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Buckling Bearing Capacity Analysis of Perforated Thin Plates under In-plane Load

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Abstract. In the design process of perforated thin plates, the buckling analysis for guaranteeing larger buckling bearing capacity and determining reasonable hole forms is essential. The nonlinear buckling behavior of the perforated plate under in-plane load is studied in this paper by using the Riks Method and the effects of geometrical parameters such as imperfection shape, hole size and hole location on buckling bearing capacity are considered. The results show that the increscent hole size or hole rotation angle always cause linear decrease of the buckling load. The buckling bearing capacity is inversely proportional to the spacing between hole center and plate center.

1. Introduction

Plates are often used as the main components in engineering practice, openings in such plates may be required. However, the presence of openings leads to change in stress distribution within the variations in buckling characteristics of plates. Many factors affect buckling bearing capacity of plates. Scholars lay emphasis on solving the buckling problem of perforated plates by Finite Element Method (FEM). Rocha et al. [1] studied the buckling behavior of rectangle plate with elliptic, square or diamond hole, optimized the structure according to the minimum principle of imperfection distribution and attained the reasonable scope of opening form. Liu et al. [2] calculated the linear buckling load of perforated plate under axis compression load and analyzed the effect law of hole size and hole location on elastic buckling load of plate. Cheng et al. investigated influences of non-dimensional parameters on compression behaviors of rectangular plates [3]. Prajapat et al. [4] studied the effect of the size, shape and eccentricity of holes on buckling loads of plates. Komur et al. investigated the buckling of perforated square and rectangular plates subjected to in-plane compressive edge loading. The plate aspect ratio, the length and location of edge loading and the diameter of circular hole were taken as the variables that have effect on the behavior of plates [5].

In this paper, nonlinear buckling analysis of square perforated plates is investigated by the Riks method, the effects of hole shape, hole size, hole angle and hole location are received and the effect laws are concluded.

2. The Approximate Analytical Solution

2.1 The Principle of Minimum Potential Energy.

The critical buckling load of the perforated square plate under in-plane load is calculated by using the principle of minimum potential energy. In the process of the plate from plane state into buckling state, the total potential energy Π of plate is

$$\Pi = U - V \tag{1}$$

where U is strain energy in buckling process and V is the work of in-plane loads. Π is the function related to the independent undetermined coefficient C_{mn} (m=1,2,3...i; n=1,2,3…j).

According to the principle of minimum potential energy, it can be defined as

$$\frac{\partial \Pi}{\partial C_{mn}} = 0 \tag{2}$$

When $\partial^2 \Pi / \partial^2 C_{mn} = 0$, Π is minimum. The necessary and sufficient condition of the elastic plate in the stable equilibrium state is that the total potential energy takes the minimum value and the corresponding load is the critical buckling load.

2.2 Buckling Load of the Plate with a Square Hole.

For the plate with a square hole in the center, L and t are the length and thickness of plate, respectively. The translational degrees of freedom perpendicular to plane are constrained in Fig. 1.

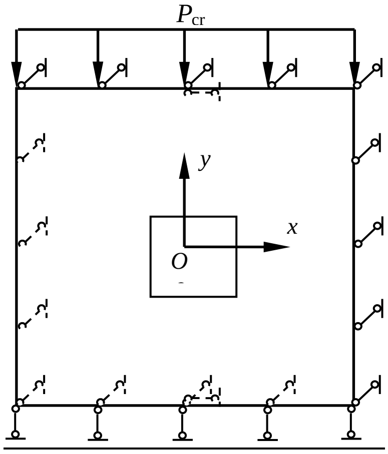


Fig. 1 The load and boundary condition of the square plate

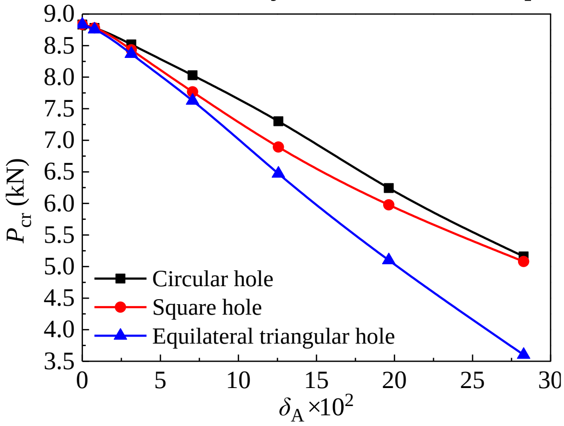


Fig. 2 Relational curves between hole size and nonlinear buckling load

The deflection equation is

$$w = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} C_{mn} \sin \frac{n\pi x}{2L} \sin \frac{m\pi y}{2L} \tag{3}$$

