The Application of Numerical Manifold Method to the Turning Point Singularity Problem in Seepage

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Abstract: The numerical manifold method with special physical patch is used to deal with the seepage problem of turning point singularity caused by sheet pile. The method is applied to four engineering examples, and is superior to the finite element method, when the results are compared with the general finite element method.

1 Introduction

Numerical Manifold method(NMM)[1] consists of three parts: (1) Mathematical Cover and Physical Cover, which allows the interpolation grid and the integral grid to be separated to facilitate the processing of strong discontinuities problem; (2) Numerical Manifold Method Space, numerical manifold method extends the finite element space, and a function space can be defined on each physical piece; (3) Unit decomposition function. Therefore, numerical manifold method is easy to deal with seepage problem[2-5], and can construct local special physical patch at singular point[4].

2 The Singular Physical Patch of NMM

This is physical patch of singularity point:

$$P_{Singular}^T = [p_1, p_2] \tag{1}$$

$$d_{Singular}^T = [d_1, d_2] \tag{2}$$

$$\phi_{Singular} = P_{Singular}^T \cdot d_{Singular} \tag{3}$$

where,

$$p_1 = 1 \tag{4}$$

$$p_2 = r^{\frac{\pi}{2\varphi}} \sin(\frac{\pi\theta}{2\varphi}) \tag{5}$$

p_1、 p_2 are basis function, d is a generalized degree of freedom, (r, θ) is the polar coordinate, and the pole is the singular corner, φ is half of the reentrant corner of the singular point.

Using the unit decomposition property of the shape function, the head function ϕ^e (potential function) at any point in the local unit satisfies:

$$\phi^e(x, y) = \sum_{i=1}^{n} N_i \phi_i = \sum_{i=1}^{n} N_i(x, y) \sum_{k=1}^{m} p_k(x, y) d_{ik} = \sum_{j=1}^{nm} T_j d_j \tag{6}$$

where, ϕ_i is the head function covered by the manifold element, N_i is the unit shape function, p_k is the basis function, $T_j = N_i p_k$ is the generalized shape function, $d_j = d_{ik}$ is the generalized degree of freedom. The rest according to the general NMM processes, not repeat the details[2-3].

