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Penalty Game with Mission Success Rates and Randomizing Mixed Nash Equilibrium Strategies

Based on Monte Carlo Simulation

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***Abstract*—This article aims at improving the practice feasibility of game theory for the couches and athletes in a football penalty game. Firstly, some mission success rates are introduced to the penalty game to depict the technical uncertainty. Secondly, direction strategies are digitalized in order to be randomized later by Matlab. Lastly, Monte Carlo(MC) simulation is adopted to randomize the players' direction strategies, according to the probability distribution of a mixed Nash equilibrium. On the statistical data, taking Messi and Dalei Wang as an imaginary example of penalty game, the mixed equilibrium distribution is ((0.193, 0.065, 0.741); (0.035,0.0262,0.703)). Theoretically, Messi is expected to selects the left, middle and right direction as the probability 0.193, 0.065 and 0.741 respectively; and Dalei Wang is expected to selects the left, middle and right direction as the probability 0.035,0.0262 and 0.703 respectively. The simulation results show that the randomizing coincides with the probability distribution of Nash equilibrium above 91%.**

Keywords-game theory; penalty game; mixed Nash equilibrium; strategy; Monte Carlo simulation

I. Introduction

The European Cup and World Cup football games attract lots of attentions every four years. Especially, the penalty game is highly concerned, because it is full of uncertainty and may be decided instantly. Scholars care the penalty game not only for entertainment but also for research. Some research are done from the viewpoint of game theory, such as analyzing the "final ball" game model and discussing its mixed Nash equilibrium(2002,Li and Xie) [1],utilizing the numerous statistical data to reason the game strategies of England football team(2010, Simon and Stefan)[2]. However, most couches and athletes usually find it is difficult to practice strategies randomly, which is expected to obey the probability distribution of a mixed Nash equilibrium. In fact, a mixed Nash equilibrium is difficult not only for practice, but also for being understood even for college students (2013, Cobb)[3].Although mixed Nash equilibria are highly significant in theory, but in practice few players can utilize it.

Some efforts are made to improve the practice feasibility of a mixed Nash equilibrium, for instance, doing economical experiment(2007, Bloomfield)[4], exploring the relationship between the decision analysis and game theory (2007, Binsbergen)[5], using diagrammatic methods for two-person non-zero-sum game (2008, Magirou)[6], by decision trees (2013,Cobb) [3],the immune algorithm (2010, *Cheheltani*and *Ebadzadeh*)[7],honey bees foraging optimization (2011, *Navidi, Ayanzadeh* and *Mousavi*)[8],adaptive agent-based algorithm (2013, *Farimani*, *Yektay* and *Mashhadi*)[9], heuristic-meta algorithm (2014, *Mohtadi* and *Nogondarian*)[10].These heuristic or evolutionary methods are helpful to compute the mixed Nash equilibria faster by computer. But it is still hard for people randomize their mixed Nash equilibrium strategies.

In 2015, Lisa worked out a way of choosing one of four pictures shown on the screen[11]. It really helps to randomize players' strategies, but it is still a little complex for the players who are nervous on the penalty pitch.

This article adopts Monte Carlo simulation (MC) to help players randomly select strategies according to probability distribution of mixed Nash equilibrium. Before doing it, we digitalize game strategies as convenience and set up a penalty game model, to which some mission success rates are introduced to depict the technical uncertainty.

II. The Penalty Game with Mission Success Rates

A. The SimplePenalty Game

There are two-players, kick player and the goalkeeper who are denoted by A and B respectively. Considering both players have three direction strategies, left (L), middle (M) and right (R). Simply, it is assumed whenever the players mismatch the direction, the kick player gains 1 shot and the goalkeeper loses 1 shot. Once they match the direction, the kick player loses 1 shot and the goalkeeper gains 1 shot. So the simple penalty game model is shown as Table I.

TABLE I. SIMPLE PENALTY GAME MODEL

Payoff of two players		Strategy of Goalkeeper B		
		L	M	R
Strategy of Kick player A	L	-1，1	1，-1	1，-1
	M	1，-1	-1，1	1，-1
	R	1，-1	1，-1	-1，1

B. Penalty Game Analysis with Mission Success Rates

As shown in table 1, the simple penalty game is just a direction matching game. It means whenever the directions don't match, the kick player can shot successfully; and once the directions match, the goalkeeper is sure to cartwheel block the shot successfully.

