

Study on Bath Water in Hot Bath

Xin Gong

Department of Electrical Engineering, North China Electric Power University (Baoding), Baoding
071000, China

1608450092@qq.com

Keywords: Hot bath, energy conservation equation, water flow rate, motions of people

Abstract: This paper determines the best strategy for people who have a hot bath by analyzing the temporal and spatial variation of the bathtub water. In order to achieve the purpose of saving water, the key research object of the model is the water flow rate from the faucet. Finally, according to the actual situation, the influence of human motion in hot water on the water flow rate is discussed.

First of all, in order to maintain the water temperature, the value of the water flow rate is assumed to be constant. Then an energy conservation equation of the bathtub is established according to the energy conservation law. After estimating some parameters, a suitable water flow rate is obtained.

In order to make the model more applicable, this paper uses the control variable method, introduces two parameters, and focuses on the dependence of the model on the motions of people. It can be concluded from the three-dimensional image that this factor has a great influence on the result of the water flow rate.

1. Introduction

A bathtub is a large container for holding water in which a person may bathe. Most modern bathtubs are made of acrylic or fiberglass, but alternatives are available in enamel on steel or cast iron.^[1]

In the real life, the average bathtub volume or the capacity to hold water when filled is determined with so many factors especially the type of tub you choose. According to Alliance for Water Efficiency, a standard non-jetted bathtub holds between 25 and 45 gallons of water approximately. The average rectangular bathtub is about 30 inches wide and 5 feet long, according to Plumbing-Basics. They come in many different shapes, however, such as circular or rounded models, and this may affect capacity.^[2]

2. Determine the flow rate of hot water

Firstly, the volume and shape of the bathtub and the shape, volume, temperature and motions of people is not considered. From a general perspective, this paper puts forward a controllable strategy for the person in the bathtub. For purpose of giving a suitable strategy for the bathing person, the core of the study is to determine the flow rate of the hot water.

In order to make the person in the bathtub have enough relaxation and do not have to adjust the flow rate of the hot water, the value of the water flow rate is assumed to be a constant. Then an equation of the water in the bathtub is established in the basis of the theory of energy conservation. After identifying the parameters according to the common bathtub in the market, a relationship between the water flow rate and the thermal conductivity coefficients of the bathtub is obtained. Finally, a suitable water flow rate is calculated.

2.1 Establish the equation of energy conservation

The water in the bathtub is considered as the object of study.

Then the heat absorbed and released by water in the bathtub when a person is bathing is summarized. Among them, the absorption of heat (Q_{in}) is relatively single, mainly from the newly added hot water. It is:

$$Q_{in} = c_w m (T_{in} - T_{out}) \quad (1)$$

Where m means the mass of the inflowing water, T_{in} means the temperature of inflow water and T_{out} means the temperature of outflow water. The releasing heat is constituted by a variety of factors, and it contains the heat released into the air, a function has been obtained as below:

$$Q_1 = h_{air} (T - T_0) S_1 \tau \quad (2)$$

In the formula, h_{air} means the convective heat transfer coefficient of air, T means the maintained temperature of the water in the bathtub, T_0 means the room temperature, S_1 means the contact area of water in the bathtub and air, and τ means time of inflow water. In addition, Q_{out} contains the heat released into the air through the bathtub wall (Q_2). Then a formula has been obtained:

$$Q_2 = k (T - T_0) S_2 \tau \quad (3)$$

Where S_2 means the contact area of water in the bathtub and the bathtub wall, k means the total heat transfer coefficient between water and air through the bathtub wall. And^[3]

$$k = \frac{1}{\frac{1}{h_w} + \frac{\delta}{\lambda} + \frac{1}{h_{air}}} \quad (4)$$

