Quantile Regression Analysis on Short-Term Measured Data of Gravity-Arch Dam Based on POD

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Abstract—**To solve the problems of little sample, multi-collinearity and bad robust ability of normal model remaining in measured dam data in process of analysis,This paper analyzed the monitoring data of measured dam crest crown cantilever and both sides of 1/4 arch of a gravity dam in 2013 using circannual monitoring data. The research shows that quantile regression analysis method based on POD can conquer the problems above when analyzing measured dam data and excavate more safety dam information.**

Keywords- POD; short-time series; quantile regression; gravity-arch dam

I. INTRODUCTION

The key problems that affect analysis of dam safety monitoring data are little sample, multi-collinearity and independent equal distribution of measuring error. Especially when dam safety behavior gets worse, the problems above are more obvious, getting inter-coupling, and more difficult to solve, by this time, it is more crucial to analyze dam safety behavior. Many scholars have done lots of research on the problems above [1-5], however, there was none effective integrative method discovered. Proper Orthogonal Decomposition (POD for short, called Karhunen-Loeve transform also) can reduce dimensionality, ensure high precision of model, save computing space and enhance computing efficiency. POD relies on data entirely and doesn`t make any apriority assumption on data [6-8]. Quantile regression estimates regression parameter of model by making weighted summation on absolute value of model residual error and letting summation reach to the minimum. Compared to classical lest square regression, quantile regression has less limiting conditions and residual error doesn`t need to meet Gaussian distribution. Quqntile regression can reveal incidence independent variable having on dependent variable on different quantiles, especially the special distributions such as tip and tail [9-13]. Aiming at the problems above, this paper combines reduction dimension analysis and random estimation theory, proposes quantile regression analysis method based on POD and builds verification test using measured data of concrete gravity dam.

II. MEASURED DATA ANALYSIS

A. Data sources

A certain reservoir, with total capacity of 2.706 billion m^3, controls drainage area of 2800km^2. The normal storage water level is 119m, limited water level is 117m in flood season and the level of dead water is 101m. The dam was heightened 1.3m and consolidated in 1978 in accordance with P.M.F water level which was under dam overtopping level. Hinge buildings composed of concrete gravity-arch dam, both sides spillway of dam crest, flood discharge mesopore, underport, power generation and water diversion pipe, and plant behind the dam. The biggest dam is 76.3m tall, including 28 dams. Dam top arc length is 419m, dam crest width is 8m and the biggest bottom width is 53.2m. This paper chooses concrete arch dam radial deformation motoring data of dam crest crown cantilever and both sides of 1/4 arch in 2013, corresponding measuring point numbers are dam crest crown cantilever No.18 and both sides of 1/4 arch No.8 and No.26. Displacement process line is shown in Figure 1. Dam operating environment was complex over the annual operating process, data analysis indicated that dam condition changed complexly [1].
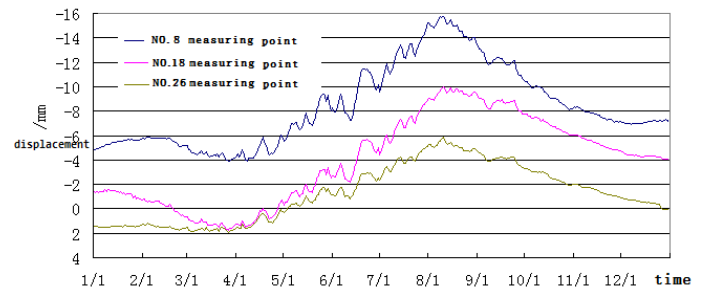

FIGURE I.   DISPLACEMENT PROCESS LINE OF THREE MEASURED POINTS IN 2013

Measured data variation diagram of No.18 gauging point, including displacement, reservoir water level and temperature, are showed in Figure 2 and Figure 3.
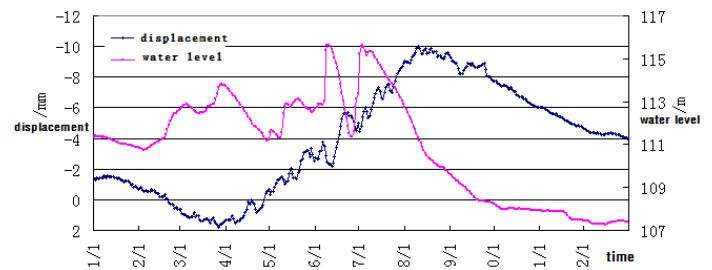

