Vibration Analysis of Sheet Metal Coupled with Concentrated Masses

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Abstract. There are many applications of sheet metal in Engineering. It is usually reduced to a sheet metal model with concentrated masses. This paper have studied the dynamics of the model, and a mechanical model of sheet metal with concentrated mass is established, the eigenvalues and modes of the model were analyzed. The results show that the concentrated mass has an influence on the natural frequency of the vibration of the sheet metal. The results can provide a theoretical basis for active vibration control of sheet metal.

1. Introduction

There is a large number of applications of the sheet metal with concentrated masses in the machinery, automobile and aviation, etc. its vibration problem is more common, it plays an important role in the application. Therefore, it is necessary to study the vibration problem of the sheet metal. There are few literatures on the free vibration of sheet metal with concentrated masses in china and abroad. The common methods include finite element, boundary element, Fu Liye decay series [1]. But the method is complicated and difficult to calculate.

JH Zhang et al. presented a method for calculating the lateral vibration of thin circular plates with concentrated mass by utilizing the structural circumferential periodicity of the inertia excitation [2]. Lin R M et al. given a new and efficient method to predict the accurate natural frequencies, and the mode shapes of plates considered arbitrary mass, stiffness and damping modifications [3]. Desheng et al. studied the mass effect of the vibration system by taking the clamped thin circular plate as an example, determined the fundamental frequency, natural frequency and mode shape function [4]. In this paper, the vibration problem of sheet metal plate with concentrated mass simply supported is analyzed, and a detailed comparison is made for different situations. The research of this problem has certain theoretical and engineering significance.

1.1 The Natural Frequencies of Sheet Metal

According to the theory of small deflection of thin plate, the differential equation of forced vibration of homogeneous thin plate:

$$D\nabla^4 w\,((x,y,t + \rho\frac{\partial^2 w(x,y,t)}{\partial t^2} = p(x,y,t) \tag{1}$$

In Eq. (1), $\nabla^4 = (\partial^2/\partial x^2 + \partial^2/\partial y^2)^2$ denotes biharmonic operator, E is Young modulus, $D = Eh^3/[12(1-v^2)]$ denotes bending rigidity, h is thickness, v is Poisson's ratio, ρ is weight of unit area, $w(x,y,t)$ is the transverse deformation of the instantaneous t at the (x, y) position, $p(x,y,t)$ is lateral load.

For the free vibration of a sheet metal coupled with concentrated mass, the inertia force of concentrated mass is considered as external excitation force. The general solution to Eq. (1) can be written as

$$p(x_l, y_l, t) = -m_l\frac{\partial^2 w(x_l, y_l, t)}{\partial t^2} = -m_l\sum_{i=1}^{n}\frac{\partial^2 q_i(t)}{\partial t^2}W^i(x_l, y_l) \tag{2}$$

For different boundary conditions, the normal mode of uniform thin plate W^I (x, y) is not the same, this method can be used to solve the vibration problem of thin plates with arbitrary concentrated mass.

2. Dynamic response calculation

The dynamic response of a sheet metal model with concentrated masses is the superposition of the n modes

$$w(x,y,t)=\sum_{r=1}^{n}\overline{w_r}(x,y)q_r(t) \tag{3}$$

where $\overline{w_r}$ as a modal function of sheet metal model with K concentrated masses, and $q_r(t)$ is generalized displacement of instantaneous t.

Consider the forced vibration of a sheet metal model with concentrated masses, take generalized initial displacement $q_r(0)=0$ and generalized initial velocity $\overline{q_r}(0)=0$, $q_r(t)$ can be expressed as;

$$q_r(t)=\frac{1}{\mathrm{m}_r w_r}\cdot\sum_{j=1}^{n}\int_0^t \overline{W}_r(x_j,y_j)F_j(\tau)\sin\overline{w_r}(t-\tau)d\tau \tag{4}$$

Combining the Eq. (3) and Eq. (4), the dynamic response of a simply supported sheet metal with K concentrated masses around n concentrated loads is obtained.

3. Numerical Simulation and Analysis

Simply supported sheet metal with one concentrated masses, The length is 300 mm, the width is 300 mm, the unit area mass ρ is 4.0x10^{3}kg/mm^2, the plate thickness h is 5 mm, Poisson's ratio v is 0.3, Young's modulus E=1.22e11 Pa, concentrated masses m_c=0.5kg. As shown in Fig.1.

