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The Model of Water Temperature Change in the Bathtub

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Abstract: This paper describes model testing of bathtub with the purpose of remaining the water temperature in the bathtub constant and reducing water waste. I establish a model and draw a conclusion. The model describes changes in the temperature of the bathtub water.

The model has two parts: the one is endothermic process that a constant trickle of hot water reheats the bathing water and the other one is exothermic process. The exothermic process is divided into three parts: on the water surface, on the bathtub wall and between bathtub bottom and ground. Finally, I get a formula about the temperature of the water according to the theory of thermology. I find that it is effective for bubble bath additive to help to keep the temperature constant in the extended model

I carry out numerical calculation by using Runge-Kutta method and compare analytic and numerical results with reality, using default parameters; I validate that my method is correct and robust.

Introduction

As shown in Fig. 1, the exothermic process is divided into three parts, and this process on the water surface is also divided into three parts: evaporation, thermal radiation and heat convection. [1] And the other two positions only contain heat conduction.

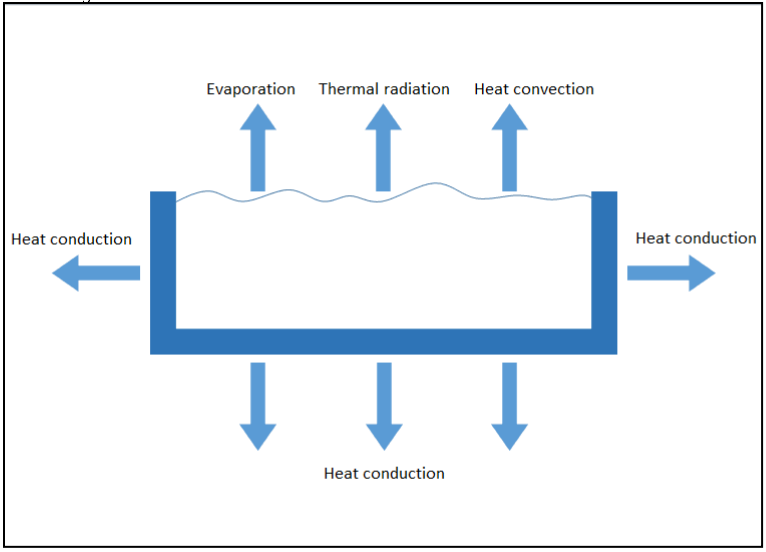

Fig. 1: Exothermic Process

Modern medicine indicates that the best temperature for bathing is between 37 and 40. So I suppose the initial temperature is 37 and the temperature of the bathtub water remains in this range.

The heat process is divided into two parts: the one is endothermic process that the constant trickle of hot water reheats the bathing water and the other one is exothermic process.

First, I calculate heat absorption of the endothermic process by calculating the heat contained in the hot water. Then, I calculate heat dissipation through the exothermic process. Next, I calculate the change of the temperature of the bathtub water by calculating the difference between the heat absorption and the heat dissipation. Then, I try and reduce them to formula.

Finally, I differentiate the formula and get the model of the temperature of the bathtub water in time.

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2. The model

Table 1：Parameter

Parameter	Meaning	Unit
F	Contact area between water and air	m^2
S_s	Area of the wall of the bathtub	m^2
S_u	Area of the bathtub bottom	m^2
T	The water temperature	K
M_0	Bathtub water's mass	kg
m	The mass flow rate of the constant trickle of hot water	Kg/s
θ	Dry bulb temperature of the air	K
β	Evaporation coefficient	$W/(m^2 \cdot hPa)$
α	Convective heat transfer coefficient	$W/(m^2 \cdot K)$;
W	Air speed above the water's surface	m/s
h	Thermal conductivity of the bathtub	$W/(m^2 \cdot K)$
$p_v^{''}$	Steam pressure of thin saturated layer of water surface	hPa
p_v	partial pressure of the wet air	hPa
k	thermal accommodation coefficient	$W/(m^2 \cdot K)$
T_0	Constant current hot water temperature	K
C	The specific heat of the hot water	$J/(kg \cdot K)$
Q_a	The heat transferred from water surface to air per second due to heat convection.	W
Q_b	The heat transferred from water surface to air per second due to evaporation.	W
Q_c	The heat transferred from water surface to air per second due to thermal radiation.	W
Q_d	The heat transferred from the bathtub wall to air per second due to heat conduction.	W
Q_e	The heat transferred from the bathtub bottom to ground per second due to heat conduction.	W
Q_f	The heat transferred from the constant trickle of hot water from the faucet to the bathtub water per second	W
Q	The heat of the bathtub water per second	W
$\varepsilon\sigma$	Radiance	

