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N-fold Darboux Transformation for a Generalized Variable-Coefficient Coupled Hirota-Maxwell-Bloch System

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***Abstract*-A generalized variable-coefficient coupled Hirota-Maxwell-Bloch system is investigated, which can describe the ultrashort optical pulse propagation in a variablecoefficient nonlinear, dispersive fiber doped with two-level resonant atoms. Based on the obtained Lax pair, *N*-fold Darboux transformation (DT) of the system is constructed. One-, two- and three-soliton solutions are derived by virtue of the obtained DT, and compiled into the determinant forms**

Keywords-dispersive fiber; lax pair; N-fold darboux transformation

I. INTRODUCTION

In recent years, the investigations of the nonlinear dynamics of variable-coefficient systems have attracted certain attention because those systems are considered to be more realistic than their corresponding homogeneous counterparts [1–4]. In this paper, accompany with the self-induced transparency (SIT) effect, a generalized variable-coefficient coupled Hirota-MB (H-MB) system is proposed [5, 6],

$$q_z+\alpha_1(z)q_t+\alpha_2(z)q+i\alpha_3(z)q_{tt}+\alpha_4(z)q_{ttt}+ \alpha_5(z)|q|^2 q_t+i\alpha_6(z)|q|^2 q+\alpha_7(z)\langle p\rangle=0 \quad (1a)$$

$$pt = 2b1(z)qh - 2ib2(z)\omega p \quad (1b)$$

$$\eta_t=-\beta_1(z)(qp^*+q^*p) \quad (1c)$$

where q(*z*, *t*) is the slowly varying envelope axial field, p(*z*, *t*) is the measure of the polarization of the resonant medium, and η(z, t) is the extent of the population inversion, the real parameter ω represents the frequency, and the asterisk denotes the complex conjugate.

II. DT FOR SYSTEM (1)

The Lax pair corresponding to System (1) can be given as [7–9]

$$\Phi_t=\mathrm{U}\Phi,\ \Phi_z=\mathrm{V}\,\Phi \quad (2.1a,b)$$

where $\mathrm{U}=\begin{pmatrix}\lambda & \beta_1 q\\ -\beta_1 q^* & -\lambda\end{pmatrix}$, $\mathrm{V}=\begin{pmatrix}V_{11} & V_{12}\\ V_{21} & -V_{11}\end{pmatrix}=[\lambda^3V_0+\lambda^2V_1+\lambda V_2+V_3]$,

$\Phi=(\phi_1,\phi_2)^T$ is the vector eigenfunction, T denotes the transpose of the vector, λ is the spectral parameter, and the matrices V_0, V_1, V_2 and V_3

$$V_0=\begin{pmatrix}-4\alpha_4 & 0\\ 0 & 4\alpha_4\end{pmatrix}$$

$$V_1=\begin{pmatrix}-2i\alpha_3 & -4\alpha_4\beta_1 q\\ 4\alpha_4\beta_1 q^* & 2i\alpha_3\end{pmatrix}$$

$$V_2=\begin{pmatrix}-\alpha_4-2\alpha_4\beta_1^2qq^* & -2\beta_1\left(i\alpha_3q+\alpha_4q_t\right)\\ 2\beta_1\left(i\alpha_3q^*-\alpha_4q_t^*\right) & \alpha_1+2\alpha_4\beta_1^2qq^*\end{pmatrix}$$

$$V3=\begin{pmatrix}A_1\eta-\frac{1}{2}i\alpha_6qq^*-\alpha_4\beta_1^2\left(q^*q_t-qq_t^*\right) & -\beta_1B_1-A_1p\\ \beta_1B_1^*-A_1-p^* & -A_1\eta+\frac{1}{2}i\alpha_6qq^*+\alpha_4\beta_1^2\left(q^*q_t-qq_t^*\right)\end{pmatrix}$$

with $A_1=-\beta_1\alpha_7/2(\lambda+i\omega\beta_2)$ and $B_1=(2\alpha_4\beta_1^2q^*q^2+\alpha_1q+i\alpha_3q_t+\alpha_4q_{tt}$

The transformation

$$\tilde{\Phi}=\mathrm{T}\Phi \quad (2.2)$$

can transform Lax Pair (2.1a, b) into

$$\tilde{\Phi}_t=\tilde{\mathrm{U}}\tilde{\Phi},\quad \tilde{\mathrm{U}}=(\mathrm{T}_t+\mathrm{TU})\mathrm{T}^{-1} \quad (2.3a)$$

$$\tilde{\Phi}_z=\tilde{\mathrm{V}}\tilde{\Phi}\quad \tilde{\mathrm{V}}=(\mathrm{T}_z+\mathrm{TV})\mathrm{T}^{1} \quad (2.3b)$$