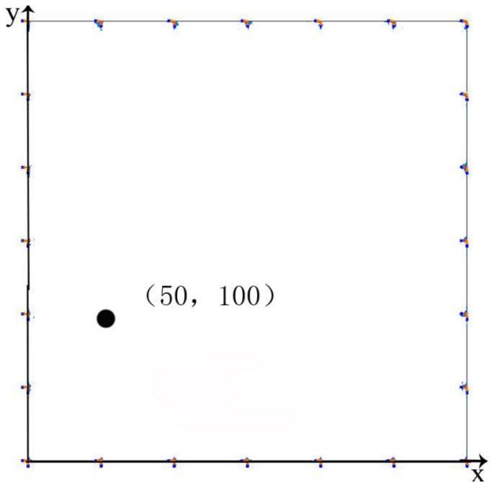

Fig.1 sheet metal with one concentrated masses

3.1 Effect of Position on Vibration Frequency

Considering the influence of the mass m_c= 0.5 kg on the natural frequency of the vibration of the plate at different positions on the plate, and compared with the natural frequency of the plate without the concentrated mass, as shown in Table 1.

Table 1 The first five frequency considering position of mass

Position	Natural frequency /Hz				
	w_1	w_2	w_3	w_4	w_5
(50,100)	52.356	70.589	117.896	154.236	189.568
(100,100)	50.897	71.986	115.869	156.489	187.354
(150,100)	77.569	126.985	160.895	190.865	228.569
(200,100)	49.156	71.986	113.431	153.562	186.721
(250,100)	51.676	71.476	116.362	153.219	188.478
No mass	59.034	78.045	128.578	160.415	190.359

It can be seen from the table that the vibration frequency of the plate with concentrated mass is lower than the natural frequency of the vibration of the plate without concentrated mass, but the natural frequency of the vibration of the concentrated mass plate is also related to the location of the concentrated mass on the plate, and the natural frequency of the vibration of the thin plate is increased in some locations. This study is helpful to the active vibration control of sheet metal.

3.2 Effect of Mass on Vibration Frequency

Considering concentrated mass on the plate fixed position (50,100), the influence of the mass on the vibration frequency of the plate at different times, and the natural frequency of the plate without the concentrated mass are compared, as shown in Table 2.

Table 2 The first five frequency considering magnitude of mass

Position	Natural frequency /Hz				
	w_1	w_2	w_3	w_4	w_5
0.1	58.567	76.569	124.256	158.367	189.964
0.3	55.598	72.986	119.869	156.489	189.869
0.5	52.356	70.589	117.896	154.236	189.568
0.7	50.567	69.549	113.579	152.574	188.548
0.9	49.168	68.583	112.469	151.897	187.875
No mass	59.034	78.045	128.578	160.415	190.359

As can be seen from the table, the natural frequency of the vibration of the plate will decrease with the increase of the mass concentration on the corresponding position.

3.3 Validation Results

The sheet metal with no concentrated mass is considered as a special case, the calculated results are compared with the results of finite element analysis, as shown in Table 3. The calculated value of analytical method is close to the finite element method. Shows that the method is correct.

Table 3. Comparison of analytic natural frequency with FEM

Method	w_1	w_2	w_3	w_4	w_5
analytical method	59.034	78.045	128.578	160.415	190.359
FEM	56.549	74.423	122.329	153.180	181.469
Relative error	4.21%	4.59%	4.86%	4.51%	4.67%

4. Summary

Under the condition of natural frequency and mode shapes of the plate with arbitrary boundary conditions, the natural frequencies and mode shapes of a sheet metal with concentrated mass are studied. And compared with finite element analysis, the results show that the method presented in this paper is correct. This study can provide a theoretical basis for the active vibration control of sheet metal.

Acknowledgments

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References

[1]. Hossain N, Islam M S, Ahshan K H N, et al. Effects on natural frequency of a plate due to distributed and positional concentrated mass[J]. Journal of Vibroengineering, 2015.

[2]. Zhang J H, Li D S. Mobility power flows of thin circular plate carrying concentrated masses based on structural circumferential periodicity [J]. ARCHIVE Proceedings of the Institution of Mechanical Engineers Part C Journal of Mechanical Engineering Science 1989-1996 (vols 203-210), 2015, 27(2):135-50.
[3]. Lin R M, Lim M K. Natural frequencies of plates with arbitrary concentrated mass and stiffness modifications [J]. Computers & Structures, 1995, 57(57):721-729.
[4]. Desheng, Zhang, Junhong, et al. Transverse Vibration Analysis of Concentrated Mass-Loaded Thin Circular Plate Based on Strcutural Circumferential Periodicity [J]. Transactions of Tianjin University, 2015, 21(2):147-152.