FIGURE II.   DISPLACEMENT AND WATER LEVEL VARIATION DIAGRAM OF NO.18 GAUGING POINT
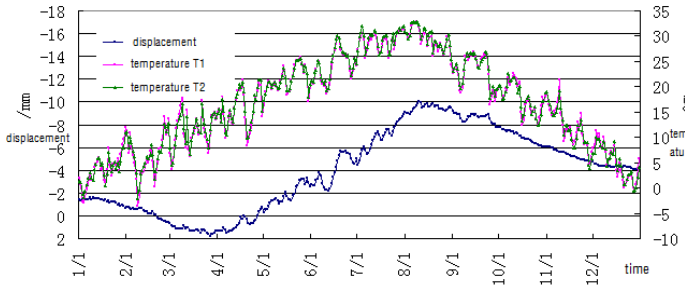
FIGURE III.   DISPLACEMENT AND TEMPERATURE
VARIATION DIAGRAM OF NO. 18 GAUGING POINT

B.   Regression Analysis Based on POD

1)   *Constructing snapshot matrix:* We choose 140 groups of monitoring data, which dates from 15th march to 1st august, to model and analyze. We choose 40 groups of data, which dates from 2nd august to 10th September, to verify predictability of the model. Regression model of displacement monitoring data is shown below:

$$y = a_0 + \sum_{i=1}^{4} a_i H^i + \sum_{j=1}^{8} b_j T_j + c_1 \theta + c_2 \ln \theta \quad (1)$$

where a_0 , a_i , b_j are coefficients, T_j is temperature monitoring value, y_T is temperature effect quantity, θ is aging factor, $\theta = t_i / 100$, t_i is number of measuring days. In order to reduce the impact of dimension, standardized treatment has been done on snapshot matrix, which include eight independent variables(H^1 , H^2 , H^3 , H^4 , T_1 , T_2 , θ , $\ln \theta$). The eight independent variables are presented as x1~x8 separately for convenience.

2)   *Calculating incidence matrix:* Using Matlab software to construct incidence matrix (that is correlation coefficient matrix) through standardized snapshot matrix. See Table 1.

TABLE I.   CORRELATION COEFFICIENT MATRIX OF INFLUENCE FACTOR

	H	**H²**	**H³**	**H⁴**	T_1	T_2	θ	$\ln \theta$
H	1.0000	1.0000	0.9998	0.9996	-0.0051	-0.0019	-0.3364	-0.3356
H²	1.0000	1.0000	1.0000	0.9998	-0.0023	0.0010	-0.3312	-0.3304
H³	0.9998	1.0000	1.0000	1.0000	0.0005	0.0040	-0.3260	-0.3252
H⁴	0.9996	0.9998	1.0000	1.0000	0.0033	0.0069	-0.3207	-0.3199
T_1	-0.0051	-0.2223	0.0005	0.0033	1.0000	0.9978	0.8156	0.8161
T_2	-0.0019	0.0010	0.0040	0.0069	0.9978	1.0000	0.8262	0.8267
θ	-0.3364	-0.3312	-0.3260	-0.3207	0.8156	0.8262	1.0000	1.0000
$\ln \theta$	-0.3356	-0.3304	-0.3252	-0.3199	0.8161	0.8267	1.0000	1.0000

From Table 1, we can find out that there is strong linear correlation between independent variables. For example, correlation between water level H and H^2, H^3, H^4 almost reaches to one, linear correlation between temperature T_1 and T_2 is strong, correlation between temperature and aging factor θ and $\ln \theta$ is strong. Multicollinearity problem exists.

3)   *Obtaining the optimum orthogonal basis of POD:* Eigenvalue and eigenvector of incidence matrix are obtained by using Matlab. Eigenvalue and corresponding eigenvector are sorted in descending order. Eigenvalue and energy`s changing conditions along with POD modality are shown as Figure 4.

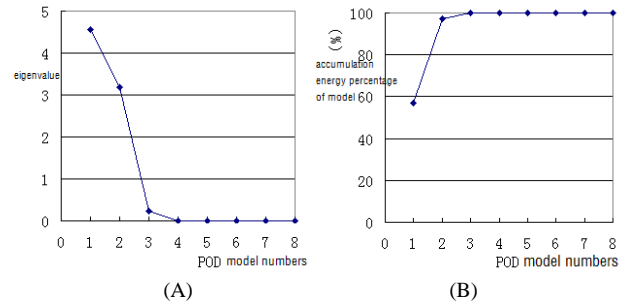(A)   (B)

FIGURE IV.   EIGENVALUE AND ENERGY'S CHANGING
CONDITIONS ALONG WITH POD MODALITY

It is shown in Figure 4 that eigenvalue descends quickly on third modality and approaches zero on fourth modality. Energy approaches 100% on third modality and is large on the first and second modality.

4)   *Establishing regression model:* Low-energy modality is truncated based on energy standard and optimum orthogonal basis is obtained according to the first two order modality of POD. Choose eigenvector to construct matrix $V = [V_1, V_2]$,carry

on the liner superposition of standardized snapshot matrix and extract the optimum orthogonal basis of POD Ψ_i (i=1,2).

$$\Psi_1 = -0.406H^1 - 0.405H^2 - 0.404H^3 - 0.403H^4 + 0.223T_1 + 0.223T_2 + \quad (2)$$
$$0.350\theta + 0.350\ln\theta$$

$$\Psi_2 = 0.277H^1 + 0.279H^2 + 0.281H^3 + 0.283H^4 + 0.472T_1 + 0.476T_2 + \quad (3)$$
$$0.344\theta + 0.345\ln\theta$$

C. Median Regression Analysis

Median regression is quantile regression whose quantile is 0.5. Analysis results are shown in Table 2.