Notice that even if the goalkeeper doesn't match the direction, the kick player may fail in his shot because of skill problems. On the other hand, even if the goalkeeper matches the exactly appropriate direction, he may also miss the shot because of his own skill. This article adopts some mission success rates to depict the uncertainty of player's skill.

First of all, for the kick player, the mission success rate of shooting to the left is defines as the probability with which he succeeds his shoot when he shoot at the left direction. It is denoted by P_L. Similarly, the mission success rates of shooting to the middle and right direction can be defined, and denoted by P_M and P_R respectively; the goalkeeper's mission success rates to the left, middle and right direction are defined as above and denoted by P_I, P_J and P_K.

Then the payoff of the game should be evaluated. Take an example, if both the kick player and goalkeeper chose the left, the kick player gains if and only if he shoots successfully and the goalkeeper fails. Namely the kick player gains $P_L(1-P_I)$.On the other hand, the goalkeeper's gains when the kick player fails or being cartwheel blocked by the goalkeeper. Namely the goalkeeper player gains$1-P_L+P_LP_I$.The two payoffs can be seen in the first crossing space.

Similarly the other payoffs can be calculated, and the penalty game model with mission success rates is shown as Table II.

TABLE II. PENALTY GAME WITH MISSION SUCCESS RATES

Payoff		**Strategy of Goalkeeper B**		
		L	M	R
A	L	$P_L(1-P_I)$,$1-P_L+P_LP_I$	P_L, $1-P_L$	P_L ,$1-P_L$
	M	P_M , $1-P_M$	$P_M(1-P_J)$,$1-P_M+P_MP_J$	P_M ,$1-P_M$
	R	P_R ,$1-P_R$	P_R ,$1-P_R$	$P_R(1-P_K)$,$1-P_R+P_RP_K$

Compare Table 2 with Table 1, it is can be obviously found that: the sum of two players in each situation of Table 2is always 1, while in Table 1 it is always 0. This indicates two aspects. Firstly, the game of Table 2is a non-zero sum game, and so the players are jus competing for the whole profit 1 not opposing to each other like the game of Table 1. Secondly, the outcome of the game depends on not only the direction match but also on their skills.

Suppose the probability distribution of the mixed Nash equilibrium of the game in Table 2 is ((r_1, r_2, r_3); (θ_1, θ_2, θ_3)), with $r_1 + r_2 + r_3 = 1$ and$\theta_1+\theta_2+\theta_3=1$.Bycomputing principle[2], r_1, r_2, r_3, θ_1, θ_2 and $\theta_3$can be evaluated as(1) to (6).

$$r_1=P_MP_JP_RP_K /(P_MP_JP_RP_K+P_LP_IP_RP_K+P_LP_IP_MP_J) \quad (1)$$

$$r_2=P_LP_IP_RP_K / (P_MP_JP_RP_K+P_LP_IP_RP_K+P_LP_IP_MP_J) \quad (2)$$

$$r_3=1- r_1- r_2 \quad (3)$$

$$\theta_1= (P_MP_JP_RP_K+P_LP_MP_J+P_LP_RP_K-P_MP_RP_K-P_MP_JP_R)/(P_MP_JP_RP_K+P_LP_IP_RP_K+P_LP_IP_MP_J) \quad (4)$$

$$\theta_2= (P_LP_IP_RP_K+P_MP_LP_I+P_MP_RP_K-P_LP_RP_K-P_RP_LP_I)/(P_MP_JP_RP_K+P_LP_IP_RP_K+P_LP_IP_MP_J) \quad (5)$$

$$\theta_3= 1-\theta_1-\theta_2 \quad (6)$$

A mixed Nash equilibrium provides an ideal probability distribution for players to randomly select strategies. It means the kick player selects the left, middle and right direction respectively at the probability r_1, r_2 and r. Meanwhile the goalkeeper selects the left, middle and right direction respectively at the probability θ_1, θ_2 and θ_3.

In theory, if players randomly select strategies according to this distribution, their benefits can reach the biggest expectations in the game. If someone alone deviates away this distribution, his own profit will reduce. But it is difficult to randomly select the strategies in accordance with mixed Nash equilibrium probability distribution. In order to help the couches and players select their directions randomly as the same as the Nash equilibrium expects, the direction strategy can be digitalized simply and then MC simulation can work.

