

# Model of Torque in Disc-Type Magnetorheological Brake Driven by Shape Memory Alloy

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**Abstract**—A magnetorheological brake driven by shape memory alloy is presented in this paper. The shape memory alloy spring is used to push the magnetorheological fluid into the working gap under thermal effect. Herschel-Bulkley model is used to describe the constitutive characteristics of magnetorheological fluids subject to an applied magnetic field. The braking torque developed by magnetorheological fluid is derived to evaluate the braking performance. The results indicate that, the braking torque changes rapidly according to strength of applied magnetic field.

**Keywords**—magnetorheological fluid; brake; shape memory alloy; braking torque

## I. INTRODUCTION

Shape memory alloys (SMAs) and magnetorheological (MR) fluids are new type of intelligent materials. MR fluids change their viscosity in response to change in strength of magnetic field when subjected to the same [1]. MR fluids have been applied in brakes [2,3], clutches [4,5], shock absorbers [6,7], valves [8], and so on. SMAs may undergo mechanical shape changes at relatively low temperatures and retain them until heated, then coming back to the initial shape [9]. This makes SMAs unique compared to other smart materials that can be used for actuator applications [10].

Based on the shape memory effect of SMA and the rheological behavior of MR fluid, combined with the methods of mechanical design, a disc-type MR brake driven by SMA is proposed. The braking torque is studied to provide the theoretical foundation for the design of new intelligent brake that have simple structure, reliable operation, fast response, in hopes of promoting the further development of research and applications of SMA and MR fluid in the brake system.

## II. OPERATIONAL PRINCIPLE

The schematic of the disc-type MR brake driven by the SMA under thermal effect is shown in Figure 1. The driving-disc turns with the angular velocity  $\omega$ . When the temperature is less than 50°C, there is a lack of the MR fluid in the working gap between two parallel discs. With the increase of the temperature, the MR fluid is pushed rapidly into the working gap because of the axial elongation of SMA spring. When the temperature is equal to 70°C, the MR fluid is filled with the working gap. The braking torque is produced by the shear stress of the MR fluid. In the application of magnetic field, the MR particles are gathered to form chain-like

structures, in the direction of the magnetic flux path. These chain-like structures increase the shear stress of the MR fluid. The braking torque can be adjusted continuously by changing the input current. When the temperature is less than 50°C, the SMA spring is shrunken back to its original state. The MR fluid is drained out of the working gap under the action of the compressed air and the centrifugal force.

## III. CHARACTERISTICS OF SMA AND MR FLUID

The output displacement  $S(T)$  of SMA spring at temperature  $T$  can be expressed as [11]

$$S(T) = \frac{(G(T) - G_L) \Delta \delta F_L \gamma_L}{(d/n\pi D^2) G(T) \Delta \delta F_L + (F_H - F_L) G_L \gamma_L} \quad (1)$$

where  $F_L$ ,  $\gamma_L$  and  $G_L$  are the axial load, shear strain and shear modulus of SMA spring at low temperature respectively,  $F_H$  is the axial load at high temperature,  $\Delta \delta$  represents the maximal displacement of the SMA spring.  $d$ ,  $D$  and  $n$  are the wire diameter, average diameter and the number of turns of SMA spring. For the application of this paper,  $F_H - F_L = \Delta P \cdot A$  ( $\Delta P$  is the pressure difference of compressed air at high and low temperatures,  $A$  is the sectional area of the piston).  $G(T)$  is the shear modulus of SMA spring at temperature  $T$ . When  $M_f \leq T \leq A_f$ , the expression of  $G(T)$  can be expressed as

$$G(T) = G_M + (G_A - G_M) / \{2[1 + \sin \phi(T - T_m)]\} \quad (2)$$

where  $G_M$  and  $G_A$  are the shear modulus of martensite and austenite, respectively. In the process of heating,  $T_m = (A_s + A_f)/2$ ,  $\phi = \pi/(A_f - A_s)$ . In the process of cooling,  $T_m = (M_s + M_f)/2$ ,  $\phi = \pi/(M_s - M_f)$ ,  $M_s$ ,  $M_f$ ,  $A_s$  and  $A_f$  are the start and finish transformation temperatures of martensite and austenite, respectively. The actual volume  $V_a$  which the SMA spring pushes the MR fluid into working gap is