The boundary conditions of the model must satisfy Eq.(3). For the perforated plate, the total potential energy can be written as

$$\Pi(C_{mn}) = U - V = \frac{D}{2} \iint_{A_0} (\nabla^2 w)^2 \mathrm{d}A - \frac{1}{2} \iint_{A_0} P_{cr} \left(\frac{\partial w}{\partial y}\right)^2 \mathrm{d}A \tag{4}$$

where D denotes the bending stiffness of the plate, $D = \dfrac{Et^3}{12(1-\mu^2)}$; E denotes elastic modulus; μ denotes passion ratio; $A_0 = A_2 - A_1$, A_2 is the area of the square plate and A_1 is the area of the hole. Substituting Eq.(3) into Eq.(4), the total potential energy of the plate can be obtained. According to Eq.(2), when m and n are equal to 2 respectively, the critical buckling load of the plate can be written as

$$P_{cr} = = \frac{[1 - (\alpha + \frac{\sin \alpha\pi}{\pi})^2 + \frac{2(1-\mu)\alpha \sin \alpha\pi}{\pi}]}{1 - (\alpha^2 - \frac{\sin^2 \alpha\pi}{\pi^2})} P_{cr0} \tag{5}$$

where P_{cr0} denotes the critical buckling load of the perfect plate, $P_{cr0} = 4D\pi^2 / L^2$; $\alpha = a / L$, a is the side length of the square hole.

3. Comparison of the Solutions of Buckling Load

3.1 Numerical Calculation Model.

The each edge length of square plates is 100 mm and the thickness is 1 mm in the following analysis. The elastic modulus of isotropic elastic-plastic ideal plates is 200GPa and passion ratio is 0.3. The plastic properties of material are presented in Table 1. The translational degrees of freedom along z-direction of four sides, y-direction of bottom side of the plate and x-direction of the center node of top and bottom side are constrained respectively. The uniformly distributed load is applied to the top side in Fig. 1.

Table 1 The plastic properties of the material

Stress/MPa	158.7	163.1	186.3	193.2	202.2	207
Plastic Strain	0	0.015	0.033	0.044	0.062	0.15

3.2 Comparison of Analytical and Numerical Solutions of Buckling Load.

The defect factor δ_A is

δA=A1/A2 (6)

where A_1, A_2 represent perforation area and plate area respectively. The position of the perforation center coincides with the center of the plate. The eigenvalue buckling load of plate with a square hole is calculated by using ABAQUS software. The numerical solution of elastic buckling load P_{cr} is compared with theoretical solution N_{cr} calculated by Eq. (5), which is shown in Table 2.

Table 2 Comparison of theoretic results and numerical results of plate with different defect factors

δA×102	0	0.785	3.141	7.068	12.566	19.634	28.274
Ncr/kN	7.2304	7.0076	6.3874	6.1363	5.5367	4.9892	4.5624
Pcr/kN	7.1845	6.9523	6.4368	6.0894	5.6365	5.4674	5.4308
Error (%)	0.63	0.78	0.77	0.76	1.80	9.57	19.03

For the perforated plate, the buckling load is inversely proportional to the hole size. When the defect factor $\delta_A \times 10^2 < 12.566$, the goodness of fit between the numerical solution and theoretical solution illustrates the rationality and high accuracy of FEM.

4. The Effects of Hole Shape and Size on Buckling Capacity

In this section, the effects of hole shape, hole size, hole angle and location on the buckling load of thin plates under in-plane load are investigated, which consider imperfection, geometrical nonlinear and material nonlinear.

4.1 The Analysis of Buckling Bearing Capacity.

The effect laws of hole shape and size on the nonlinear buckling load of thin plates is concluded. The relational curves between defect factor and buckling load are obtained in Fig. 2.