In the formula, h_w means convective heat transfer coefficient of water in the bathtub, δ means the thickness of the bathtub and λ means the thermal conductivity coefficient of the bathtub wall. At last, the heat transfer Q_3 between people and the water in the bathtub are considered, and we obtain that:

$$Q_3 = h_w (T - T_p) S_p \tau \quad (5)$$

Where T_p means the temperature of people, S_p is the contact area of between people and the water in the bathtub. Thus the releasing heat consists of the 3 parts above:

$$Q_{out} = Q_1 + Q_2 + Q_3 \quad (6)$$

The goal is to keep the temperature of the water in the bathtub as close as the initial temperature, so the heat exchange equilibrium condition is as below:

$$Q_{in} = Q_{out} \quad (7)$$

Then, the total requirements are integrated and a formula is obtained:

$$c_w m (T_{in} - T_{out}) = h_{air} (T - T_0) S_1 \tau + k (T - T_0) S_2 \tau + h_w (T - T_p) S_p \tau \quad (8)$$

The suitable flow rate of the water can be calculated, and the paper derivatives the variable τ at the two sides of the equation. The parameters at the right side of the equation in addition of τ can be considered as constants which have nothing to do with time. The temperature of outflow water is equal to the temperature of the water in the bathtub, which means $T = T_{out}$. Finally an equation is obtained:

$$\dot{m} = \frac{1}{c_w (T_{in} - T_{out})} [h_{air} (T - T_0) S_1 + k (T - T_0) S_2 + h_w (T - T_p) S_p] \quad (9)$$

Among them, $\dot{m} = \frac{dm}{dt}$ means the flow rate of the water, in other words, it means the discharge

of water for per second. As a result, \dot{m} is the variable which we need to calculate.

2.2 Estimate the value of each parameter

Assume that the researching bathtub is of 1.7m long, 0.8m wide and 0.7m high. From the data obtained, the contact area of the water in the bathtub and air is $S_1 = 1.36\text{m}^2$, and the area of the inner surface of the bathtub wall is $S_2 = 4.86\text{m}^2$. The contact area of the water in the bathtub and person is

$S_p = 1.49\text{m}^2$, and the thickness of the bathtub is $\delta = 0.05\text{m}$. Combined with the common requirements for bathing, the temperature of the hot water flowing into the bathtub is assumed as $T_{in} = 50^\circ\text{C}$. The initial temperature of the water is $T = 40^\circ\text{C}$, which means the temperature of outflow water is $T_{out} = 40^\circ\text{C}$. Based on common sense, the temperature of each person is $T_p = 37^\circ\text{C}$ and the temperature of the room is $T_0 = 25^\circ\text{C}$. Finally, the values of convection heat transfer coefficients are assigned that $h_{air} = 6\text{ W}/(\text{m}^2 \cdot \text{K})$ and $h_w = 3000\text{ W}/(\text{m} \cdot \text{K})$.^[4]

2.3 Find the relation between flow rate and thermal conductivity

The thermal conductive coefficient of the bathtub wall (λ) is considered as the independent variable, and the flow rate \dot{m} as the dependent variable. The estimated parameters are put into the Eq.9 and then a functional relation is obtained:

$$\dot{m} = 0.3222 + \frac{0.104\lambda}{3 + 10\lambda} \quad (10)$$

It can be seen that \dot{m} is an increasing function of λ . A figure of the function is that:

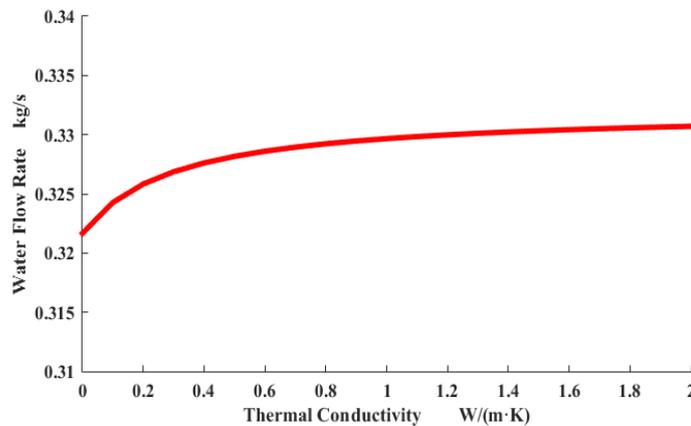


Figure 1: The relation between water flow rate and thermal conductivity

According to the figure, some results can be gotten as below:

As for a certain bathtub, there is a relationship between the flow rate and thermal conductivity, and the flow rate stays around $0.328\text{ kg} / \text{s}$.