$$V_a = S(T) \cdot A \quad (3)$$

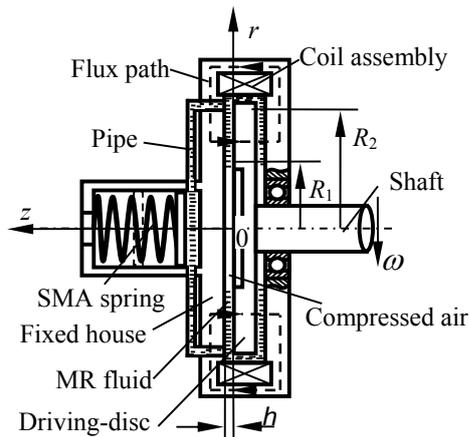


FIGURE I. DISC-TYPE MR BRAKE DRIVEN BY SMA

MR fluids exhibit a controllable yield stress-like behavior in shear, whereby the application of a magnetic field transverse to the flow creates a resistance to flow which increases with an increasing magnetic field. To accommodate the shearing thinning observed in MR fluids, the Herschel-Bulkley model [12] can be used to describe the flow behavior of MR fluid:

$$\begin{cases} \tau = \tau_y(H) + K|\dot{\gamma}|^m \text{sgn}(\dot{\gamma}) & \tau \geq \tau_y(H) \\ \dot{\gamma} = 0 & \tau < \tau_y(H) \end{cases} \quad (4)$$

where,  $\tau$  is the total shear stress,  $\tau_y(H)$  is the yield strength caused by the applied magnetic field,  $\dot{\gamma}$  is the shear rate,  $m$ ,  $K$  are constants. In the Herschel-Bulkley model, the constants  $m$ ,  $K$  and the function  $\tau_y(H)$  are empirically determined from experiments.

#### IV. MATHEMATICAL MODEL OF TORQUE

##### A. Flow Equation

The diagram of the operational mode of a disc-type MR brake is shown in Figure 2. In order to derive the equation of the fluid flow in the gap between two parallel circular plates, the following assumptions are given: the fluid is incompressible. There is no flow in radial direction and axial direction, but only tangential flow. The flow velocity of MR fluid is a function of radius. The pressure in the thickness direction of MR fluid is constant. The strength of magnetic field in the gap of the activation region is well-distributed. In cylindrical coordinates  $(r, \theta, z)$ , the distribution of the flow velocity is

$$V_r = 0, V_\theta = r\omega(z), V_z = 0 \quad (5)$$

where  $V_r$ ,  $V_\theta$  and  $V_z$  are the flow velocity of the fluid in the  $r$ -direction, the  $\theta$ -direction and  $z$ -direction, respectively;

$\omega(z)$  is the rotation angular velocity of the fluid in the  $\theta$ -direction. The angular velocity  $\omega(z)$  is the function of  $z$ -coordinate. The flow equation of the MR fluid in the  $\theta$ -direction may be approximated by

$$d^2\omega(z)/dz^2 = (1/\eta)d\sigma_{\theta\theta}/d\theta \quad (6)$$

where  $d\sigma_{\theta\theta}/d\theta$  is the gradient of the pressure in the  $\theta$ -direction. It is assumed that the fluid between two parallel circular plates is a steady-state flow,  $d\sigma_{\theta\theta}/d\theta = \text{const}$ .

Integrating the (6), the rotation angular velocity  $\omega(z)$  can be indicated as:

$$\omega(z) = (z^2/2\eta)d\sigma_{\theta\theta}/d\theta + c_1z + c_2 \quad (7)$$

where  $c_1$  and  $c_2$  are the two integrating constant. It is assumed that the fluid in touch with the surface of the turning plate has the same velocity as the turning plate. The angular velocity of the MR fluid increases with  $z$ -direction. It is assumed that there is no magnetic flux line at the upper ( $r = R_2$ ) and the lower ( $r = R_1$ ) portions of the domain. So, applying the boundary conditions of  $\omega(z) = 0$  at  $z=0$  and  $\omega(z) = \omega$  at  $z=h$ , the flow velocity  $\omega(z)$  can be mathematically manipulated to yield the flow as follows:

$$\omega(z) = \omega z/h + (1/2\eta)(z^2 - zh)d\sigma_{\theta\theta}/d\theta \quad (8)$$

where  $h$  is the gap between two parallel circular plates.  $\omega$  is the rotational velocities of the rotor-plate.