III. RANDOMIZING PRINCIPLE OF MIXED EQUILIBRIUM STRATEGIES

A. Introduction to Monte Carlo simulation (MC)

Monte Carlo method is also known as random sampling technique or statistical test methods. The basic theory is the law of large numbers and so its main idea is to use the frequency to replace the corresponding probability. This idea can be traced back to the 17th century, the earliest experiments date back to the 18th century the famous Buffon random needle cast[12]. Since the Monte Carlo method can simulate realistically the actual physical process, it has many applications.

In this paper, both the kick player and goalkeeper have exactly three direction strategies: In order to randomize direction strategies later by Matlab, we digitalize strategies as the next part.

B. The Digitalizing of Stratedgies

For convenience, three directions left, middle and right are respectively map into -1, 0 and 1. These numbers are very easy to understood, where 0 is referred to as the middle. Thus, the distribution of Nash equilibrium can be converted into two three-point discrete random strategy variable and X_B whose distribution laws are shown in Table III and Table IV.

TABLE III. DISTRIBUTION OF KICK PLAYER'S STRATEGY VARIBLE XA

	Value of The Digitalized kick player's Strategy		
	-1	0	1
Probability	r_1	r_2	r_3

From Table III, X_A has the same probability distribution as the kick player A is expected by the mixed Nash equilibrium $((r_1, r_2, r_3);(\theta_1, \theta_2, \theta_3))$. If the kick player randomly selects strategies just according to the random samples of variable X_A, then the kick player selects the left, middle and right direction respectively at the probability r_1, r_2 and r_3. Therefore X_A can be called the strategy variable of the kick player.

TABLE IV. DISTRIBUTION OF GOALKEEPER'S STRATEGY VARIBLE XB

	Value of The Digitalized Goalkeeper's Strategy		
	-1	0	1
Probability	θ_1	θ_2	θ_3

Similarly, from Table IV, X_B has the same probability distribution as the goalkeeper B is expected by the mixed Nash equilibrium $((r_1, r_2, r_3); (\theta_1, \theta_2, \theta_3))$, which expects goalkeeper selects the left, middle and right direction respectively at the probability θ_1, θ_2 and θ_3.

If the goalkeeper randomly selects strategies just according to the random samples of variable X_B, then the kick player selects the left, middle and right direction respectively at the probabilityθ_1, θ_2 and θ_3. Therefore X_B can be called the strategy variable of the goalkeeper.

C. Randomizing the Strategies on Distributions of X_A and X_B

Take randomizing kick player's strategies as an example. It means to randomly sample a three point discrete random variable X_A whose distribution law is shown in Table 3. The random sample of X_A can be done as three steps.

Step 1. R=rand. It get a random sample from the uniform

distribution in the interval (0,1).

Step 2. Set $p^{(1)}= r_1$, $p^{(2)}= r_1 + r_2$, $p^{(3)}= r_1 + r_2+ r_3$ and $r_1 + r_2+ r_3=1$. If $0<R<p^{(1)},X_A=-1$; If $p^{(1)}<R<p^{(2)},X_A=0$; If $p^{(2)}<R<p^{(3)},X_A=1$.

Step 3. Go to Step1, repeat till you get enough samples.

It can be proven the random sample R which is get by the above 3 steps methods obeys the same distribution law as X_A.

Notice that $P(0<R<p^{(1)})=p^{(1)}-0=p_1-0=p_1=P(X=1)$, and

$P(p^{(1)}<R<p^{(2)})=p^{(2)}-p^{(1)}=p_2 = P(X=0)$, and

$P(p^{(3)}<R<p^{(2)})=p^{(3)}-p^{(2)}=p_1+p_2+p_3-p_1-p_2=p_3=P(X=1)$.

It is shown that the probability of events {P (0) <R<p(1)},{ P (1)R< P (2)}and { P (3) <R<p (2)}are the same as events{X=-1},{X=0}and {X=1}. And so the probabilities of variable R falling in three intervals (0, P (1)) (P, (1), P (2), (P) (2), P (3)) are the same as the variable X_A taking the value of -1, 0 and 1.

Therefore randomizing kick player's strategies can be simulated by sampling X_A as the above 3 steps.

