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Sparse Recovery-based Multi-Static Radar Multi-Target Projection Localization

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Abstract. Multi-static radar multi-target projection localization method overcomes data association problem in the viewpoint of the imaging technique, in which the receivers are considered as a sparse antenna array that causes 2-D spatial resolution, and the transmitted broadband signal causes the range resolution. However, the range resolution is decreased due to the main-lobes broadening and the side-lobes crosstalk after the pulse compression system. The main-lobes and the side-lobes of the targets of high scatter coefficient might be selected as the targets, which lead to some false targets. In order to eliminate the main-lobes broadening and the side-lobes crosstalk, in this paper we present a sparse recovery-based multi-static radar multi-target projection localization method. Exploiting the sparsity feature of the targets under the surveillance scene, the orthogonal matching pursuit algorithm is used to reconstruct signals of each receiver. Then, the reconstruction signals are projected by BR projection to image space in which the multi-target localization is performed by the PGC algorithm. The simulated results confirm that the proposed method eliminates the main-lobes broadening and the side-lobes crosstalk and improves the range resolution.

Introduction

In multi-static radar system, a transmitter emits signals with a specific pulse repetition frequency (PRF) to the surveillance region and the receivers deployed in a vast area receive the echoes of the region. Consider that there are only the time of arrival (TOA) measurements can be obtained, such as the transmitter and the receivers equip the omnidirectional antenna and work with the flood-light transmitting mode. For the localization of multi-target, data association is required and extremely complex. Typical data association algorithms include multiple hypothesis tracking (MHT) [1] and joint probabilistic data association (JPDA) [2] for multi-target localization.

As another method which can avoid data association, multi-target positioning via projection in the viewpoint of the imaging technique have been proposed in [3, 4]. The receivers are considered as a sparse antenna array that causes 2-D spatial resolution, and the transmitted broadband signal causes the range resolution. Thus, the multi-target positioning problem is solved by the bistatic range space (BR space) projection algorithm and Positioning via Greedy and Cleaning (PGC) algorithm jointly. However, the range resolution should decline because of the broadened main-lobe and the crosstalk side-lobe caused by the ambiguity function after the pulse compression system. This might lead to some false targets in the image space.

In this paper, we extend the approach of multi-target positioning via projection to compressed sensing (CS). Considering the surveillance task, the range cells occupied by the echoes of the targets are few compared with the range domain. Thus, the echoes are sparse in the range domain. The sparse recovery methods, such as the orthogonal matching pursuit (OMP) algorithm [5, 6], basis pursuit (BP) [7] and regularization technique [8], can be applied to eliminate the side-lobes and increase the range resolution.

The organization of this paper is as follows. In section II, we introduce the multi-static radar signal model and sketch it as a linear inverse problem. In section III, the OMP-based multi-static radar multi-targets projection localization is proposed. The performance is analyzed via numerical experiment in section IV. The conclusions are presented in section V finally.

Model and Problem Formulation

Signal Model. Assuming that a target locates at $\boldsymbol{P}_w$, the TOA from the transmitter by the target to the i th receiver is,

$$\frac{1}{C}\left\|\boldsymbol{P}_T-\boldsymbol{P}_w\right\|_2+\frac{1}{C}\left\|\boldsymbol{P}_w-\boldsymbol{P}_R(i)\right\|_2=t\left(i;\boldsymbol{P}_w\right),\ i=1,\ldots,N \tag{1}$$

where, $\boldsymbol{P}_T$ denotes the transmitter's position, $\boldsymbol{P}_R(i)$ denotes the positions of the i th receiver, C is the speed of light in air, $t\left(i;\boldsymbol{P}_w\right)$ denotes the TOA of the i th receiver that can be measure directly. The TOA is proportional to the bistatic range (BR). Thus, Eq. 1 can be rewritten as,

$$\left\|\boldsymbol{P}_T-\boldsymbol{P}_w\right\|_2+\left\|\boldsymbol{P}_w-\boldsymbol{P}_R(i)\right\|_2=R\left(i;\boldsymbol{P}_w\right),\ i=1,\ldots,N \tag{2}$$

where $R\left(i;\boldsymbol{P}_w\right)=Ct\left(i;\boldsymbol{P}_w\right)$ denotes the bistatic range of the i th receiver.

The received echo of the i th receiver for $\boldsymbol{P}_w$ can be expressed as follows,

$$S\left(i,l;\boldsymbol{P}_w\right)=s\left(i;\boldsymbol{P}_w\right)\exp\left(-j2\pi f_c t\left(i;\boldsymbol{P}_w\right)\right)\exp\left[j\pi f_{dr}\left(t(l)-t\left(i;\boldsymbol{P}_w\right)\right)^2\right],\ l=1,\ldots,Nrange,\ i=1,\ldots,N \tag{3}$$

where $s\left(i;\boldsymbol{P}_w\right)$ denotes the scattering coefficient of the target located at $\boldsymbol{P}_w$ at the ith receiver, l is the index of fast time, $Nrange$ is the number of samples in the bistatic range domain, f_c denotes the carrier frequency, f_{dr} is the LFM chirp rate.