For the plates with three hole shapes, failure mode is mainly inward bulge. The displacement around hole edge is greater than that of other area. The displacement around hole edge near the loaded side is greatest, which is prone to buckling. The distribution of stresses at the edge of a circle hole is more gentle than the other holes edge. The existence of the sharp angle for the plates with a square hole or triangular hole results in obvious stress concentration, the plates are extremely unstable and prone to buckling.

4.2 The Effects of Hole Angle on Buckling Bearing Capacity.

For the plates with a square hole or triangular hole located in the center of plates, the difference of the hole angle will affect buckling behavior.

(1) The effect of square hole angle on buckling of plates

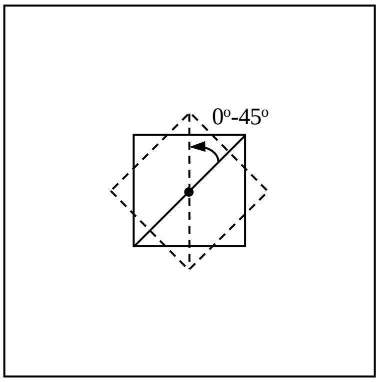


Fig. 3 Rotation angle α of square hole

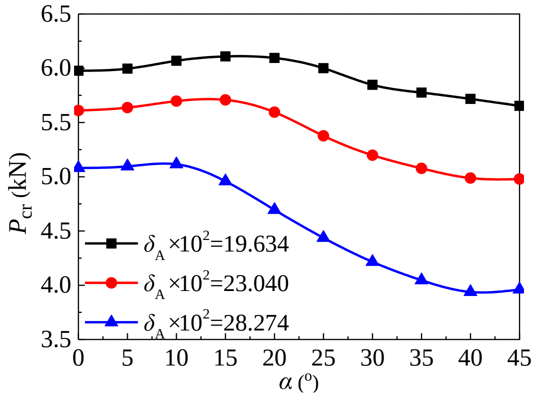


Fig. 4 Relation curves between hole angle and buckling load

When the square hole area is lesser, the effect of hole angle on buckling bearing capacity of plates is not obvious. However, for the bigger hole, it is quite clear. Buckling analysis of the square hole plates with different rotational angles α are investigated in Fig. 3. The relational curves between buckling load and hole angle for different hole sizes are shown in Fig. 4. For the square hole with same area, when α is lesser (α<15°), the change of buckling load with hole angle is not obvious; when α is bigger (15°<α<45°), the buckling load presents a trend of linear decrease with the increase of hole angle. When $\delta_A \times 10^2=28.274$ and the hole rotates counter clockwise from 0° to 45°, the buckling load drops 22.57%.

Buckling bearing capacity is closely related to the cross section stiffness. When the hole rotates, the minimum section size declines obviously and the total section stiffness is impaired. For the plate under less external load, the minimum section closed to external load is prone to buckling.

(2) The effect of equilateral triangle hole angle on buckling of plates

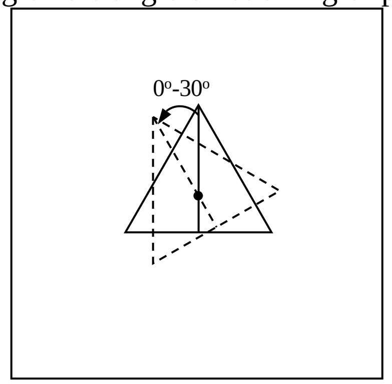


Fig. 5 Rotation angle α of equilateral triangle hole

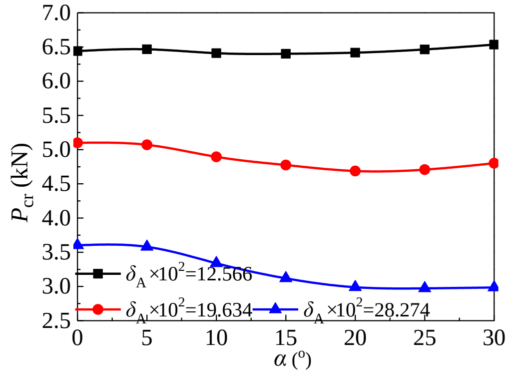


Fig. 6 Relation curves between hole angle and buckling load

Buckling analysis of square plates with different equilateral triangle hole angle shown in Fig. 5 is investigated, $\delta_A\times10^2$ takes 12.566, 19.634 and 28.274 separately. For the equilateral triangular hole with same area, when α is lesser, the change of buckling load with hole angle is not obvious; otherwise, it is clear in Fig. 6. When α is greater ($5^o<\alpha<20^o$), the buckling load of plates presents a trend of linear decrease with the increase of hole angle. When α is greater than 20^o but less than 30^o, the descend range of buckling load is prone to flat. When $\delta_A\times10^2$ =19.634 and the hole rotates counter clockwise from 0^o to 30^o, the buckling load drops 17.46%.

