Thermodynamics Temperature Field Model for Bathtub

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Abstract: Our basic model has two parts: to study effects of physical factors on the temperature and water consumption of water in bathtub, and to explore the influence of human action on water temperature distribution. Finally, we give the best strategy to maintain initial water temperature without wasting too much water when people take a bath. We find there is a certain relationship between water consumption and water temperature, bath size and human action.

First, we conduct a heat dissipation model by obtaining and solving a differential equation based on energy conservation to find the relationship between water temperature and water heat loss. Then, we analyze the influence on heat loss due to the shape, volume of bathtub and shape, volume and temperature of people. Shortly afterwards, we conduct an optimization model and get the optimal volume of tub as well as the velocity of tap water.

1. Introduction

With the development of society, more and more people begin to seek a more comfortable life. We have developed a number of new technologies in order to obtain a better quality of life. However, it is undeniable that this also resulted in accelerated depletion of natural resources. Our main task is to establish a bathtub model, so that users can continue to enjoy its appropriate temperature without wasting too much water. After careful analysis, the problem can be described as the following:

- Determine the extent to which the strategy depends upon some factors;
- Describe specific strategies when bathing obtained by analyzing the model;

Our main goal is to build a water temperature model by considering various factors, and get to the user bathtub usage policies. We solve problem step by step.

2. The Model

2.1 Model Ⅰ

We understand the optimal temperature of the bath should be 308K to 313K [1]. So that we want to determine an optimum temperature which could not only to meet people's requirements but also to enable cooling bath slowly as possible.

According to the Energy Conservation, we can establish a differential equation [2] to study the vintage water temperature:

$$\begin{cases} cm\dfrac{dT}{dt} = -H_1S_1(T-T_2) - \dfrac{S_1\lambda(T-T_2)}{D} - \varepsilon\sigma_1S_1(T^4-T_2^{\ 4}) - H_2S_2(T-T_2) - \varepsilon\sigma_2S_2(T^4-T_2^{\ 4}) - W - Q \\ T_{(t=0)} = T_0 \end{cases}$$

2.2 Solutions to Model Ⅰ

We find solutions of the equation by simulation so that we could know the relationship between time and water temperature which shows in Figure 1.Aparrently, the minimum slope of the curve means the lowest point of pyrexia. We observe the curve while water temperature is between 308K to

313K and discover the curve has smallest slope when water temperature is 308K .So, we finally determine the symbol T1=308K.

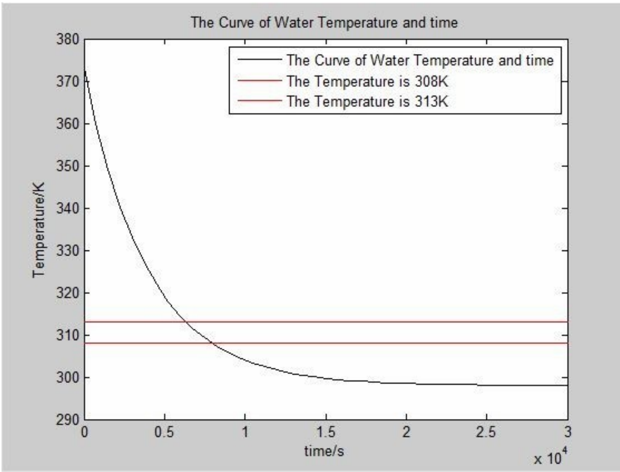

Figure 1: The curve of water temperature and time

2.3 Model Ⅱ

In this model, we try to find some factors' influence on heat dissipation. There is no doubt that the amount of water which flow into bathtub equal to the amount of water that outflow the bathtub. At the same time, we know that the heat is abiding because it's asked for keeping water temperatureas close as possible to the initial temperature. Thus, we develop a differential equation [3] as followings:

$$\begin{cases} cm\frac{dT}{dt} = f\rho c(T_1 - T) - H_1 S_1 (T - T_2) - \frac{S_1 \lambda (T - T_2)}{D} - \varepsilon\sigma_1 S_1 (T^4 - T_2^{\ 4}) - \\ H_2 S_2 (T - T_2) - \varepsilon\sigma_2 S_2 (T^4 - T_2^{\ 4}) - W - \mu N \\ T(t=0) = T_0 \end{cases}$$

Note: (The first term on the right represents the energy difference between tap water and the water that outflow bathtub.)

