Interfacial Slip Calculation of RC Beams Strengthened with FRP Plate under Symmetry Concentrated Loading

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Abstract. Debonding of fiber reinforced polymer (FRP) is a main premature failure of RC beams strengthened with FRP plate, which is mainly due to the interfacial slip between FRP and concrete beam. The interfacial slip is one of the key factors to affect the behaviors of RC beams strengthened with FRP plate. Under external loading, the relative slip occurring on the interface between FRP plate and concrete beam causes the redistribution of stress and then the bearing capacity of the member is reduced. According to elasticity theory, this study analyzes the theoretical analysis of interfacial slip of RC beams strengthened with FRP plate under symmetry concentrated loading. The analytical model is established, and the differential equation of interfacial slip of RC beams strengthened with FRP plate is established. Using the boundary conditions, the calculated formulas of interfacial slip are deduced under symmetry concentrated loading.

Introduction

In civil engineering, FRP has mainly been used to repair and to upgrade reinforced concrete structure decade ago, and is being increasingly used as an alternative to steel for strengthening RC beams. FRP plates can be bonded to the surface of RC beam through a thin adhesive layer. In this strengthening method, the performance of the FRP-concrete interface in providing an effective stress transfer is of crucial importance. At present, a large number of investigations have focused on finding possible method which used FRP sheets or plates to strengthen structures and research on its debonding failure modes and mechanical behavior[1-6]. But the research about the interfacial slip of the FRP-concrete interface is relatively less. Under external loading, the relative slip occurred on the interface between FRP plate and concrete causes the redistribution of stress and then influences mechanical behavior of component. Therefore, this paper studies on the slip of FRP-concrete interface. Based on the analytical model, the differential equation of interfacial slip is established. And the calculated formulas of interfacial slip are deduced under symmetry concentrated loading.

Analytical Model

Governing Differential Equation

A differential section (dx) is cut out from the RC beams strengthened with FRP plate. The composite beam is made of three materials: RC concrete, adhesive layer and FRP plate. In the present analysis linear elastic behavior is assumed for all the materials. The adhesive is considered only in transferring the stresses from the concrete to the FRP reinforcement and the stresses in its layer do not change through the direction of the thickness.

In this study, the following assumptions are made: 1. The beam, adhesive, and FRP plate are having elastically and linearly. 2. Stresses in the adhesive layer do not change with the thickness. 3. Deformations of FRP plate and concrete beam are due to bending moments, axial and shear forces. 4. The curvatures of FRP plate and concrete beam are assumed to be the same. 5. The axial force, bending moment and shear force of adhesive layer are ignored. So the adhesive layer is exposed only to the interfacial shear and normal stresses. Adhesive layer is mainly transmitted shear force and shear deformation.

The interfacial shear stress is denoted t. The constitutive equations for the interface can be expressed as follow.

$$t = f(d) \tag{1}$$

In Eq. 1, the interfacial slip d is defined as the relative displacement between the FRP plate and concrete beam, and then d can be written as below.

$$d = u_f(x) - u_c(x) \tag{2}$$

Where $u_c(x)$ and $u_f(x)$ are the longitudinal displacements at the base of concrete beam and the top of FRP plate respectively.

After differentiating, the Eq. 2 can be expressed in terms of mechanical strain of concrete beam and FRP plate as follow.

$$\frac{dd}{dx} = e_f(x) - e_c(x) \tag{3}$$

Where $e_c(x)$, $e_f(x)$ are the strain at the bottom of concrete beam and the top of FRP plate respectively.

The strain formulas of concrete beam and FRP plate are given as follow.

$$e_c(x) = \frac{M_c(x)y_1}{E_c I_c} - \frac{N_c(x)}{E_c A_c} \tag{4}$$

$$e_f(x) = -\frac{M_f(x)y_2}{E_f I_f} + \frac{N_f(x)}{E_f A_f} \tag{5}$$

Where E_c, E_f, I_c, I_f are elastic modulus, inertia moments of RC beam and FRP plate respectively; A_c, A_f are cross-sectional areas of RC beam and FRP plate respectively; $M_c(x), M_f(x), N_c(x), N_f(x)$, $V_c(x), V_f(x)$ are the bending moment, axial and shear forces in each adherend while y_1 and y_2 are the distances from the bottom of RC beam and the top of FRP plate to their respective centroid.