Similarly, randomizing goalkeeper's strategies can be simulated by sampling X_B as the above 3 steps.

IV. NUMERICAL EXAMPLE OF AN INAGINARY GAME

A. Introduction to Monte Carlo Simulation (MC)

There are few existing data on few players with the repeated kick penalty game .But thanks to the current big data environment, the authors collected some penalty data of the famous kick player Messi and goalkeeper Dalei Wang.

According to the Barcelona's official website and FC Barcelona's official website statistics[13], and some videos date which is collected by the authors directly, Messi's mission success rates to the left and right direction are respectively $P_L=0.75$ and P_R =0.8.Since the middle direction for the outstanding player Messi is lowest, it is supposed that $P_M=1$.

According to related reports of Sina Sports[14]together with some videos date which is collected by the authors directly, Dalei Wang's mission success rates to the left and right are respectively $P_I=0.45$and P_K =0.11.Since the middle direction for the outstanding goalkeeper Dalei Wang is lowest, it is assumed that $P_M=1$.Thus, the penalty game between Messi and goalkeeper Wang Dalei can be modeled as Table 5.

TABLE V. PENALTY GAME MODELWITH MISSION SUCCESS RATES

Payoff of two players		Strategy of Dalei Wang		
		L	M	R
Strategy of Messi	L	0.412,0.588	0.75,0.25	0.75,0.25
	M	1,0	0,1	1,0
	R	0.8,0.2	0.8,0.2	0.712,0.288

According to Equation (1), (2) and (3), Messi's left-middle-right strategy probabilities of the mixed equilibrium is $(r_1, r_2, r_3) = (0.193,0.065, 0.741)$.According to Equation (4), and (6), Dalei Wang's left-middle-right strategy probabilities of the mixed equilibrium$(\theta_1, \theta_2, \theta_3) = (0.035,0.262, 0.703)$.

Update Table. 3 by $(r_1, r_2, r_3) = (0.193, 0.065, 0.741)$, Update Table. 4 by $(\theta_1, \theta_2, \theta_3) = (0.035, 0.262, 0.703)$. According to the three steps method, two simulation programs are written, and simulation of 500 times is done by Matlab. The random numbers are transmitted to the players, and they select the directions just mapping -1, 0 and 1 into the direction left, middle and right respectively.

To illustrate the simulation, the histograms of Messi and Dalei Wang are shown as Figure I.

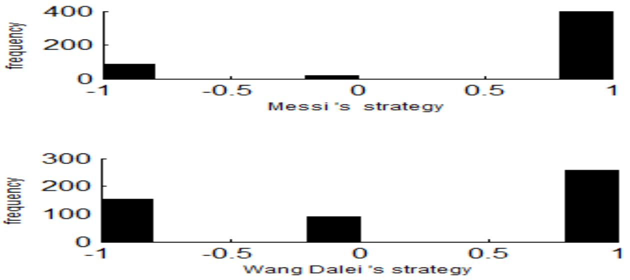

FIGURE I. HISTROGRAM OF RANDOMAL STRATEGIES

From Figure I, the frequencies of Messi selecting left, middle and right are respectively 97, 33 and 370; the frequencies of Dalei Wang choosing left, middle and right are respectively 18,131 and 351. And so, for the simulation of 500 times, the simulation probabilities of Messi selecting left, middle and right are respectively0.194, 0.066 and0.74; the probabilities of Wang Dalei choosing left, middle and right are respectively0.036, 0.262 and 0.702.

Compared with the theory probabilities (0.193, 0.065, 0.742) of Messi selecting left, middle and right in accordance with mixed strategy equilibrium, the coincidence rate is above 98%. Compared with the theory probabilities (θ_1, θ_2, θ_3) = (0.035,0.262, 0.703) of Dalei Wang choosing left, middle and right in accordance with mixed strategy equilibrium, the coincidence rate is above 97%.

Therefore the randomizing coincides with the probability distribution of Nash equilibrium above 97%.

In order to visualize the simulation of game, the two players' game and payoff curve are shown as Figure II.