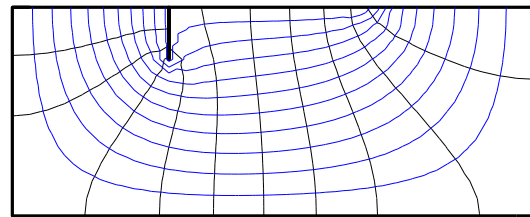## 3 Engineering Application Examples

3.1 Example One. There is a weir problem with sheet piles[7] in Fig.1,A is singulary point.Flow net shown in Fig.2 to Fig.3, which is calculated separately by NMM and finite element method(FEM).
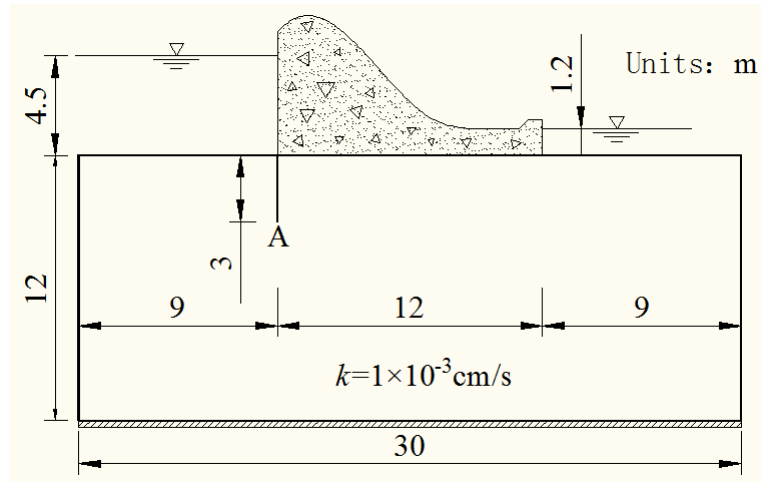
Fig.1　Example one Weir problem with sheet piles A
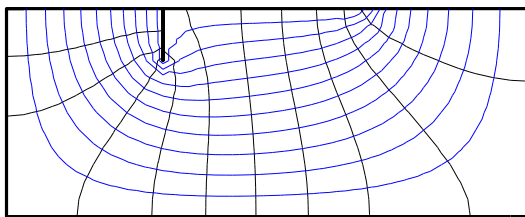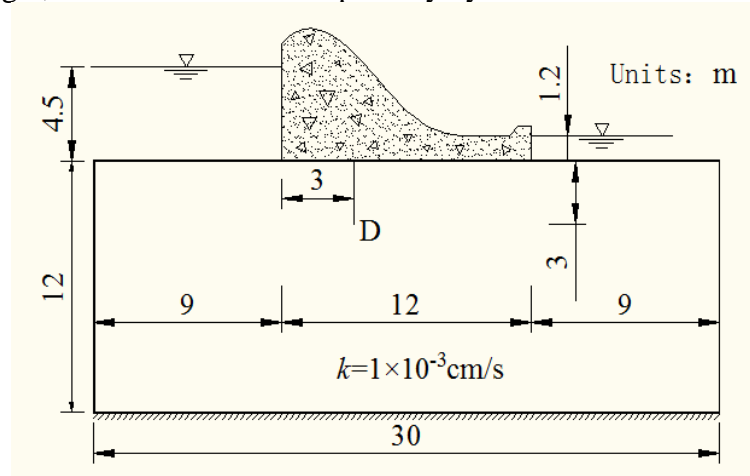

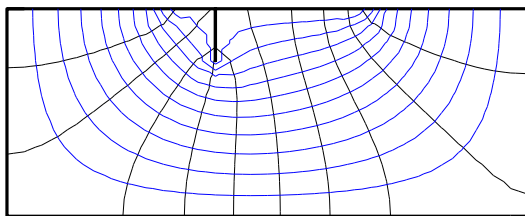**Fig.2**　Flow net with NMM of example one
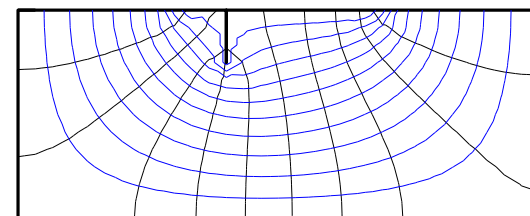

Fig.3　Flow net with FEM of example one

3.2 Example Two. There is a weir problem with sheet piles in Fig.4, D is singular point.Flow net shown in Fig.5 to Fig.6, which is calculated separately by NMM and FEM.

Fig.4　Example two Weir problem with sheet piles D
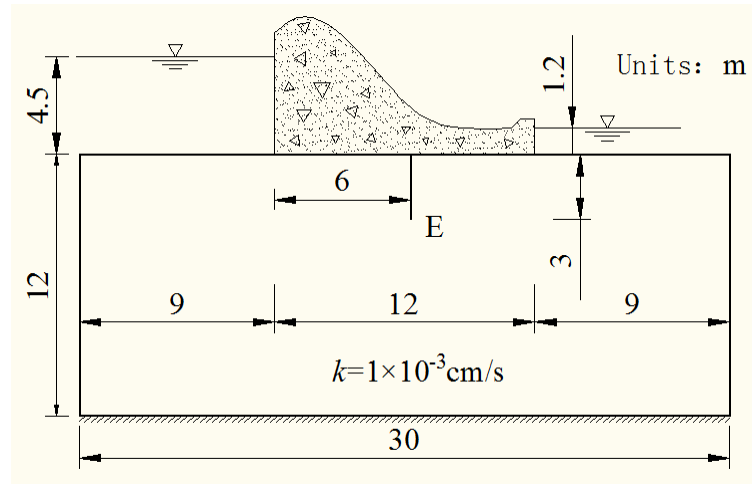