TABLE II. MEDIAN REGRESSION ANALYSIS RESULTS OF NO.18 GAUGING POINT DATA

parameter	Degree of freedom	Estimated value	Standard error	95%confidence interval		t value	Pr > |t|
Constant term	1	-16765.5	34064.09	-84065.7	50534.69	-0.49	0.6233
x1	1	435.3213	842.5165	-1229.23	2099.876	0.52	0.6061
x2	1	-3.9753	7.4526	-18.6993	10.7487	-0.53	0.5945
x3	1	0.0121	0.0220	-0.0313	0.0555	0.55	0.5829
x4	0	0.0000	0.0000	0.0000	0.0000	.	.
x5	1	0.2963	0.1283	0.0428	0.5498	2.31	0.0223
x6	1	-0.3877	0.1401	-0.6646	-0.1108	-2.77	0.0064
x7	1	-10.5848	12.2732	-34.8329	13.6633	-0.86	0.3898
x8	1	505.9433	1963.319	-3372.97	4384.861	0.26	0.7970

(Pr(Probability) represents accompany probability(p value) of statistical magnitude(t), Pr > \|t\|=P(\|t\| random >\|t\|）)

It can be known from calculating results that freedom degree of independent variable x4 and x8 on No.8 gauging point and No.26 gauging point is zero. Freedom degree of independent variable x4 on No.18 gauging point is zero. It means that variables whose degree of freedom are zero aren`t included in fitting regression equation. These variables are considered to have no influence on displacement in model. Meanwhile, regression coefficients of four variables connected with water level haven`t passed the significance testing, temperature coefficients have passed the significance testing ($p < 0.05$). Obviously, regression model of median is unreasonable. It follows that linear dependence among variables affects precision and actual availability of regression analysis model seriously.

D. Quantile Regression Based on POD

In order to conquer multilinear problem, we do quantile regression analysis on displacement y and orthogonal basis Ψ_1 and Ψ_2 directly. Typical results are shown in Table 3. Typical quantile regression equation coefficients are shown in Table 4.

TABLE III. REGRESSION ANALYSIS RESULTS OF QUANTILE 0.5

Gauging point number	parameter	Degree of freedom	Estimated value	Standard error	95% confidence interval		t value	Pr > |t|
8	Constant term	1	-9.0536	0.1347	-9.3196	-8.7877	-67.23	<.0001
	Ψ1	1	-1.0213	0.0518	-1.1237	-0.9190	-19.71	<.0001
	Ψ2	1	-1.6786	0.0435	-1.7645	-1.5926	-38.56	<.0001
18	Constant term	1	-3.4304	0.0745	-3.5775	-3.2833	-46.06	<.0001
	Ψ1	1	-0.9880	0.0354	-1.0579	-0.9181	-27.91	<.0001
	Ψ2	1	-1.7061	0.0306	-1.7667	-1.6456	-55.67	<.0001
26	Constant term	1	-1.5430	0.0397	-1.6213	-1.4646	-38.90	<.0001
	Ψ1	1	-0.6752	0.0242	-0.7230	-0.6274	-27.91	<.0001
	Ψ2	1	-1.0389	0.0156	-1.0697	-1.0082	-66.73	<.0001

Pr(Probability) represents accompany probability(p value) of statistical magnitude(t), Pr > \|t\|=P(\|t\| random >\|t\|）

TABLE IV. REGRESSION ANALYSIS COEFFICIENT TABLE OF EACH QUANTILE ON NO.18 GAUGING POINT

Quantile τ	Constant term	Orthogonal basis Ψ_1	Orthogonal basis Ψ_2	Whether has passed the significance testing
0.1	-4.5261	-0.8374	-1.6004	yes
0.2	-4.2037	-0.8409	-1.5807	yes
0.3	-3.7672	-0.9552	-1.6351	yes
0.4	-3.6337	-0.9500	-1.6517	yes
0.5	-3.4304	-0.9880	-1.7061	yes
0.6	-3.1564	-1.0529	-1.7705	yes
0.7	-2.8752	-1.0579	-1.8350	yes
0.8	-2.5619	-1.0815	-1.9125	yes
0.9	-2.3022	-1.1332	-1.9870	yes