Since the various parameters selected are generally the value of the case, the conclusion is general and suits for most people when bathing. However, the flow rates for different people are fluctuating at the center of the determined value.

3. The dependence on the motions of people

The motions of people are a factor which affects the water temperature. If people have movements when bathing, there will be the following effects:

- The convection heat transfer coefficient of the water in the bathtub (h_w) will change.
- The speed of heat dissipation from the contact of water and air is quicker.
- The total thermal resistances of water and air through the bathtub wall (k) will change.
- The contact area of water and air (S_1) will change.

In this case, k is a function of h_w and h_{air} . S_1 also changes a lot, and it can make h_{air} add a lot of times. Now the energy conservation equation has become a multivariate function of h_w and h_{air} . It is not easy to quantify the motions of the person, as a result, α and β are introduced to describe the motions of the person. α means the times of the motions of people to make a multiplier of h_{air} (due to the contact area of water and air will greatly increase the adding times of h_{air} , so the adding times

as 60 which means the value of α changes from 1 to 60 is considered.) β means the adding times that the motions of people make to the h_w . Finally, the functional relation of \dot{m} with α and β is obtained:

$$\dot{m} = 0.0029\alpha + 0.3187\beta + 0.0065 \quad (11)$$

Then a three-dimensional image of the relationship between the three parameters is made as below:

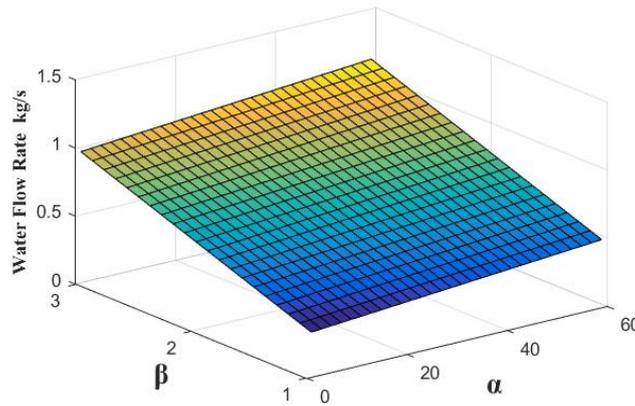


Figure 2: The relationship between \dot{m} , α and β

From the three-dimensional image we can see that minor changes in the parameter α do not result in large changes of \dot{m} . However, due to the impact of S_1 and the flow of the air which make large changes in α , great changes have taken place in the water flow rate. At the same time, small changes of β can lead to a great change in water flow rate, which is the direction of the ground to take a larger slope. As a result, a pertinent strategy can be given to the bathing person: do not always shake the body, so that the heat of water will not release to the body and air too much. If the motions are too severe, the amount of water needed is large.

4. Conclusions

The purpose of all of the work is to find a good strategy for those who have a bath. The strategy requires to keep temperature of the bathtub water constant and not to waste too much water. So the strategy is displayed:

The subject investigated of our model is water flow rate from the faucet. The flow rate of average person is 0.328 kg/s, and it remains unchanged in the whole process of bathing. The value has certain universality and it is applicable for most people. Considering different person and different bathtub, the equation (10) is still applicable. Only some of the parameters may vary.

If different people have a hot bath, water flow rate may be greatly different. It is related to the motions of the person greatly. The suggestion is that, if a person wants to conserve water in a bath, try to reduce the large motions in the water. And do not have severe motions when cleaning the body because it will cause great dissipation of the water heat.

References

- [1] Hirt C W, Nichols B D. Volume of fluid method for the dynamics of free boundary [J]. Journal of Computational Physics, 1981,39(1):201-225
- [2] Naiping Gao, Jianlei Niu. CFD study on micro-environment around human body and personalized ventilation [J]. Building and Environment, 2004,39(7):795-805
- [3] S-rensen D N, Voigt L K . Modeling flow and heat transfer around a seated human body by computational fluid dynamics [J]. Building and Environment,2003,38(6):753-762

[4] Haitao Hu, Zhizhong Li, Hui Xiao. Study on the convection heat transfer coefficients [J]. Heat Transmission Science, 2003, 12(1): 39-42