4.3 The Effect of Hole Location on Buckling Bearing Capacity.

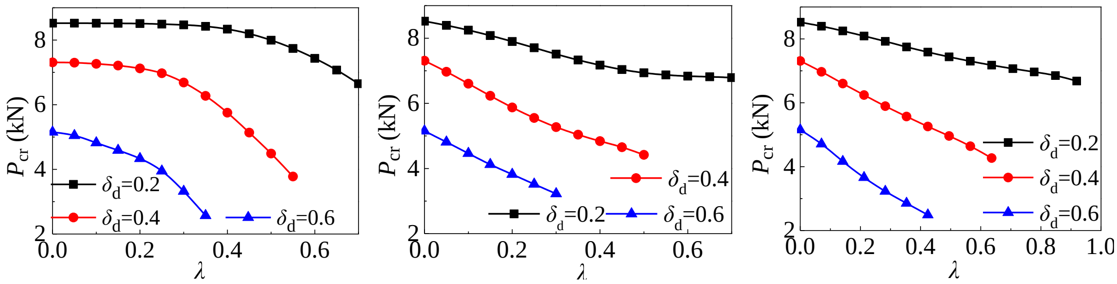
The effects of circular hole location on buckling bearing capacity are investigated in this section. The location of circular hole center moves along *x*-axis, *y*-axis and 45^o-direction respectively in the first quadrant shown in Fig. 1. The dimensionless local geometric imperfection δ_d is

$$\delta d=d/a \tag{7}$$

where *d* denotes circular hole diameter and *a* denotes side length of plate. The hole diameters are 20mm, 40mm, 60mm respectively. The dimensionless hole location λ is

$$\lambda=2l/a \tag{8}$$

where *l* denotes the distance from hole center to plate center.



(a) Move along X axis (b) Move along Y axis (c) Move along 45o-direction

Fig. 7 Comparison of buckling load with the hole in diffenrent positions

For the plate with same hole area, the buckling load presents a decrease trend with the increase of hole location in Fig. 7. When hole center moves along *x*-axis, the relation between buckling load and hole location is cubical; when it moves along *y*-axis or 45^o-direction, the relation is linear. The greater the hole size is, the more obvious the effect of hole location on buckling load is.

When hole size is same, but hole loaction is different, for example δ_d=0.4, the buckling load of the plate drops 8.52% whose hole location changes from 0 to 0.3 and hole center is located in *x*-axis, the buckling load drops 27.89% while hole center is located in *y*-axis. The change along *y*-axis of the hole center has the greatest impact on the buckling load.

5. Conclusions

The effects of hole parameters on the nonlinear buckling load of square perforated plates subjected to uniform compression load in the *y*-direction are studied by FEM. The goodness of fit between numerical solution and theoretical solution of critical buckling load is better. The curves between defect factor and buckling load are obtained, some general conclusions and recommendations are summarized as follows.

(1) For the plates with same hole area and different hole shape, the buckling bearing capacity of plate with a circular hole is greatest. The critical buckling load decreases linearly with the increase of defect factor.

(2) When hole shape is same and hole area is smaller, the effect of hole angle α on the buckling load of plate is not obvious. For the plate with a square hole($15^o<\alpha<45^o$) and the plate with an equilateral triangular hole($5^o<\alpha<20^o$), the buckling load of plate both presents a trend of linear decrease with the increase of hole angle.

(3) For the plates with same circular hole area, the buckling bearing capacity is inversely proportional to the spacing between hole center and structure center. When the center of hole is located in x-axis, the relation between buckling load and hole location is cubical; when it is located in y-axis or 45^o-direction, the relation is linear, and when it is located in y-axis, the change of hole location has the greatest impact on the buckling load.

If the opening on plate structure is inevitable, the opening should be a circular hole and located in the center of the plate as far as possible.

Acknowledgements

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