Table 4 has presented regression estimated value and 95% confidence interval on different quantiles based on POD, as well as significance testing results of regression coefficient. From quantile regression analysis results based on POD, regression coefficient on every quantile point has passed the significance testing, which is different from quantile regression

analysis of original data. Regression equation is reasonable and this method has eliminated the bad influence of linear correlation among independent variables. It is known from the calculation results that regression coefficients are different of each variable on different quantile points. It means POD orthogonal basis has different influence on displacement on different quantile points, that is to say, independent variables, such as water level, temperature and aging, have different influence for arch dam displacement on different quantile points. Each measuring point`s coefficient value is different on the same quantile point. so, influence factor has different influence on arch dam displacement in different positions. Therefore, more different positions should be chosen for monitoring data analysis during arch dam deformation analysis, which can make evaluating results more precise and comprehensive. Figure 5 has shown the regression coefficient change of each independent variable on different quantile points.
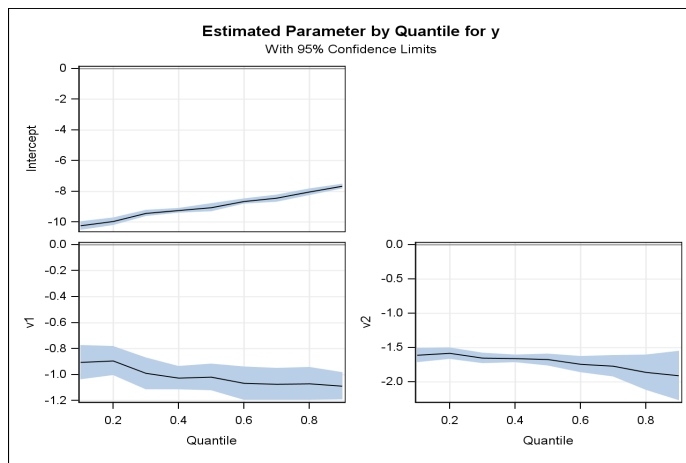
(where v_1, v_2 represent Ψ_1, Ψ_2 respectively, abscissa represents quantile point size, ordinate represents regression coefficient value of independent variable)

FIGURE V. REGRESSION COEFFICIENT CHANGE CHART OF EACH INDEPENDENT VARIABLE ON DIFFERENT QUANTILE POINT OF NO.8 GAUGING POINT

It can be known from calculation results that regression coefficient absolute value of constant term decreases along with quantile point increase on No.8 gauging point, Ψ_1 regression coefficient absolute value increases continually between quantile point 0.2 and quantile point 0.4, while decreases between quantile point 0.4 and quantile point 0.5 and then increases gradually. Increasing degree is relatively large between quantile point 0.5 and quantile point 0.6. It means there is switching changes of influence that Ψ_1 having on arch dam displacement between quantile point 0.4 and quantile point 0.6, while changing tendency is smooth between quantile point 0.6 and quantile point 0.9. Coefficient changing gradient is small. Regression coefficient absolute value of Ψ_2 has a laxly increasing tendency, while increasing gradient is small. Regression coefficient absolute value of Ψ_2 is larger than that of Ψ_1. It means Ψ_2 has larger influence on displacement than that of Ψ_1. Calculation results show that absolute value of constant term coefficient decreases along with quantile point increase on NO.18 gauging point, regression coefficient absolute value of Ψ_1 increases largely between quantile point

0.2 and quantile point 0.3 and increases along with quantile increase between quantile point 0.4 and quantile point 0.9, progressive increasing degree is relatively small. In total, influence degree that Ψ_1 having on arch dam displacement increases gradually along with quantile point increase. Different from regression coefficient changing degree of Ψ_1, regression coefficient absolute value of Ψ_2 increases along with quantile point increase and changes smoothly. Regression coefficient absolute value of constant term on No.26 gauging point, which is the same with absolute value on No.8 gauging point and No.18 gauging point, decreases along with quantile point increase. Regression coefficient absolute value of Ψ_1 reaches to the maximum on quantile point 0.6.It means Ψ_1 has the largest impact on displacement. Changing degree of Ψ_2 regression coefficient absolute value which is similar to changing degree on No.18 gauging point increases smoothly along with quantile point increase. It appears totally that regression coefficient versions of each variable are similar on three gauging points, regression coefficient absolute value of constant term increases along with quantile point increase, fluctuation range of Ψ_1 regression coefficient is relatively large, Ψ_2 regression coefficient changes smoothly.

95% confidence interval represents estimated value spanning scope when confidence level reaches to 95%. Cograph shadow part represents confidence interval size when confidence level reaches to 95% confidence degree, the narrower confidence interval is, the more precise estimated value is and the more information related to population parameter can be provided. From copragh ,estimated value of constant term precision is high since confidence interval of constant term is narrow . Confidence interval of Ψ_1 is largish, while width remains fairly consistent along with quantile point changing. It means precision of Ψ_1 regression coefficient estimated value remains the same on each quantile point. Width of Ψ_2 confidence interval is narrow and changes a little between quantile point 0.1 and quantile point 0.5, while width grows bigger gradually between quantile point 0.6 and quantile point 0.9.It means estimated value precision between quantile point 0.1 and quantile point 0.5 is higher than that between quantile point 0.6 and quantile point 0.9.