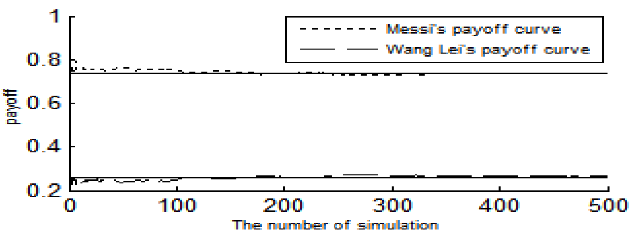

FIGURE II. SIMULATION OF PLAYERS' PAYOFF

From Figure II, it can be seen the average gains of Macy and Dalei Wang stabilizes at 0.73 and 0.26 respectively. The simulation of average gains coincides with the theoretical payoff of mixed Nash equilibrium above 91%.

Therefore the simulation of random strategies by the method is not only simple but coincides with the theoretical value at high level.

V. CONCLUSIONS

To improve the practice feasibility of game theory for the couches and players in a football penalty game, two important works are done. Firstly, a penalty game with some mission success rates introduced is set up to analyze the technical and direction uncertainty. Secondly, Monte Carlo simulation is adopted to randomize the digitalize players' direction strategies. The simulated probabilities, according to which the strategies are randomized, can be proven theoretically to ve consistent with the probabilities of the mixed Nash equilibria. The simulation of penalty game example of Messi and Dalei Wang show the randomizing coincides with the probability distribution of Nash equilibrium above 91%.

However, there are still further researches to be done in two aspects. First, the game model established in this paper is a complete information static game, but reality often encounter many incomplete and asymmetric information. And so how to model and simulate the situation remains to be further discussed. On the other hand, MC simulation only generates random sampling data independently. But in practice the strategies may affect each other, so dependent sampling simulations is worth looking forward to, such as Markov Chain Monte Carlo(MCMC),etc.

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REFERENCES

[1] Li Yiqun.Sports Game Theory[M].BeiJing:Beijing Sports University Press, 2002

[2] K. Simon, S. Stefan.(Writers), R. Ma(Translator). "Football Economics".Beijing: Chinese Light Industry Press,2010.

[3] B. R.Cobb , and T. Sen, "Finding mixed strategy Nash equilibria with decision trees," Int. Rev. Econ. Educ, vol. A20, pp.43-50, March,2013.

[4] B. Robert, "Learning a mixed strategy equilibrium in the laboratory".Jour. of Eco. Beh. and Org,vol.25, pp.411-436, December, 1994.

[5] V. Binsbergen , J.H., Marx, L.M..,"Exploring relations between decision analysis and game theory". Dec Ana,vol.4, pp.32-40, March, 2007.

[6] Magirou E.F., Vassilakis D.K., I. Androutsopoulos, "Teaching nonzero sum games using a diagrammatic determination of equilibria ".IN. Tra. on Educ., vol.3, pp.11–124, May, 2008.

[7] Cheheltani SH,Ebadzadeh SM, "Immue Based Approach to Find Mixed Nash Equilibrium in Normal Form Games". Journal of Applied Sciences, vol.10, pp.487-493, 2010.

[8] NavidiH,Ayanzadeh R,Mousavi A S, "Honey Bees Foraging Optimization for Mixed Nash Equilibrium Estimation".Trends in Applied Sciences Research,vol.6, pp.1353-1359, 2011.

[9] Farimani FD.,Yektay N.,Mashhadi HR., "A novel adaptive agent-based algorithm to find mixed Nash equilibrium".21s Ira. Con. on Elec. Eng., vol.5, pp.1-5, May, 2013.

[10] Mohtadi MM,Nogondarian K, "Presenting an algorithm to find Nash equilibrium in two-person static games with many strategies".App. Math.& Com.,,vol.2, pp.442-452, January , 2015.

[11] L. Bruttel,T. Friehe, " A note on making human randomize". Jou. of Beh. and Exp. Eco., pp.40-45, June, 2015.

[12] George S. Fishman, "Monte Carlo-Concepts, Algorithms and Applications", Berlin:Springer, 1996.

[13] Sina Sports."Drawings of Messi's missed penalties. All over the world know where to he play".http://sports.sina.com.cn/g/laliga/2015-01-23/11577494188.shtml,2015-01-23.

[14] Sina Sports."UncoverWang lei's penalty secret:This recruit is unique".http://sports.sina.com.cn/n/2015-01-12/01487478917.shtml, 2015-01-12.