After the range-compression for the pulse compression system, the compressed echo can be expressed as,

$$S\left(i,l;\boldsymbol{P}_w\right)=s\left(i;\boldsymbol{P}_w\right)sinc\left(r(l)-R\left(i;\boldsymbol{P}_w\right)\right)\exp\left(-jkR\left(i;\boldsymbol{P}_w\right)\right),\ l=1,\ldots,Nrange,\ i=1,\ldots,N \tag{4}$$

where r denotes the bistatic range domain, $r(l)$ denotes the l th bistatic range call in bistatic range domain, $k=2\pi f_c/C$ the wave number, $sinc\left(r(l)-R\left(i;\boldsymbol{P}_w\right)\right)$ is the range ambiguity function. For multi-target environment, the echoes signal is the sum of all targets, i.e.,

$$S(i,l)=\sum_w S\left(i,l;\boldsymbol{P}_w\right)=\sum_w s\left(i;\boldsymbol{P}_w\right)sinc\left(r(l)-R\left(i;\boldsymbol{P}_w\right)\right)\exp\left(-jkR\left(i;\boldsymbol{P}_w\right)\right),\ w=1,\ldots,K \tag{5}$$

where K denotes the number of targets.

From Eq. 5, we can find that the main-lobes might be broadened and the side-lobes might crosstalk because of the Sinc function, which will lead to the range resolution declines. Because the range cells occupied by the echoes of K targets are far less than the number of the range domain, it can be considered that the echoes signal is sparse in the range domain. Thus, the CS theory [9] can be used and the sparse recovery algorithms can be applied to eliminate the side-lobes. For satisfy CS theory, first of all, a linear model of echo signal must be created.

Create a Linear Model for Echo Signal. Due to the position and the number of the targets are unknown for multi-static radar multi-target localization, we assume that $Nrange$ point targets are located at all of bistatic range cells in bistatic range domain. Thus, Eq. 5 is rewritten as,

$$S(i,l)=\sum_m S\left(i,l;r_m\right)=\sum_m s(i;m)\,sinc\left(r(l)-r(m)\right)\exp\left(-jkr(m)\right),\ m=1,\ldots,Nrange \tag{6}$$

where $s(i;m)$ denotes the scattering coefficient at *m*th bistatic range cell of the *i*th receiver. Let $s(i)=\left\{s(i;m),m=1,\ldots,Nrange\right\}$ denotes an $Nrange$-elements scattering coefficient vector of the *i*th receiver. Ideally, the number of nonzero of $s(i)$ equals the number of targets K. Since the number of bistatic range cells always far greater than the number of targets, i.e., $K \ll Nrange$, the scattering coefficient vector $s(i)$ can be expressed in a space orthogonal basis $\Psi \in \mathbf{R}^{Nrange\times Nrange}$ as

$$s(i) = \mathrm{Y} a(i) \tag{7}$$

where $a(i)$ is defined as the vector whose nonzero components are corresponding to the complex amplitudes of K point targets. Thus, $s(i)$ is K-sparse in Y. Further, we can parameterize Eq. 6 in terms of the scattering coefficient vector $s(i) \in \square^{Nrange \times 1}$ and $\mathrm{F}(l) \in \square^{Nrange \times 1}$ as follow

$$S(i,l) = \mathrm{F}(l)^T s(i),\ l = 1, \mathbf{L}, Nrange,\ i = 1, \mathbf{L}, N \tag{8}$$

where $\mathrm{F}(l) = \{sinc(r(l) - r(m))\exp(-jkr(m)), m = 1, \mathbf{K}, Nrange\}$ is interpreted as an $Nrange$-element measurement vector at the bistatic range cell l. We rearrange the echo signal Eq. 8 to a vector as,

$$\boldsymbol{S}(i) = \{S(i,l); l = 1, \mathbf{K}, Nrange\} \tag{9}$$

where $\boldsymbol{S}(i)$ is an $Nrange$-element vector. When we take an additive noise $\boldsymbol{v}(i)$ (e.g., assuming a white Gaussian noise with zero-mean and variance $\mathrm{S}(i)$), the relationship between $s(i)$ and the observed signal vector $\boldsymbol{S}(i)$ can be compactly expressed as a linear model,

$$\boldsymbol{S}(i) = \mathbf{A} s(i) + \boldsymbol{v}(i) \tag{10}$$

where $\mathbf{A} \in \square^{Nrange \times Nrange}$ denotes the measurement matrix of the observed signal at the i th receiver.

