number function $G_r(u_1, u_2)$ be defined in the C.

$$G_r(\mu_1,\mu_2)=\begin{cases}\dfrac{1}{C_2}+\dfrac{\mu_2}{C_3}, & if \quad r=2\\ \dfrac{\mu_1}{C_{r-1}}+\dfrac{1}{C_r}+\dfrac{\mu_2}{C_{r+1}}, & if\ 3\le r\le n-1,\\ \dfrac{\mu_1}{C_{n-1}}+\dfrac{1}{C_n}, & if \quad r=n.\end{cases} \tag{1}$$

The independent variables $u_1 \in [0, 1)$ and $u_2 \in [0, 1)$, also called membership degree of the $G_r(u_1, u_2)$. When $u_1=u_2=u$, SFBDR's fuzzy number function is denoted as $G_r(u_1, u_2)$ $(r=2,3\ldots n)$.

Definition 2: for the time series prediction problem, Let B be the universe of discourse of the historical data, $B = \{B_1, B_2\ldots B_n\}$, Let C be the difference rate for the universe of discoursed of the historical data, $C = \{C_2, C_3\ldots C_n\}$, let fuzzy number function $G_r(u_1,u_2)$ be defined in the C, the inverse fuzzy function $E_r(u_1,u_2)$ defined on $G_r(u_1,u_2)$ as follows:

$$E_r(\mu_1,\mu_2)=\begin{cases}\dfrac{1+\mu_2}{\dfrac{1}{C_2}+\dfrac{\mu_2}{C_3}}, & if\ r=2,\\ \dfrac{\mu_1+1+\mu_2}{\dfrac{\mu_1}{C_{r-1}}+\dfrac{1}{C_r}+\dfrac{\mu_2}{C_{r+1}}}, & if\ 3\le r\le n-1,\\ \dfrac{\mu_1+1}{\dfrac{\mu_2}{C_{n-1}}+\dfrac{1}{C_n}}, & if\ r=n.\end{cases} \tag{2}$$

The independent variables $u_1 \in [0,1)$ and $u_2 \in [0,1)$, also called as degree of membership of the $E_r(u_1, u_2)$.When $u_1=u_2=u$, SFBDR's inverse fuzzy number function is denoted as $E_r(u_1, u_2)$ $(r=2,3,\ldots,n)$. Comparison of different forecasting models is provided in Table I and Table II.

TABLE I. COMPARISON OF DIFFERENT FORECASTING MODELS

Year	Enroll-ments	Wang, Guo, Feng, Jin.[5]	Saxena, Sharma, Easo. [6]	Wang, Guo, Wang, Feng.[8]	Feng, Guo, Wang Zhang.[9]	Wang, Guo, Feng , Jin.[11]	Wang, Guo, Feng, Jin.[12]
1971	13055	-	-	-	-	-	-
1972	13563	-	13486	13563	13563	-	-
1973	13867	13809	13896	13867	13867	13745	13845
1974	14696	14610	14698	14695	14696	14531	14729
1975	15460	15422	15454	15460	15461	15575	15412
1976	15311	15299	15595	15311	15312	15446	15317
1977	15603	15642	15600	15603	15604	15555	15620
1978	15861	15901	15844	15861	15860	15901	15895
1979	16807	16782	16811	16796	16804	16933	16786
1980	16919	16935	16916	16919	16920	16950	16961
1981	16388	16328	16425	16388	16387	16601	16334
1982	15433	15362	15657	15432	15430	15456	15461
1983	15497	15496	15480	15497	15496	15544	15497
1984	15145	15077	15214	15144	15143	15165	15094
1985	15163	15274	15184	15163	15163	15187	15133
1986	15984	15966	15995	15982	15976	15953	15972
1987	16859	16849	16861	16859	16858	16849	16805
1988	18150	18312	17965	18150	18150	18211	18183
1989	18970	18974	18964	18970	18974	19077	18990
1990	19328	19236	19329	19327	19326	19344	19338
1991	19337	19299	19378	19337	19338	19200	19346
1992	18876	18951	18984	18874	18872	18851	18822
AFER		0.273%	0.3406%	0.0026%	0.0099%	0.462%	0.171%
MSE		2912	9169	1	7	8963	1121