Step1.Endothermic Process**:**I need to supplement the loss of the water's temperature by adding water from the water tap. So I get a formula like this:

$$Q_f=c(T_0-T)m, \tag{1}$$

Step2.Exothermic Process**:** As mentioned above, the exothermic process is divided into three parts, and this process on the water surface is also divided into three parts: evaporation, thermal radiation and heat convection.

The heat that the bathtub water loses by heat convection can be represented as:

$$Q_a=\alpha(T-\theta)F \tag{2}$$

The heat that the bathtub water loses by evaporation can be represented as:

$$Q_b=\beta(p_v^{''}-p_v)F \tag{3}$$

According to literature[1] The evaporation coefficient ("β") can be represented as:

$$\beta=[22.0+12.5W^2+2.0(T-\theta)]^{2} \tag{4}$$

The heat that the bathtub water loses by thermal radiation can be represented as:

$$Q_c=\varepsilon\sigma T^4 F \tag{5}$$

The heat that the bathtub water loses by heat conduction on the bathtub wall can be represented as:

$$Q_d=h(T-\theta)S_s \tag{6}$$

The heat that the bathtub water loses by heat conduction between the bathtub bottom and ground can be represented as:

$$Q_e=k(T-\theta)S_u \tag{7}$$

Step3.Energy Analysis**:** For the bathtub water, its heat change process is related to the change of the temperature of it. So I get an expression like this:

$$Q=Q_a+Q_b+Q_c+Q_d+Q_e-Q_f.$$

If I want to keep the temperature constant, I should make Q=0.

Numerical Computation

The coefficient of convective heat transference between water and air is generally 5-200. Thus, I generally assume α= 100.

According to modern medical research, the most suitable indoor temperature of human habitation is 25, so I assume dry bulb temperature (θ) of the air is 25 that Kelvin temperature is 298K.

Because I assume that the bathtub is placed in a small, enclosed room, so I assume that the air speed (W) is 0.1.

According to the reference, I assume $\varepsilon\sigma$=0.97, p_v''=42hPa and p_v=33.6hPa.

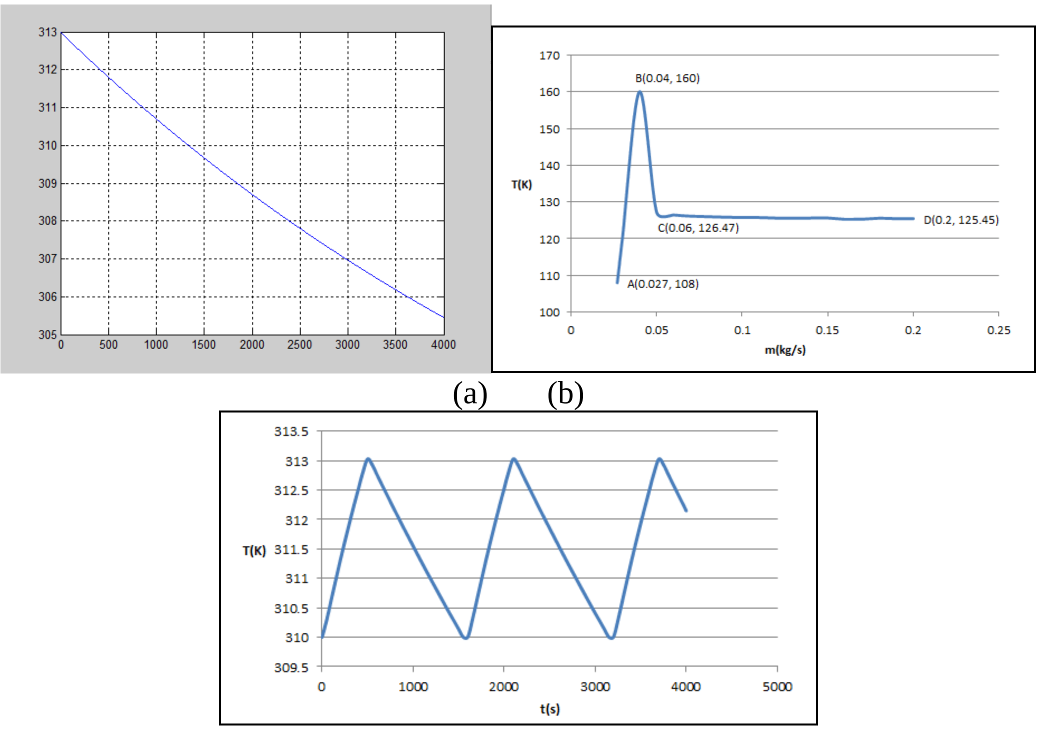
Because most of the bathtub material is acrylic, so I assume h=k=0.19.