From Eq. 10, we can see that estimating or recovering the scattering coefficient vector $s(i)$ from a linear equation with the given measurement matrix $\mathbf{A}$ and the measured signal $\boldsymbol{S}(i)$ will eliminate the side-lobes. Since the scattering coefficient vector $s(i)$ is sparse according to Eq. 7 and Eq. 10 satisfies CS theory [9], the sparse recovery techniques, such as the OMP, BP and regularization technique, can be applied to recover $s(i)$ and eliminate the side-lobes. Among them, the OMP algorithm is the most effective one, which is suitable for multi-static radar multi-target localization application.

Recovery via Orthogonal Matching Pursuit

OMP is an iterative greedy algorithm that selects at each step the column of $\mathbf{A}$ which is most correlated with the current residuals. This column is then added into the set of selected columns. The algorithm updates the residuals by projecting the observation $\boldsymbol{S}(i)$ onto the linear subspace spanned by the columns that have already been selected and the algorithm then iterates. The pseudo-code of the OMP-based multi-static radar multi-target projection localization is summarized in Algorithm 1.

Algorithm 1: OMP for multi-static radar multi-target projection localization

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Parameter: Given the measurement matrix $\mathbf{A}$, the measurements $\boldsymbol{S}(i)$, and the error threshold e_0.

Initialization: Initialize $k = 0$, $s^{(0)}(i) = 0$, $\boldsymbol{r}^{(0)}(i) = \boldsymbol{S}(i)$, $\mathrm{W}^{(0)}(i) = f$.

Main Iteration: Increment k by 1 and perform the following steps:

1). $j^{(k)}(i) = \arg\max_j \left\| \mathrm{A}_j^T \boldsymbol{r}^{(k-1)}(i) \right\|_2$.

2). $\mathrm{W}^{(k)}(i) = \mathrm{W}^{(k-1)}(i) \cup j^{(k)}(i)$.

3). $s^{(k)}(i)\big|_{\Omega^{(k)}} = \mathbf{A}_{\Omega^{(k)}}^H \boldsymbol{S}(i)$, $s^{(k)}(i)\big|_{\bar{\Omega}^{(k)}} = 0$.

4). $\boldsymbol{r}^{(k)}(i) = \boldsymbol{S}(i) - \mathbf{A} s^{(k)}(i)$.

5). Repeat steps 1-4, until all sensors are processed.

6). $\bar{\boldsymbol{r}}^{(k)} = \sum_{i=1}^{N} \boldsymbol{r}^{(k)}(i) \Big/ N$.

7). Stopping rule: If $\left\| \bar{\boldsymbol{r}}^{(k)} \right\|_2 < e_0$, stop. Otherwise, apply another iteration.

Outputs: k - sparse approximation $s(i) \leftarrow s^{(k)}(i)$

Once the reconstructed scattering coefficient vector $s(i)$ is obtained, multi-target positioning via projection method calculates the existence probability for each of $s(i)$, projects the existence probability which is larger than the threshold to the BR image space via BR space projection, and extracts and locates the targets via PGC algorithm in BR image space.

Performance Analysis and Discussion

Assume that the transmitter is placed at the origin, and there are 20 receivers, three of them are located at [-25, 0, 0]km, [25, 0, 0]km and [0, 43, 0]km, and the others distribute uniformly in the triangle determined by the three receivers. Five targets are distributed uniformly in a 3-D cube with size 400×400×400 m^3 and centered at [50, 50, 10] km. The RCSes of the targets vary from 1 to 30 at random, some Gauss noises with standard deviation 0.2 are added in the echoes. The system's range resolution is 10m.

The original signal and the recovery signal in the bistatic range domain of the 5th sensor and the 10th sensor are plotted in Fig. 1 and Fig. 2, respectively. The subfigures in Fig. 1 and Fig. 2 are the zoom-in figure of the bistatic range cells which are occupied by the targets. From them, we can first find that the targets occupied bistaice range cells are different of each receiver due to the different geographic position of the receivers. Next, we observe the main-lobe broadening and the side-lobes crosstalk of the original signal as discussed in section II. Due to this reason, there are a dozen of locations that might be considered as targets for the original signal. On the contrary, there are only five spikes of the recovery signals which correspond to the five targets. Obviously, the side-lobes are eliminated and range resolution is improved by spares recovery technique. Fig. 1, Fig. 2 and other simulation results (not shown here for brevity) confirm that the OMP algorithm for multi-static radar multi-target projection localization algorithm eliminates the side-lobes and improves the range resolution. Fig. 3 plots the location result of the OMP-based multi-static radar multi-target projection localization that matches the actual locations soundly.