From (2.3a) and (2.3b), one can get

$$\tilde{\mathrm{U}}_z-\tilde{\mathrm{V}}_t+[\tilde{\mathrm{U}},\tilde{\mathrm{V}}]=\mathrm{T}(\mathrm{U}_z-\mathrm{V}_t+[\mathrm{U},\mathrm{V}])\mathrm{T}^{-1} \quad (2.4)$$

Suppose

$$\mathrm{T}=\mathrm{T}(\lambda)=\begin{pmatrix}A & B\\ C & D\end{pmatrix} \quad (2.5)$$

where

$$A=\lambda^N+\sum_{k=0}^{N-1}A_k\lambda^k,\ B=\sum_{k=0}^{N-1}B_k\lambda^k \quad (2.6)$$

$$C=\sum_{k=0}^{N-1}C_k\lambda^k\quad D=\lambda^N+\sum_{k=0}^{N-1}D_k\lambda^k \quad (2.7)$$

and

$$\sum_{k=0}^{N-1}\left(A_{k}+B_{k}\alpha_{j}\right)=-\alpha_{j}^{N},\quad \sum_{k=0}^{N-1}\left(C_{k}+D_{k}\alpha_{j}\right)=-\alpha_{j}^{N} \tag{2.8}$$

with

$$\alpha_{j}=\frac{\phi_{2}(\lambda_{j})-\gamma_{j}\psi_{2}(\lambda_{j})}{\phi_{1}(\lambda_{j})-\gamma_{j}\psi_{1}(\lambda_{j})}\quad (1\le j\le 2N) \tag{2.9}$$

where $\phi(\lambda)=\left[\phi_{1}(\lambda),\phi_{2}(\lambda)\right]^{T}$ and $\psi(\lambda)=\left[\psi_{1}(\lambda),\psi_{2}(\lambda)\right]^{T}$ are two basic solutions of Lax Pair (2.1a, b), λ_k and $\gamma_{j}\left(\lambda_{k}\neq\gamma_{j}\text{ as }k\neq j\right)$ are parameters suitably chosen such that the determinant of coefficients for Eq. (2.8) is non-zero, thus, A_k, B_k, C_k and D_k are uniquely determined by Eq. (2.8).

From Eq. (2.5), one can see that detT(λ) is a 2*N* polynomial of λ, and

$$\det T\left(\lambda_{j}\right)=A\left(\lambda_{j}\right)D\left(\lambda_{j}\right)-B\left(\lambda_{j}\right)C\left(\lambda_{j}\right)$$

From Eq. (2.8),

$$A\left(\lambda_{j}\right)=-\alpha_{j}B\left(\lambda_{j}\right)\ \ C\left(\lambda_{j}\right)=-\alpha_{j}B\left(\lambda_{j}\right) \tag{2.10}$$

Therefore

$$\det T(\lambda_{j})=0$$

Thus, $\lambda_{j}(1\le j\le 2N)$ are 2*N* roots for the determinant of matrix T(λ), i.e.,

$$\det T(\lambda)=\prod_{j=1}^{2N}(\lambda-\lambda_{j}) \tag{2.11}$$

Proposition 1. The matrix $\tilde{U}$ defined by Eq. (2.3a) has the same form as U

$$\tilde{U}=\begin{pmatrix}\lambda & \beta_{1}\tilde{q}\\ -\beta_{1}\tilde{q}^{*} & -\lambda\end{pmatrix} \tag{2.12}$$

where the transformation from the old potential q to new one $\tilde{q}$ is given by

$$\tilde{q}=q-\frac{2}{\beta_{1}}B_{N\text{-}1} \tag{2.13}$$

Proof. Let $T^{-1}=T^{*}/\det T$ and

$$(T_{t}+TU)T^{*}=\begin{pmatrix}f_{11}(\lambda) & f_{12}(\lambda)\\ f_{21}(\lambda) & f_{22}(\lambda)\end{pmatrix} \tag{2.14}$$