I know that the size of the majority of bathtub is 1.7*0.8*0.7 by asking bathtub companies. I assume that m_0 (which represents the mass of the water in the bathtub after the person is lying in the bathtub) =230kg. And I conservatively assume that S_s=3.5m^2, F=1.5m^2 and S_u=1.36m^2.

As I assumed, T_0=333K.

Numerical calculation was carried out by using Runge-Kutta method, and Fig. 2 shows the output figure from numerical simulation.

As Fig. 2(a) shows, only considering the heat dissipation process, the relationship between the temperature and time is a smooth concave curve. And I see that the temperature decreases by 3K for about 1000 seconds.



(a) (b)

(c)

Fig.2: (a): The relationship between the temperature and time about the heat dissipation process.
(b): The relationship between m and M.
(c): The relationship between time and temperature if people add water intermittently.

As Fig. 2(b) shows, I can get the relationship between *m* and *M*. The curve between A and B describes the relationship between *m* and *M* when people add water constantly, and the curve between C and D describes the relationship between *m* and *M* when people add water intermittently. As Fig. 2(b) shows, *M* is proportional to*m* when people add water constantly, and if the temperature keeps at 309.5K, the total water injection is minimized. In addition, *M* is inversely proportional to*m* when people add water intermittently, and *m* should range from 0.06 kg/s to 0.2 kg/s to make water consumption minimal.

As Fig. 2(c) shows, if people add water intermittently, he should run the faucet periodically, and the time of the cycle is 1550s.

As Fig. 3 **s**hows, if the temperature of the constant hot water changes, the total water injection will also changes and the total water injection will reduce when the temperature of the constant hot water increases.

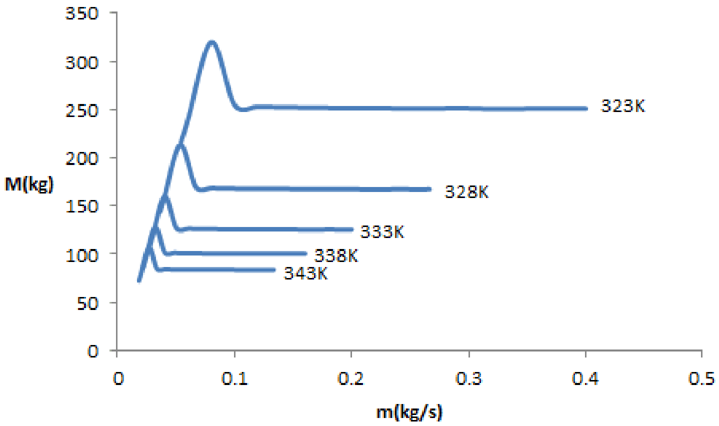


Fig. 3**:** The relationship between *m* and *M* under different temperature.

Conclusions

The total heat of water will be affected by many complex factors in reality. By and large, the water of the bathtub distributes heat in three ways:

The water distributes heat through the bath, but this process would be affected by the bath materials and the actual room temperature.

The water lose heat by evaporation,in fact, the amount of evaporation will change with the increase of air humidity in the bathroom.

Water has radiation effect, but air has a thermal radiation effect on water, too.

If people want to add the hot water constantly, the total water injection is minimized when the mass flow rate of the constant trickle of hot water is 0.027kg/s.

If people want to add water intermittently, the total water injection is minimized when the mass flow rate of the constant trickle of hot water ranges from 0.06kg/s to 0.2 kg/s. In addition, people should switch the faucet 1550 seconds.

According to my model, the more heat you add, the more water you can save. But in order to prevent hot water scald you, I recommend that you can use a higher temperature of water without burning your case.

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