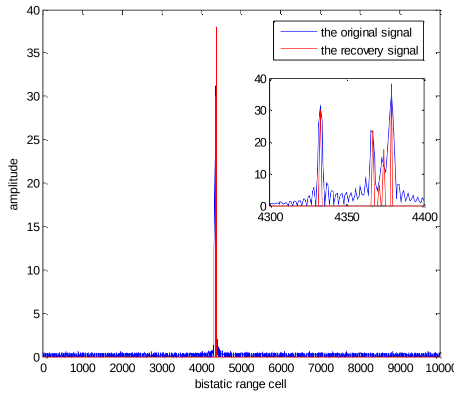

Fig. 1. The original signal and the recovery signal of 5th sensor.

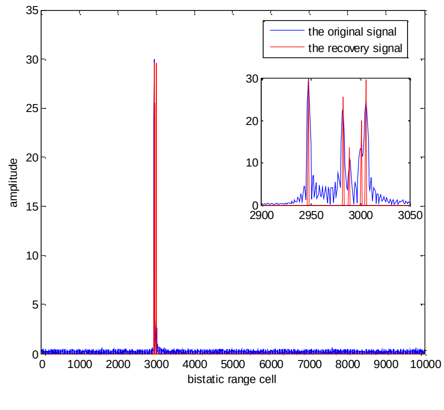

Fig. 2 The original signal and the recovery signal of 10th sensor

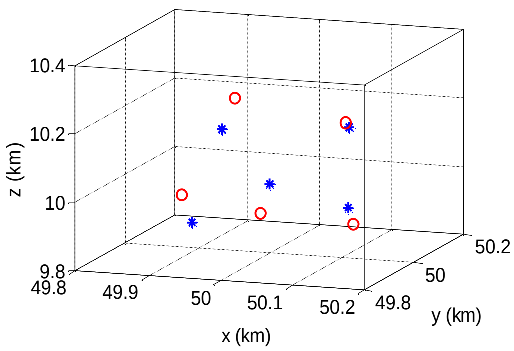

Fig. 3. Location result of OMP-based multi-static radar multi-target projection localization

Conclusions

In this paper, we present a multi-static radar multi-target projection localization based on sparse reconstruction to eliminate the main-lobes broadening and the side-lobes crosstalk and increase the range resolution. Since the echoes in bistatic range domain are sparse, the linear model for echo signal is created and OMP algorithm is used to reconstruction echo signal. Simulated results show that the main-lobes broadening and the side-lobes crosstalk are eliminated and the range resolution is improved by the presented method. However, there are still some challenges need to be overcome, for example the error threshold e_0 in the OMP algorithm should be further discussed.

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References

[1] S. Blachman: Multiple hypothesis tracking for multiple target tracking. IEEE Trans. Aerosp. Electron. Syst., vol. 19 (2004), p. 5-17.

[2] Y. Bar-Shalom, T. Kirubarajan, and X. Lin: Probabilistic data association techniques for target tracking with applications to sonar, radar and EO sensors. IEEE Trans. Aerosp. Electron. Syst., vol. 20 (2005), p. 37-55.

[3] J. Shi, W.C. Zhao, and X. L. Zhang: Space-invariant projection method for multi-target positioning. Proceedings of the TENCON 2013-2013 IEEE Region 10 Conference (2013), Xi' an, p. 22-25.

[4] J. Shi, L. Fan, X. L. Zhang, and T. Y. Shi: Multi-target positioning for passive sensor network via bistatic range space projection. Sci. China Ser F- Inf. Sci., vol. 59 (2016), p. 019302:1-019302:3.

[5] Y. C. Pati, R. Rezaiifar, and P. S. Krishnaprasad: Orthogonal matching pursuit: Recursive function approximation with applications to wavelet decompositions. Proc. 27th Asilomar Conference on Signals, Systems and Computers, A. Singh, ed., 1993.

[6] D. L. Donoho, M. Elad, and V. Temlyakov: Stable Recovery of Sparse Overcomplete Representations in the Presence of Noise. IEEE Trans. Inform. Theory, vol. 52 (2006), p. 6-18.

[7] Sadegh Jokar and M. E. Pfetsch: Exact and Approximate Sparse Solutions of Underdetermined Linear Equations. SIAM J. SCI. Comput., vol. 31 (2008), p. 23-44.

[8] A. M. Bruckstein, D. L. Donoho, and M. Elad: From Sparse Solutions of Systems of Equations to Sparse Modeling of Signals and Images. SIAM Review, vol. 51 (2009), p. 34-81.

[9] R. Baraniuk: Compressive sensing. IEEE Signal Processing, vol. 24 (2007), p. 118–121.