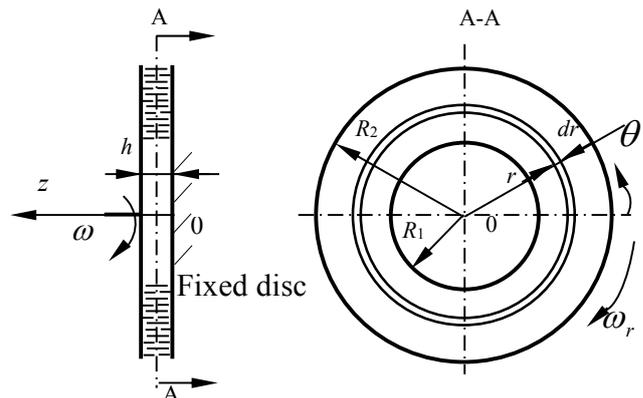


FIGURE II. CIRCULAR FLOW MODE OF MR FLUID BETWEEN TWO CIRCULAR PLATES

##### B. Governing Equation

It is assumed that there is no body force and neglecting the effects of gravity. The momentum equation of the MR fluid in

the  $\theta$ - direction for the cylindrical coordinates may be approximated by

$$\frac{\partial \tau_{r\theta}}{\partial r} + \frac{1}{r} \frac{\partial \sigma_{\theta\theta}}{\partial \theta} + \frac{\partial \tau_{z\theta}}{\partial z} + \frac{2}{r} \tau_{r\theta} = 0 \quad (9)$$

It is assumed that the MR fluid between two parallel circular plates is continuous flow. The gradient of the pressure in the  $\theta$ -direction is

$$d\sigma_{\theta\theta}/d\theta = (6\eta\omega/h^3)(h-h_0) \quad (10)$$

where  $h_0$  is the thickness of the MR fluid at maximum pressure. Because the two plates in the clutch are parallel each other ( $h = h_0$ ), so the gradient's value of the pressure in the  $\theta$ -direction is zero ( $d\sigma_{\theta\theta}/d\theta=0$ ). The flow angular velocity  $\omega(z)$  in the  $\theta$ -direction is constant ( $\tau_{r\theta}=0$ ). Equation (9) is simplified to

$$d\tau_{z\theta}/dz = 0 \quad (11)$$

The fluid shear strain rate of (4) may be approximated by

$$\dot{\gamma} = r(d\omega(z)/dz) \quad (12)$$

### C. Torque Equation

The torque transmitted by the disk-type brake is calculated by integrating the shear stress of the MR fluid. In Figure 2,  $R_1$  and  $R_2$  are the inner and outer radius of two circular plates, respectively. Take a micro-unit at distance  $r$  location in a circle, micro shear torque of the unit imposed on disc is as follows:

$$dJ = dF \cdot r = (\tau \cdot dS) \cdot r \quad (13)$$

The total transmission torque is:

$$J = N \int_r dJ = 2\pi N \int_r \tau_{z\theta} r^2 dr \quad (14)$$

where  $N$  is the number of working gap of MR fluid. Equations (4), (8), (12) and (14) can be mathematically manipulated to yield the torque as follows:

$$J = \frac{2\pi N}{3} (R_2^3 - R_1^3) \tau_y(H) + \frac{2\pi KN}{m+3} (R_2^{m+3} - R_1^{m+3}) \left(\frac{\omega}{h}\right)^m \quad (15)$$

## V. COMPUTATIONAL RESULTS AND DISCUSSIONS

Figure 3 shows the effect of the temperature on the MR fluid volume pushed by the SMA spring into the working gap. In this calculation,  $A_s=50^\circ\text{C}$ ,  $A_f=70^\circ\text{C}$ ,  $G_M=7.5\text{GPa}$ ,  $G_A=25\text{GPa}$ ,  $d=3.7\text{mm}$ ,  $D=22\text{mm}$ ,  $n=3$ ,  $\Delta P=0.743\text{MPa}$ ,  $A=500\text{mm}^2$ ,  $\Delta\delta=16\text{mm}$ . When the temperature is higher than  $50^\circ\text{C}$ , SMA spring begins to push the MR fluid into the working gap. When the temperature is equal to  $70^\circ\text{C}$ , the MR fluid can be entirely pushed by the SMA spring into the working gap.

For this example, we choose the commercial MR fluid (MRF-132DG) manufactured by Lord Corporation [13]. Figure 4 shows the yield strength of MRF-132DG under different magnetic field strength, measured by experiment.