TABLE II. COMPARISON OF DIFFERENT FORECASTING MODELS

Year	Enroll-ments	Jilani, Burney, Ardil. [13]	Wang, Guo, Feng, Jin. [14]	Stevenson, Porter. [15]	Wang, Guo, Feng Zhang.[17]	SFBDR (0.000001, 0.000004)	SFBDR (0.000004, 0.000001)
971	13055	13579	-	-	-	-	-
1972	13563	13798	-	13410	-	13563	13563
1973	13867	13798	13813	13932	13867	13867	13867
1974	14696	14452	14681	14664	14691	14696	14696
1975	15460	15373	15525	15423	15461	15460	15460
1976	15311	15373	15189	15847	15311	15311	15311
1977	15603	15623	15685	15580	15607	15603	15603
1978	15861	15883	15895	15877	15861	15861	15861
1979	16807	17079	16878	16773	16797	16807	16807
1980	16919	17079	16839	16897	16920	16919	16919
1981	16388	16497	16505	16341	16375	16388	16388
1982	15433	15737	15349	15671	15436	15433	15433
1983	15497	15737	15511	15507	15497	15497	15497
1984	15145	15024	15026	15200	15136	15145	15145
1985	15163	15024	15051	15218	15163	15163	15163
1986	15984	15883	15980	16035	15861	15984	15984
1987	16859	17079	16805	16903	16858	16859	16859
1988	18150	17991	18246	17953	18154	18150	18150
1989	18970	18802	18926	18879	18976	18970	18970
1990	19328	18994	19275	19303	19329	19328	19328
1991	19337	18994	19428	19432	19338	19337	19337
1992	18876	18916	19046	18966	18766	18876	18876
AFER		1.02%	0.442%	0.57%	0.085%	0%	0%
MSE		41426	6825	21575	1384	0	0

In table I and table II, the mean square error MSE (Mean Square Error) and the average forecasting error rate AFER (Average Forecasting Error Rate) are calculated respectively as follows:

$$AFER=\left(\frac{1}{n}\sum_{r=1}^{n}\left|D_r-B_r\right|/B_r\right)\times 100\%$$

$$MSE=\frac{1}{n}\sum_{r=1}^{n}(D_r-B_r)^2$$

Definition 3: For the time series prediction problem, Let B be the universe of discourse of the historical data, $B=\{B_1, B_2\ldots B_n\}$, Let C be the difference rate for the universe of discoursed of the historical data, $C=\{C_2, C_3\ldots C_n\}$. For each given r, $r\in\{2, 3\ldots n\}$, the predicted function $D_r(u_1, u_2)$ is defined as follows:

$$D_r(\mu_1,\mu_2) = B_{r-1}\left(1+E_r(\mu_1,\mu_2)\right) \tag{3}$$

The independent variables $u_1 \in [0,1)$ and $u_2 \in [0,1)$, also called membership degree of the $D_r(u_1, u_2)$. B_{r-1} is *r-1* years of historical data, $E_r(u_1, u_2)$ is *r* years of inverse fuzzy number function (2), When $u_1=u_2=u$, $D_r(u_1,u_2)$ $(r=2,3,\ldots,n)$ is called SFBDR's predicted formula, and also denoted as SFBD(u_1,u_2)R.

In the study of time series forecasting problems, when membership degree u_1 and u_2 are the specific values of the [0,1), and the forecasting model SFBDR(u_1,u_2) can be obtained by the predicted formula (3), its application steps are as follows:

Step1: Input time series prediction of the historical data table;

Step2: Input historical data in the field of *B* and the differential rate domain *C*;

Step3: Input predicted formula SFBDR (u_1, u_2);

Step4: SFBDR (u_1,u_2) to calculate the historical data predicted value.