E. Quantile Regression of Each Independent Variable

In order to analyze influence degree independent variable having on arch dam displacement on different quantile point further, quantile regression analysis has been done on the relationship between independent variable and dependent variable, without considering interaction between the independent variables. According to the analysis results, independent variable has different influence on displacement at different times. In order to analyze dependent variable impact precisely, we divide monitoring data into two phases: one is 160 groups of monitoring data dating from March 15th to August 21st, the other is 120 groups of monitoring data dating from September 1st to December 29th. See Figure 6.
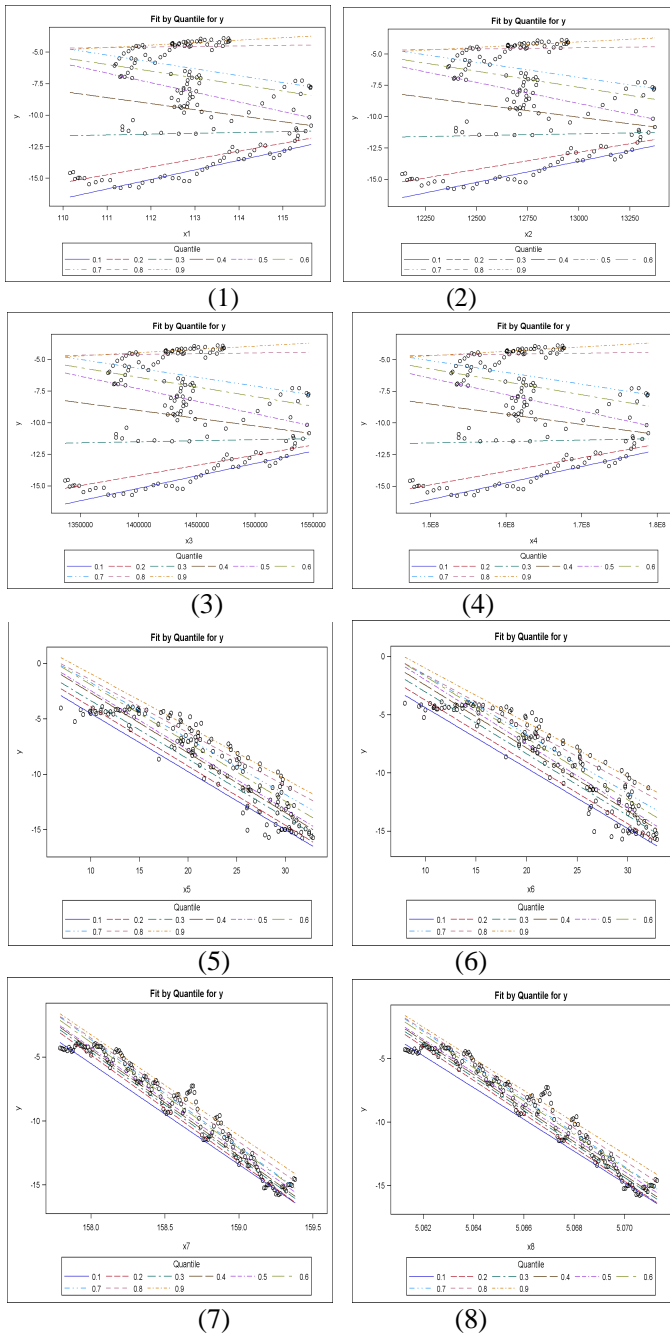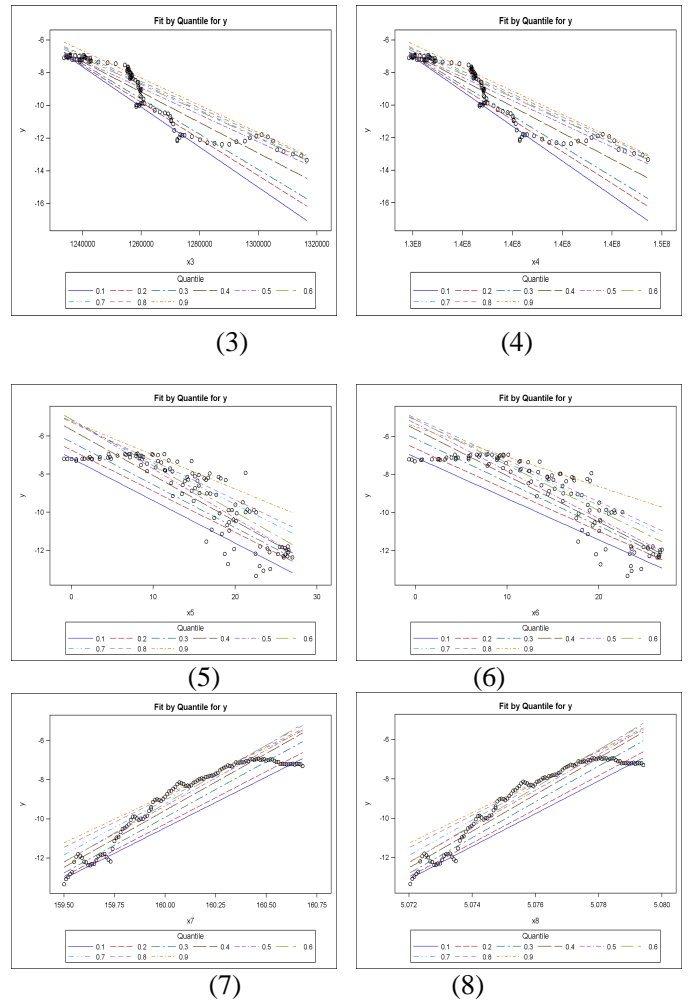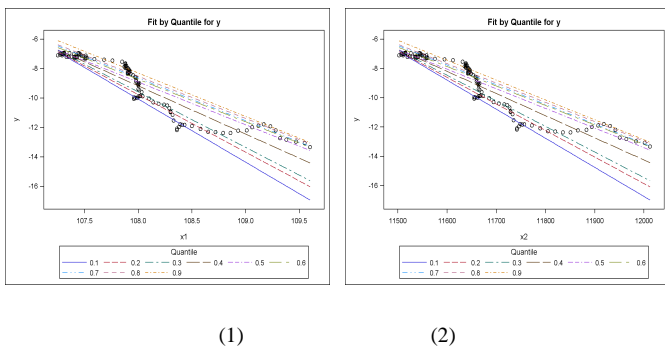
(1) (2)