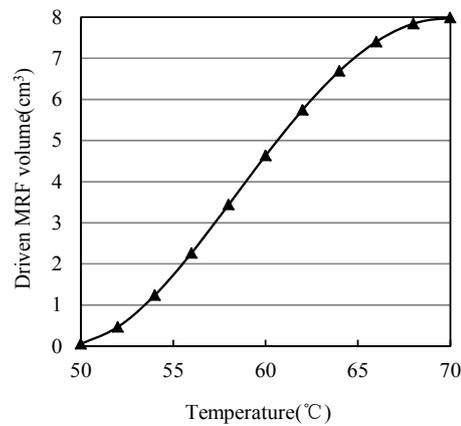


FIGURE III. THE DRIVEN MR FLUID VOLUME VERSUS TEMPERATURE

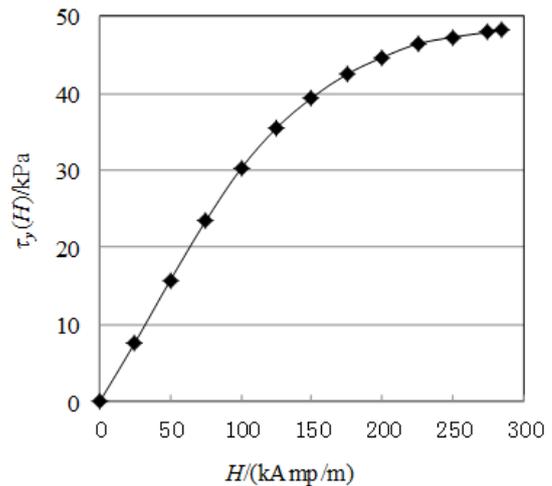


FIGURE IV. YIELD STRENGTH VERSUS MAGNETIC FIELD STRENGTH(MRF-132DG)

From the figure 4, we can find that the dynamic yield stress is proportional to the square of the magnetic field strength. MRF-132DG exhibits dynamic yield stresses of 0~44

kPa for the applied magnetic field strength of 0~200 kAmp/m. The ultimate strength of MR fluid is limited by magnetic saturation. The result shows that with the increase of the applied magnetic field strength, the dynamic yield stress goes up rapidly.

According to (15), the effect of magnetic field strength in transmission torque of the brake be analyzed, show in Figure 5. In this study, a commercially available MR fluid, LORD MRF-132DG, is used. The viscosity of MRF-132DG,  $\eta=0.0925\text{Pa}\cdot\text{s}$ . It is assumed that the constants  $m=1$ ,  $K=\eta$  in the Herschel-Bulkley model. Geometric parameters of the brake are: inner radius  $R_1=20\text{mm}$ , outer radius  $R_2=50\text{mm}$ , working gap  $h=1\text{mm}$ . The input angular velocity is,  $\omega = 100 \text{ rad/s}$ . When the magnetic field is applied, the torque is 7.7 N.m, 14.8 N.m and 19.3 N.m at the strength of magnetic field of 50kAmp/m, 100kAmp/m and 150kAmp/m, respectively. When MR fluid is saturated at the strength of magnetic field of 200kAmp/m, the torque is 21.9 N.m. The results indicate that with the increase of the applied magnetic field, the torque developed by MR fluid goes up rapidly.

## VI. CONCLUSIONS

The braking torque of a MR brake driven by SMA is investigated theoretically in this paper. The mathematical model of braking torque developed by the MR fluid is derived. The torque of the brake under different magnetic field strength is analyzed. With the increase of the applied magnetic field strength, the braking torque of the brake is increased rapidly.

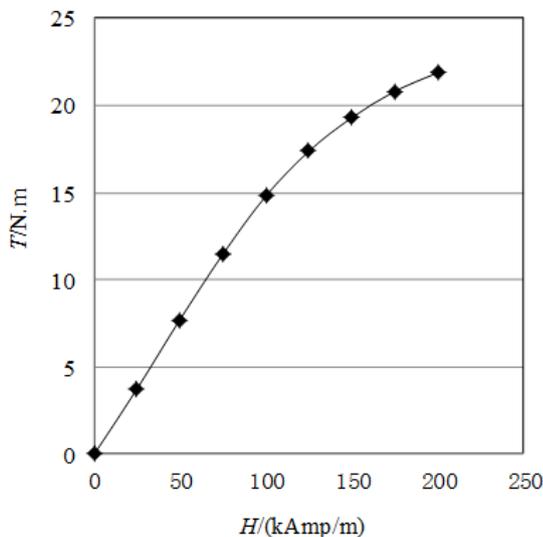


FIGURE V. BRAKING TORQUE VERSUS MAGNETIC FIELD STRENGTH

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