It can be seen that when a predicted formula SFBDR (u_1, u_2) is determined, so predicted model SFBDR (u_1, u_2) can be determined.

Definition 4: For the time series prediction problem, Let *B* be the universe of discourse of the historical data, $B = \{B_1, B_2... B_n\}$, the difference rate for the universe of discoursed of the historical data is $C = \{C_2, C_3... C_n\}$, when the u_1 and u_2 are all values times on the half open interval [0,1), the infinite time series predicted model SFBDR(u_1,u_2) can obtained. All of the time series predicted model SFBDR(u_1, u_2) consists of a collection of fuzzy time series forecasting model of difference rate the set SFTSFMBDR (The Set of Fuzzy based on Time Series Forecasting Models Based on the Difference Rate), simplify for SFBDR . General element of SFBDR is SFBDR(u_1,u_2). SFBDR(u_1,u_2), which represents a fuzzy time series predicted formula also said to SFBDR (u_1,u_2) fuzzy time series predicted formula of the predicted model.

Theorem: For the time series prediction problem, the universe of discourse of the historical data $B=\{B_1, B_2... B_n\}$, the differential rate domain of the historical data is $C=\{C_2, C_3... C_n\}$. For each set of $r \in \{2, 3... n\}$, then

1).SFBDR's fuzzy number function $G_r(u_1,u_2)$, SFBDR's the inverse fuzzy number function $E_r(u_1,u_2)$ and SFBDR's the predictive function $D_r(u_1, u_2)$ are continuous functions;

2).When $\mu_1 \to 0$, $\mu_2 \to 0$, the inverse fuzzy number function $E_r(u_1,u_2)$ converges to the differential rate of the historical data C_r:

$$\lim_{\mu_1\to 0,\mu_2\to 0} E_r(\mu_1,\mu_2) = C_r$$

3).When $\mu_1 \to 0$, $\mu_2 \to 0$, the predicted function

$D_r(u_1, u_2)$ converges to t the historical data B_r:

$$\lim_{\mu_1\to 0,\mu_2\to 0} D_r(\mu_1,\mu_2) = B_r$$

4).When the membership degree u_1 and u_2 are small enough, the predicted function value $E_r(u_1,u_2)$ of SFBDR equal to historical data B_r of the *r* years.

Table I and table II show the predicted results obtained by using other applications of defuzzification technique proposed the fuzzy time series predicted model. Because the prediction accuracy of the prediction model using the defuzzification technique is higher, because the length limits, the forecast results proposed other types of fuzzy time series prediction models are not listed. Table II give the predicted results of SFBDR (0.00001,0.00004) and SFBDR(0.00004,0.00001) to predict the historical enrollments of the University of Alabama, obtained MSE=0 and AFER=0.0%,respectively the highest predicted accuracy. The predicted results in Table I and Table II show AFER and MSE of the SFBDR (0.00001, 0.00004) and SFBDR (0.00004, 0.00001) are the smallest

III. CONCLUSIONS

In the study of predicted problem historical enrollments of the University of Alabama, applying the prediction model proposed SFBDR(0.000001, 0.000004) and SFBDR(0.000004, 0.000001) in this paper, to get the Mean Square Error (MSE=0) and the Average Forecasting Error Rate (AFER=0%) for the predicted value of the enrollments (see Table II), to finish off the problem that the prediction accuracy of fuzzy time series prediction model is not high. The following work should be study the practical application using the prediction model.

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REFERENCES

[1] Q Song, B S Chissom. Fuzzy series and its models. Fuzzy Sets and Systems, Vol. 54, pp.269-277, 1993.

[2] L A Zadeh. Fuzzy set .Fuzzy Sets and Systems, Vol. 8, pp. 338-353, 1965.

[3] Q Song, B S Chissom. Forecasting enrollments with fuzzy time series—Part I.Fuzzy Sets and Systems, Vol.54, pp. 1-9,1993.