(3) (4)

(5) (6)

(7) (8)

FIGURE VI. (A) QUANTILE REGRESSION ANALYSIS GRAPH OF EACH INDEPENDENCE VARIABLE ON DIFFERENT QUANTILE AT THE FIRST PHASE

(1) (2)

(3) (4)

(5) (6)

(7) (8)

(abscissa represents independence variable, ordinate represents displacement (unit: mm))

FIGURE VI. (B) QUANTILE REGRESSION ANALYSIS GRAPH OF EACH INDEPENDENCE VARIABLE ON DIFFERENT QUANTILE AT THE SECOND PHASE

Figure 6 shows that quantile regression analysis obtains a bunch of curve, which reflects regression relationship between independent variable and dependent variable on different quantile points. It shows independent variable affects displacement differently on different quantile points because regression curve is different on different quantile points. The bigger angle between regression curves is, the greater difference of influence degree is. This paper divides cograph into three classes. According to changing of influence degree that independent variable having on displacement at different quantile points, this paper divides cograph into three classes. See Table 5 for details.

From Table 5, regression curves of temperature and aging factor in the first and second analysis phase are approximately parallel on each quantile point. It means temperature and aging factor have little influence on displacement with the change of quantile point. Regression curve of water level distributes irregularly and differs a lot in the first phase on each quantile point. It means water level factor has complicated influence on displacement and is irregular.

TABLE V. INDEPENDENCE VARIABLE REGRESSION COEFFICIENT ESTIMATED TABLE ON DIFFERENT QUANTILE POINTS

sort	Number	Sort basis
A	a-5, a-6, a-7, a-8, b-5, b-6, b-7, b-8	Sample points are relatively centralized, regression lines are parallel basically
B	b-1, b-2, b-3, b-4	Regression line changes as a sector
C	a-1, a-2, a-3, a-4,	Sample point is dispersive, regression line is irregular

Regression curve distributes as a fan of water level factor in the second phase on each quantile point, angle differs more along with quantile point changing. It means influence degree water level having on displacement changes along with quantile point changing. There is one curve of regression equation which is obtained using general regression analysis method, regression relationship described is one-sided and can`t reflect influence degree independent variable having on dependent variable precisely, so, it is necessary to conduct quantile regression analysis for dam monitoring data.