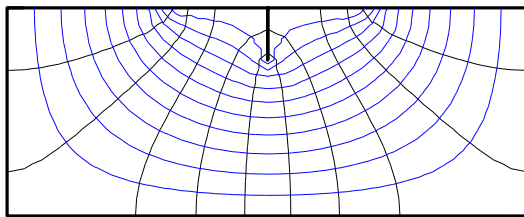**Fig.5**　Flow net with NMM of example two
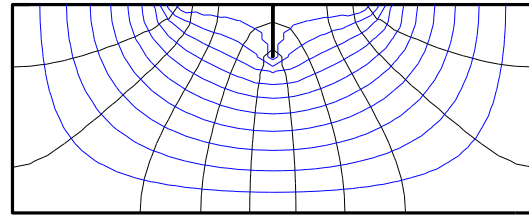

Fig.6　Flow net with FEM of example two

3.3 Example Three. There is a weir problem with sheet piles in Fig.7, E is singular point.Flow net shown in Fig.8 to Fig.9, which is calculated separately by NMM and FEM.

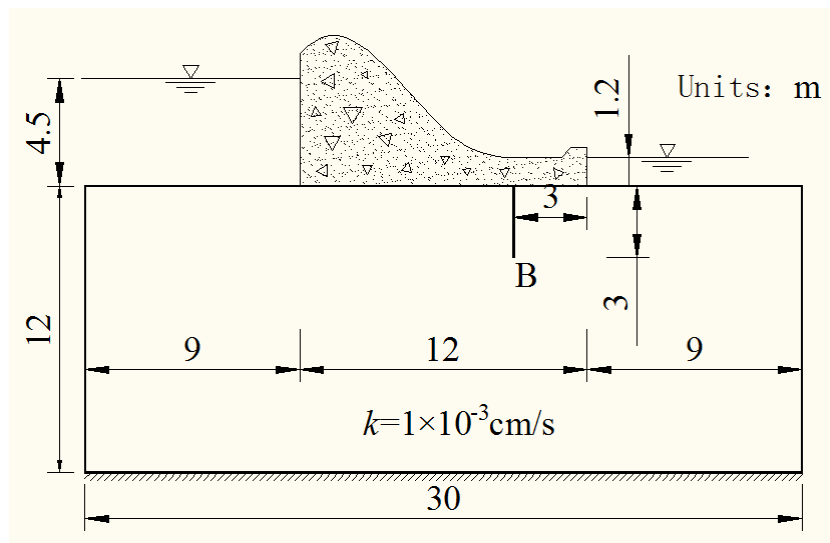**Fig.7** Example three Weir problem with sheet piles E
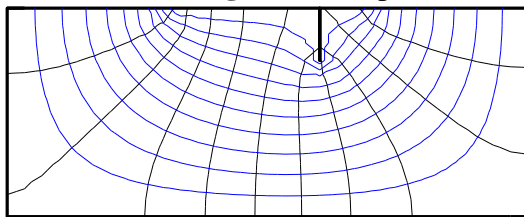

Fig.8 Flow net with NMM of example three

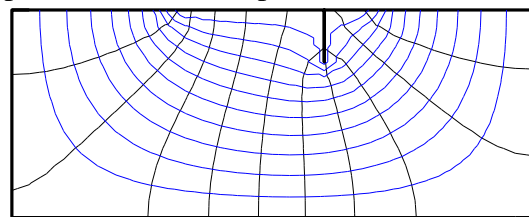**Fig.9** Flow net with FEM of example three

3.4 Example Four. There is a weir problem with sheet piles in Fig.10, B is singulary point.Flow net shown in Fig.11 to Fig.12,which is calculated separately by NMM and FEM.

Fig.10 Example four Weir problem with sheet piles B

Fig.11 Flow net with NMM of example four

Fig.12 Flow net with FEM of example four

4 Conclusions

Sum up the above analysises, The streamlines calculated by NMM rise slowly near the singularity, while the streamlines that are computed by FEM rise steep near the singularity. This is because FEM

does not consider singularity. The singularity here means that the potential function is continuous and the partial derivative is discontinuous. And the equipotential lines calculated by the manifolds are smoother.

NMM is more suitable for solving seepage problems, especially for some local cases where there are special solutions. NMM can construct special local physical patch function.

Acknowledgements

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