[4] Q Song, B S Chissom. Forecasting enrollments with fuzzy time series—Part II.Fuzzy Sets and Systems, Vol.62, pp. 1-8,1994.

[5] Wang Hongxu, GuoJianchun, Feng Hao, Jin Hailong. A new forecasting model of fuzzy time series. 2014 3rdInternational Conference on Mechatronics and Control Engineering (ICMCE 2014), Applied Mechanics and Materials, Vol. 678(2014), PP: 59-63, 2014.

[6] Preetika Saxena, Kalyani Sharma, Santhosh Easo. Forecasting enrollments based on fuzzy time series with higher forecast accuracy rate. Int. J. Computer Technology& Applications, Vol.3 (3), pp, 957-961, 2012.

[7] Wang Hongxu, Wu Zhenxing. Preliminary Theory of Set SDR of Fuzzy Time Series Forecasting Model. 2016 International Conference on Mathematical, Computational and Statistical Sciences and Engineering (MCSSE2016), pp:261-265, 2016.

[8] Hongxu Wang, JianchunGuo, Hui Wang, HaoFeng. A fuzzy time series forecasting model based on yearly difference of the student enrollment number [C]. 2014 2nd International Conference on Soft Computing in

Information Communication Technology (SCICT2014), The Authors- Published by Atlantis Press, pp. 172-175, 2014.

[9] Hao Feng, Jianchun Guo, Hongxu Wang, Fujin Zhang. A modified method of forecasting enrollments based on fuzzy time series [C]. 2014 2nd International Conference on Soft Computing in Information Communication Technology (SCICT2014), The Authors- Published by Atlantis Press, pp. 176-179 , 2014.

[10] Tahseen A Jilani，S M Aqil Burney, and C Ardil. Multivariate high order fuzzy time series forecasting for car road accidents. World Academy of Science, Engineering and Technology, Vol. 1, pp: 288-293, 2007.

[11] Hong Xu Wang, JianChun Gum, HaoFeng, HaiLong Jin. A fuzzy time series forecasting model based on percentages. 2nd International Conference on Frontiers in Computer Education (ICFCE2014), December 24-25, 2014, Wuhan, China.1 ICT IN EDUCATION.Frontiers in Computer Education.pp: 11-14, 2014.

[12] Wang Hongxu, GuoJianchun, FengHao, Jin Hailong. An improved forecasting model of fuzzy time series.2014 3rdInternational Conference on Mechatronics and Control Engineering (ICMCE 2014), Applied Mechanics and Materials, Vol. 678(2014), PP: 64-69, 2014

[13] T A Jilani，S M A Burney, C Ardil. Fuzzy metric approach for fuzzy time series forecasting based on frequency density based partitioning. Proceedings of World Academy of Science, Engineering and Technology, Vol. 34, pp, 333-338, 2007.

[14] Hong Xu Wang, JianChun Guo, Hao Feng, HaiLong Jin. A fuzzy time series forecasting model based on data differences. 2ndInternational Conference on Frontiers in Computer Education (ICFCE2014), December 24-25, 2014, Wuhan, China.1 ICT IN EDUCATION. Frontiers in Computer Education. pp: 15-18, 2014.

[15] Meredith Stevenson and John E. Porter. Fuzzy time series forecasting using percentage change as the universe of discourse. Proceedings of World Academy of Science, Engineering and Technology, Vol. 55, pp, 154-157, 2009.

[16] Wang Hongxu，Wu Zhenxing. Preliminary Theory of Set DR of Fuzzy Time Series Forecasting Model. 2016 International Conference on Mathematical, Computational and Statistical Sciences and Engineering (MCSSE2016)，pp:256-260, 2016.

[17] H X Wang, J C Guo, H Feng, F J Zhang.A new model of forecast enrollment using fuzzy time series. Education Management and Management Science, 2014 International Conference on Education Management and Management Science (ICEMMS 2014), 7-8 August,2014,Tianjin, China, pp: 95-98,2014.