2.4 Solutions to Model Ⅱ

We simulate the differential equation by using MATLAB software. After solutions obtained equations, we further analysis some factors' influence on the water temperature T by taking derivative which shows in the Table 1:

Table 1 results

PARAMETERS	N	D	S_1	S_2	T_1
RESULTS of DERIVATION	$\frac{\partial T}{\partial N} < 0$	$\frac{\partial T}{\partial D} > 0$	$\frac{\partial T}{\partial S_1} < 0$	$\frac{\partial T}{\partial T_1} > 0$	$\frac{\partial T}{\partial S_2} < 0$

Through the contents of **Table 2,** we can draw the following conclusions:

- The more aggressive action, water temperature at a reduced speed the faster;
- The thicker the wall of bathtub ,bath water temperature, the more difficult it is to cooling;

- The larger the surface area of bathtub wall, the faster the water temperature is cooling;
- The larger the surface area of water, the faster the water temperature decreases;
- The water temperature in the faucet is higher, the much harder the water temperature lowers.

Obviously, the above conclusions are consistent with the actual situation. We can see through our studying and analyzing, that it is necessary for us to determine the proper shape and size of the bathtub to select an ideal bathtub with the least heat dissipation. According to the actual situation, we discussed two common types of bathtub---rectangular shape and cylinder shape .We calculate their heat dissipation by meanings of the differential equation[4]:

$$\begin{cases} cm\dfrac{dT}{dt} = -H_1S_1(T-T_2) - \dfrac{S_1\lambda(T-T_2)}{D} - \varepsilon\sigma_1S_1(T^4-T_2^{\,4}) - H_2S_2(T-T_2) - \varepsilon\sigma_2S_2(T^4-T_2^{\,4}) - W - Q \\ T(t=0)=T_0 \end{cases}$$

Differentiating both sides and solving the resulting fourth-order linear ordinary differential equation, we get value of heat dissipation individually. Ultimately, we know the cylindrical bathtub save more heat by comparing two values. So, the cylindrical bathtub is better to conserve water under the same conditions.

2.5 Model Ⅲ

Taking into account that we need to maintain the temperature of the water in the bathtub is close to the initial water temperature without wasting too much water, naturally we want to find out the relationship between the flow of water per unit time
and the volume of the bathtub.Therefore, we establish an optimization model to find
the way to save water. We have the objective function:

$$\min V = ft + V_1 - V_2 .$$

From this equation, we arrive an expression of the flow of tap water per unit time f which contain the volume of bath V1 . Following this, we use the expression to replace the f in objective function. On the other hand, the size of the common bath is about 280 to 340 liters. We get:

$$280 \le V_1 \le 340 .$$

At last, the relationship between the volume of the bath and the total water consumption is obtained. We demonstrate it in the Figure 2:

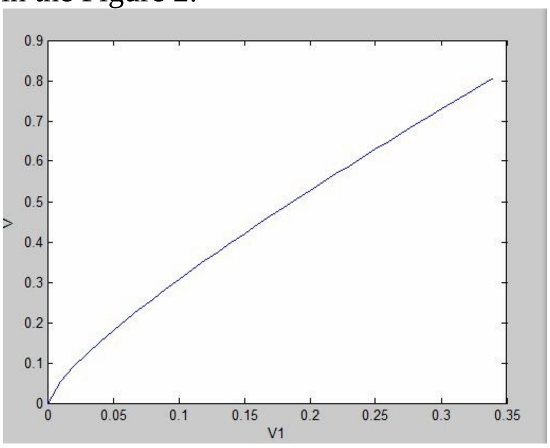

Figure 2. The curve of volume of the bath and the total water consumption.

By the image we can see the less the volume of bath is, the more water we can conserve. So we determine the volume of bath as 280L. Furthermore, we calculate the flow of tap water per unit time:

$$f=142.9g/s.$$

4. Conclusions

Based on the above models and the solutions to them, we get the best strategy of hot bath. Taking into account the size of the bathtub on the market today, as well as the conclusion that the smaller the surface area of the bathtub insulation better performance, we recommend users to choose a similar column of the bathtub. We recommend a volume of 280 liters of bath, bath water temperature remained at about 308 Kelvin, the water into the hot water temperature is about 343 Kelvin.

References

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