The following equilibrium equations are established as below.

$$\frac{dM_f(x)}{dx} = V_f - t b_f y_2 \tag{6}$$

$$\frac{dM_c(x)}{dx} = V_c - t b_f y_1 \tag{7}$$

$$\frac{dN_c(x)}{dx} = \frac{dN_f(x)}{dx} = t b_f \tag{8}$$

Substituting Eqs. 4-8 into Eq.3, and get derivative, then the differential equation refers with Eq.9.

$$\frac{d^2 d}{dx^2} - a^2 f(d) + bV(x) = 0 \tag{9}$$

Where $a^2 = b_f \left[\dfrac{1}{E_f A_f} + \dfrac{1}{E_c A_c} + \dfrac{(y_1 + y_2)^2}{EI} \right]$, $b = \dfrac{y_1 + y_2}{EI}$, $EI = E_f I_f + E_c I_c$ and $V(x) = V_f + V_c$.

Interfacial bond-slip model

Various bond-slip models have been considered in previous work. The model of stress-slip relationship is considered to be possible in representing the nonlinear interfacial behavior.

The mathematical expression is as follow.

$$f(d) = \begin{cases} \dfrac{t_f^2}{2G_f} d ....for...0 \le d \le d_f \\ 0 ....for...d > d_f \end{cases} \tag{10}$$

Where , t_f is the local bond strength, and d_f is the maximum slip, and G_f is the interfacial fracture energy. Substituting Eq.10 into Eq.9, when $0 \le d \le d_f$, the differential equation of interfacial slip refers with Eq.11.

$$\frac{d^2 d}{dx^2} - w^2 f(d) + bV(x) = 0 \tag{11}$$

Where $w^2 = \dfrac{b_f t_f^2}{2G_f} \left[\dfrac{1}{E_f A_f} + \dfrac{1}{E_c A_c} + \dfrac{(y_1 + y_2)^2}{EI} \right]$.

Calculation formulas

The differential equation of interfacial slip has different solutions under different loads. Under the effect of symmetry concentrated loading, the calculation formulas of interfacial slip are established.

Under symmetry concentrated loading P (beam length is L, length of FRP plate is l_f), the beam can be divided into bending-shear part and pure bending part, and the general solutions of differential equation can be established respectively. The solutions to differential equation of interfacial slip refer with Eq.12-13.

On bending-shear part, the general solution of differential equation refers with Eq.12.

$$d_1 = A_1 e^{wx} + A_2 e^{-wx} - \frac{bP}{w^2} \tag{12}$$

On pure bending part, the general solution of differential equation refers with Eq.13.

$$d_2 = A_3 e^{wx} + A_4 e^{-wx} \tag{13}$$

Where A_1, and A_2, and A_3 , and A_4 are constant.

The solutions to differential equations of interfacial slip can be obtained by using the following boundary conditions. The boundary conditions are respectively:

1. when $x = 0$, there is $d_2 = 0$; 2.when $x = 0.5l_0$, there are $d_1 = d_2$ and $\dfrac{dd_1}{dx} = \dfrac{dd_2}{dx}$; 3.

when $x = 0.5l_f$, there is $\dfrac{dd_1}{dx} = -\dfrac{P(L - l_f)(y_1 + y_2)}{2EI} = -d_a$.

The formulas of the constants respectively refer with Eq.14, Eq.15 and Eq.16 and Eq.17.

$$A_1 = \frac{bP(e^{-\frac{wl_0}{2}} + e^{\frac{wl_0}{2}}) - 2wd_a e^{\frac{w}{2}l_f}}{2w^2(1 + e^{wl_f})} \tag{14}$$

$$A_2 = \frac{bP(e^{-\frac{wl_0}{2}} + e^{\frac{wl_0}{2}})e^{wl_f} + 2wd_a e^{\frac{w}{2}l_f}}{2w^2(1 + e^{wl_f})} \tag{15}$$

$$A_3 = \frac{bP(e^{\frac{wl_0}{2}} - e^{wl_f - \frac{wl_0}{2}}) - 2wd_a e^{\frac{w}{2}l_f}}{2w^2(1 + e^{wl_f})} \tag{16}$$

$$A_4 = -\frac{bP(e^{\frac{wl_0}{2}} - e^{wl_f - \frac{wl_0}{2}}) - 2wd_a e^{\frac{w}{2}l_f}}{2w^2(1 + e^{wl_f})} \tag{17}$$

Conclusions

Based on the analytical model, the differential equation of interfacial slip of RC beams strengthened with FRP plate is established. And the calculated formulas of interfacial slip are deduced under symmetry concentrated loading. The calculation formulas of interfacial slip are rational in theory, but the applicability of the calculation formulas needs more experiments to verify.

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