It can get the regression coefficient estimated value changing condition of each independent variable on different quantile point and whether it has passed the significance testing by SAS software. from the results , regression coefficient significance of four water level factors passed the significance testing only on quantile point 0.1 and quantile point 0.2. It means water level factor has little impact on displacement. When observing regression coefficient of water level factor on each quantile point, regression coefficient is positive between quantile point 0.1 and quantile point 0.3, there is positive correlation between displacement and water level; regression coefficient turns into negative number between quantile point 0.4 and quantile point 0.7,there is negative correlation between displacement and water level, regression coefficient turns into positive number again between quantile point 0.8 and quantile point 0.9, there is positive correlation between displacement and water level. It means influence water lever having on displacement changes. Variables x5, x6, x7 and x8, which are the regression coefficients of temperature T_1 , T_2 and aging θ , $\ln\theta$, have passed the significance testing on each quantile point. It means that influence temperature and aging having on arch dam displacement is obvious. In table 4.24-2, all influence factors have passed the significance testing. It means that each influence factor have obvious impacts on dam displacement at this analysis stage. Different from the first analysis stage, water level factor has passed the significance testing on each quantile point and regression coefficients are minus. It means that influence water level factor having on dam displacement is negatively correlative at this phase. In order to make out changing condition of regression coefficient intuitively, this paper shows the regression coefficient variation of temperature T_1 , T_2 and aging θ , $\ln\theta$ on each quantile point at the first analysis phase as well as regression coefficient variation of eight impact factors on each quantile point at the second phase. See Figure 7.
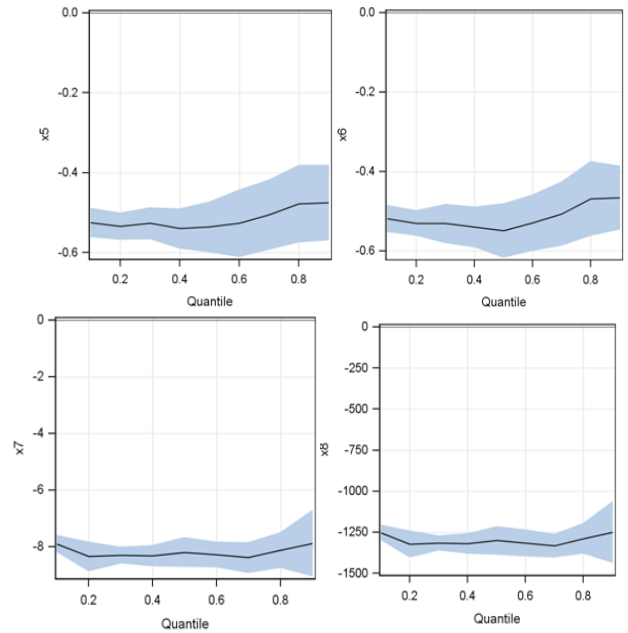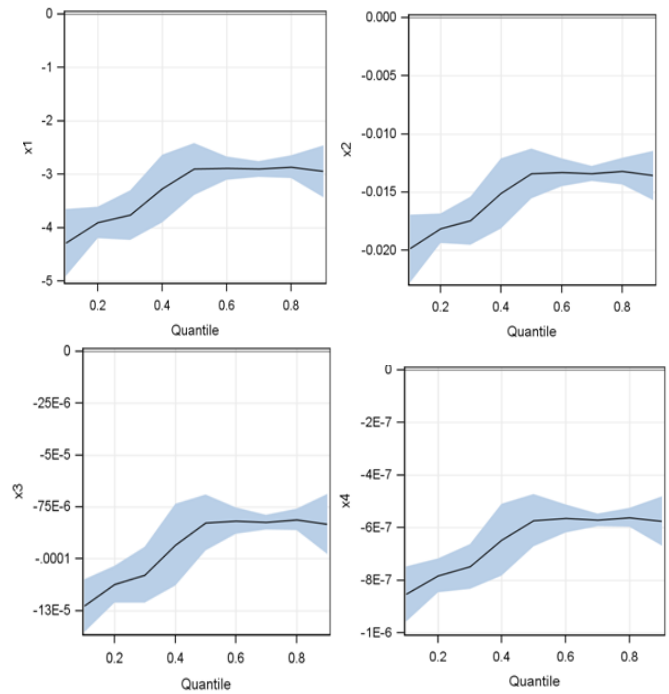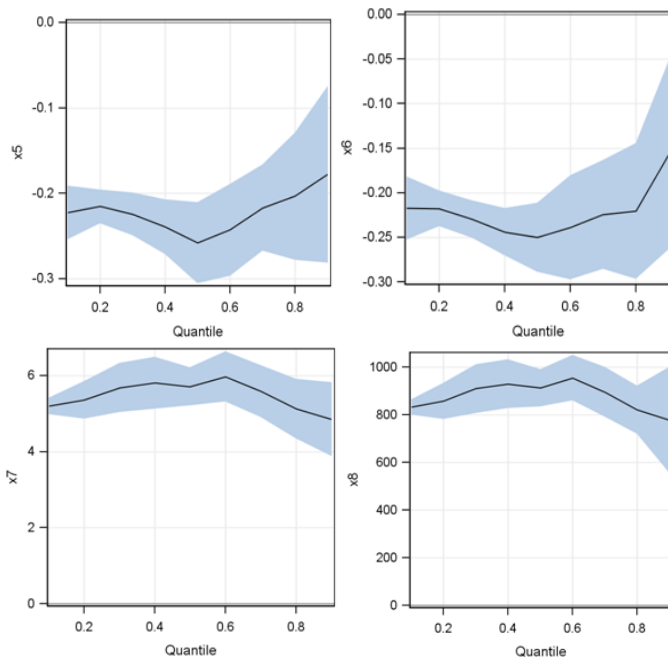