$f_{11}(\lambda)$ and $f_{22}(\lambda)$ are (2*N* + 1) th-order polynomials in λ, and $f_{12}(\lambda)$ and $f_{21}(\lambda)$ are 2N th-order polynomials in λ. It can be obtained from Eqs. (2.1a, b) and (2.9)

$$\alpha_{j,t}=-\beta_{1}q^{*}-2\lambda\alpha_{j}-\beta_{1}q\alpha_{j}^{2}$$

It can be verified that all $\lambda_{j}(1\le j\le 2N)$ are roots of $f_{kj}(\lambda)$ (k, j = 1, 2). Noting Eq. (2.11), we can prove that

$$\det(T\mid f_{kj}(\lambda)\quad (k,j=|1,2)$$

Therefore, together with Eq. (2.14), we have

$$(T_{t}+TU)T^{*}=(\det T)P(\lambda) \tag{2.16}$$

with

$$P(\lambda)=\begin{pmatrix}P_{11}^{(1)}\lambda+P_{11}^{(0)} & P_{12}^{(0)}\\ P_{21}^{(0)} & P_{22}^{(1)}\lambda+P_{22}^{(0)}\end{pmatrix}$$

Eq. (2.16) can be written as

$$(T_{t}+TU)=P(\lambda)T \tag{2.17}$$

Comparing the coefficients of λ^{N+1} and λ^{N} in Eq. (2.17), we can obtain

$$\begin{aligned}&P_{11}^{(1)}=-P_{22}^{(1)}=1,\quad P_{12}^{(0)}=\beta_{1}q-2B_{N-1}=\beta_{1}\tilde{q}\\ &P_{11}^{(0)}=P_{22}^{(0)}=0,\quad P_{21}^{(0)}=-\beta_{1}q^{*}+2C_{N-1}=-\beta_{1}\tilde{q}^{*}\end{aligned} \tag{2.18}$$

From Eqs. (2.12) and (2.18), one can see that $P(\lambda)=\tilde{U}$, the proof is completed.

Similarly, we can prove that the matrix $\tilde{V}$ has the same form as V under Transformations (2.2) and (2.13). In summary, we arrive at the following theorem.

Theorem 1. The solutions (q, p, η) of System (1) are mapped into their new solutions ($\tilde{q},\tilde{p},\tilde{\eta}$) under DT(2.2).

III. Solutions and Applications in Inhomogeneous Erbium-Doped Optical Bers

In this section, we discuss the DT for System (1) and give its explicit solutions. First, we take

$$\Phi(\lambda)=\left[\phi_{1}(\lambda),\phi_{2}(\lambda)\right]^{T},\ \ \Psi(\lambda)=\left[-1\phi_{2}^{*}(-\lambda^{*}),\phi_{1}^{*}(-\lambda)\right]^{T} \tag{3.1}$$

and parameters

$$\lambda_{2j}=-\lambda_{2j-1,}^{*}\quad \gamma_{2j}=-\gamma_{2j-1}^{*-1},\quad 1\le j\le N \tag{3.2}$$

and

$$\alpha_{2j}^{-1}=-\alpha_{2j-1}^{*},\ \ D_{k}^{*}=-A_{k},\ \ C_{k}^{*}=B_{k},\ \ 1\le k\le N\text{ - }1 \tag{3.3}$$

In this way, the solutions for Eqs. (2.8) and (2.9) reduce to

$$\sum_{k=0}^{N-1}\left(A_{k}+\alpha_{2j-1}B_{k}\right)\lambda_{2j-1}^{k}=-\lambda_{2j-1}^{N} \tag{3.4}$$

$$\sum_{k=0}^{N-1}\left(-\alpha_{2j-1}^{*}A_{k}+B_{k}\right)\lambda_{2j-1}^{*k}=-\alpha_{2j-1}^{*}\lambda_{2j-1}^{*N},\quad 1\le j\le N \tag{3.5}$$

with

$$\alpha_{2j-1} = \frac{\phi_2\left(\lambda_{2j-1}\right) - \gamma_{2j\text{-}I}\psi_2(\lambda_{2j-1})}{\phi_2\left(\lambda_{2j-1}\right) - \gamma_{2j\text{-}I}\psi_1(\lambda_{2j-1})} \quad (3.6)$$

IV. CONCLUSIONS

In this paper, we have investigated a generalized inhomogeneous coupled H-MB system, i.e., System (1), which describes the ultrashort optical pulse propagation in an inhomogeneous nonlinear, dispersive fiber doped with two-level resonant atoms. Based on the obtained Lax Pair (2.1a, b), we have constructed the N-fold DT for System (1) and compiled the obtained solutions into the determinant forms. Our analysis and results will be of certain value in realizing soliton-type pulse propagation in an inhomogeneous erbium-doped fiber with the higher-order effects.

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