FIGURE VII. (A) REGRESSION COEFFICIENT ESTIMATED VALUE CHANGING GRAPHS OF INDEPENDENT VARIABLE ON DIFFERENT QUANTILE POINTS AT THE FIRST ANALYSIS PHASE

(abscissa represents quantile, ordinate represents regression coefficient of each independent variable)

FIGURE VII. (B) REGRESSION COEFFICIENT ESTIMATED VALUE CHANGING GRAPHS OF INDEPENDENT VARIABLE ON DIFFERENT QUANTILE POINTS AT THE SECOND ANALYSIS PHASE

From Figure 7(a), regression coefficient absolute value of temperature factor x5 decreases firstly and then increases between quantile point 0.2 and quantile point 0.4, then reaches to the maximum on quantile point 0.4.It means temperature factor x5 has the largest impact on arch dam displacement on quantile point 0.4. Absolute value of regression coefficient decreases gradually along with quantile point increasing between quantile point 0.4 and quantile point 0.9.It means impact that temperature factor x5 having on displacement decreases gradually. Regression coefficient absolute value of temperature factor x6 decreases firstly and then increases between quantile point 0.2 and quantile point 0.5,then reaches to the maximum on quantile point 0.5.It means temperature factor x6 has the largest impact on arch dam displacement on quantile point 0.5. Absolute value of regression coefficient decreases gradually along with quantile point increasing between quantile point 0.5 and quantile point 0.9. It means impact temperature factor x6 having on displacement decreases gradually. Temperature factors have different impact on displacement. Regression coefficients of aging factor x7 and x8 have similar version, they both have gone through two fluctuating changes. Absolute value of regression coefficient increases between quantile point 0.1 and quantile point 0.2, decreases between quantile point 0.2 and quantile point 0.5 and then gradually increases between quantile point 0.5 and quantile point 0.7, finally decreases between quantile point 0.7 and quantile point 0.9. Visibly, aging factors have vibrations impact on displacement. Regression coefficients of water level factor x1, x2, x3 and x4 change similarly in Figure 7(b). Absolute value of regression coefficient decreases along with quantile point increasing between quantile point 0.1 and

quantile point 0.5.It means impact water level having on displacement decreases gradually. Absolute value of regression coefficient heads for steady and changes very little between quantile point 0.5 and quantile point 0.9.It means water level factor has little impact on displacement. Regression coefficients of temperature factor x5 and x6 have similar versions. Absolute value of regression coefficient increases gradually between quantile point 0.2 and quantile point 0.5, decreases between quantile point 0.5 and quantile point 0.9, and reaches to the maximum on quantile point 0.5.It means temperature factors have the greatest influence on displacement. Regression coefficient change of two aging factors x7 and x8 is the same, absolute value of regression coefficient increases gradually between quantile point 0.1 and quantile point 0.4, decreases on quantile point 0.5, reaches to the maximum on quantile point 0.6 and decreases gradually between quantile point 0.6 and quantile point 0.9. Comparing Figure. 7(a) and Figure. 7(b), it shows that one impact factor has different impact on dam displacement in different phases. so, in order to analyze dam deformation more precisely and keep dam safety, different phases should be chosen to analyze monitoring data. Confidence interval which belongs to regression coefficient estimated value of each variable changes a lot and is divided into three classes. See in Table 6 for details.

TABLE VI. REGRESSION COEFFICIENT ESTIMATED TABLE OF INDEPENDENT VARIABLE ON DIFFERENT QUANTILE POINTS

sort	Independent variable	Sort basis
A	1-x7, 1-x8	Confidence interval is narrow and changes little
B	1-x5, 1-x6	Confidence interval increases gradually
C	2-x1, 2-x2, 2-x3, 2-x4, 2-x5, 2-x6, 2-x7, 2-x8,	Confidence interval changes a lot

For class-A, confidence interval of aging factor x7 and x8 is relatively narrow and changes little on different quantile points at first analysis phase.It means regression coefficient estimated value of aging factor is precise. For class-B, confidence interval of temperature factor x5 and x6 increases along with quantile point increase at first analysis phase. It means precision of estimated value of regression coefficient decreases gradually. For class-C, confidence interval of each independent variable regression coefficient changes a lot on different quantile points at second analysis phase, some quantile interval is narrow while some quantile interval is wide. Estimated value precision of regression coefficient changes a lot on different quantile points.

III. CONCLUSIONS

This paper combines step-by-step regression analysis and quantile regression analysis method based on POD. Instance analysis on short sequence length of a concrete dam shows that quantile regression based on POD can solve linear dependence problem among independent variables commendably, which can improve model precision and robustness, reveal different independent variables impact degree during complex water engineer transformation condition efficiently, and find out that an independent variable has different impact on dependent variable on different quantile points and at different phases, this is consistent with dam actual safety condition numerical value.

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