

# Modified Synchronization Analysis of Complex System

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**Abstract.** This paper studies the modified projective synchronization of complex system, focusing on fractional-order hyper-chaotic nonlinear system. Based on the stability theory of nonlinear system and control method, effective scheme of synchronization for complex system is designed. Numerical simulations results showed that the proposed scheme is effective and feasible.

**Keywords:** Complex system, Synchronization, Nonlinear control

## Introduction

In recent years, many researchers have focused their attention on hyperchaotic systems in many fields [1,2]. Also, Fractional calculus has considered as the generalization of the conventional calculus. It can be seen that the fractional calculus has applications in different fields, such as chaos synchronization, secure communication [3]. Moreover, complex dynamic behaviors including bifurcations and chaos exist in fractional-order systems [4].

Recently, many researchers investigated the dynamics of chaotic systems with complex variables. Modified projective synchronization (MPS) of systems in real space has been investigated are extended to synchronize fractional-order chaotic complex systems [5,6]. Motivated by the above discussion, this paper is to study the fractional-order complex system.

## Synchronization scheme

In this section, an scheme is presented via the idea of nonlinear technique and the fractional-order stability theory to study the MPS of two identical fractional-order complex nonlinear systems. Suppose the derivative order  $q_1 = q_2 = q_3 = q_4 = q$ .

The fractional-order complex drive as follows:

$$D_*^q y = f(y), \quad (1)$$

The response system after coupling is given by:

$$D_*^q z = f(z) + P(z, g), \quad (2)$$

where  $q \in (0,1]$  is the derivative order, nonlinear vector functions  $f$  is continuous and a goal dynamics  $g = My = [m_1 y_1, m_2 y_2, \dots, m_n y_n]^T$ , where  $M = \text{diag}(m_1, m_2, \dots, m_n)$  denotes an arbitrary scaling matrix.

The coupling function is obtained as :

$$P(z, g) = D_*^q g - f(g) + (H - \frac{f}{g})(z - g) \quad , \quad (3)$$

Define the synchronization error vector as  $e = z - g$ , the MPS of drive and response systems is achieved if the following condition is satisfied

$$\lim_{t \rightarrow \infty} \|e\| = \lim_{t \rightarrow \infty} \|z - g\| = 0 \quad , \quad (4)$$

Then the error dynamics is described as:

$$D^q e = He \quad (5)$$

## Simulation results

The drive system

$$\begin{cases} D_*^{q_1} y_1 = a(y_1 - y_2) + y_4 \\ D_*^{q_2} y_2 = by_2 - y_1 y_3 + y_4 \\ D_*^{q_3} y_3 = \frac{1}{2}(\bar{y}_1 y_2 + y_1 \bar{y}_2) - cy_3 \\ D_*^{q_4} y_4 = \frac{1}{2}(\bar{y}_1 y_2 + y_1 \bar{y}_2) - dy_4 \end{cases} \quad (6)$$

and the response system is

$$\begin{cases} D_*^q z_1 = a(z_1 - z_2) + z_4 \\ D_*^q z_2 = bz_2 - z_1 z_3 + z_4 \\ D_*^q z_3 = \frac{1}{2}(\bar{z}_1 z_2 + z_1 \bar{z}_2) - cz_3 \quad , \\ D_*^{q_4} z_4 = \frac{1}{2}(\bar{z}_1 z_2 + z_1 \bar{z}_2) - dz_4 \end{cases} \quad (7)$$

The arbitrary Hurwitz matrix is given by:

$$\begin{cases} D_*^q x'_1 = a(x'_3 - x'_1) + x'_6 + m_1[a(x_3 - x_1) + x_6] - a(m_3 x_3 - m_1 x_1) - m_6 x_6 + (a-1)e_1 + ae_3 - e_6 \\ D_*^q x'_2 = a(x'_4 - x'_2) + x'_6 + m_2[a(x_4 - x_2) + x_6] - a(m_4 x_4 - m_2 x_2) - m_6 x_6 + (a-1)e_2 + ae_4 - e_6 \\ D_*^q x'_3 = bx'_3 - x'_1 x'_5 + x'_6 + m_3[bx_3 - x_1 x_5 + x_6] - bm_3 x_3 - m_1 m_5 x_1 x_5 - m_6 x_6 + m_5 x_3 e_1 - (1+b)e_3 + m_1 x_1 e_5 - e_6 \\ D_*^q x'_4 = bx'_4 - x'_2 x'_5 + x'_6 + m_4[bx_4 - x_2 x_5 + x_6] - bm_4 x_4 - m_2 m_5 x_2 x_5 - m_6 x_6 + m_5 x_3 e_2 - (1+b)e_4 + m_2 x_2 e_5 - e_6 \\ D_*^q x'_5 = x'_1 x'_3 + x'_2 x'_4 - cx'_5 + m_5[x_1 x_3 + x_2 x_4 - cx_5] - m_4 x_1 (m_3 x_3 + e_3) - m_4 x_4 (m_2 x_2 + e_2) - cm_5 x_5 + (c-1)e_5 \\ D_*^q x'_6 = x'_1 x'_3 + x'_2 x'_4 - dx'_6 + m_6[x_1 x_3 + x_2 x_4 - dx_6] - m_1 x_1 (m_3 x_3 + e_3) - m_4 x_4 (x_2 m_2 + e_2) - dm_6 x_6 + (d-1)e_6 \end{cases} \quad (8)$$

The error dynamical system can be rewritten as:

$$\begin{bmatrix} D_*^q e_1 \\ D_*^q e_2 \\ D_*^q e_3 \\ D_*^q e_4 \\ D_*^q e_5 \\ D_*^q e_6 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \\ e_6 \end{bmatrix} \quad (9)$$

The numerical simulations show that the presented method for the system is effective.

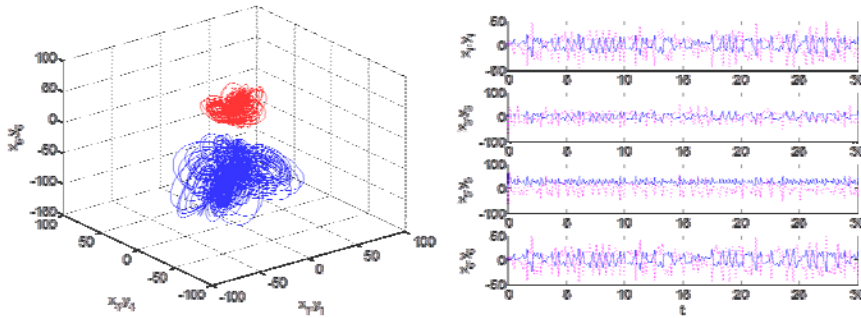


Fig.1. Hyperchaotic attractors and state variables of drive system and response system for scale factors  $m_i = 2(i=1, \dots, 6)$

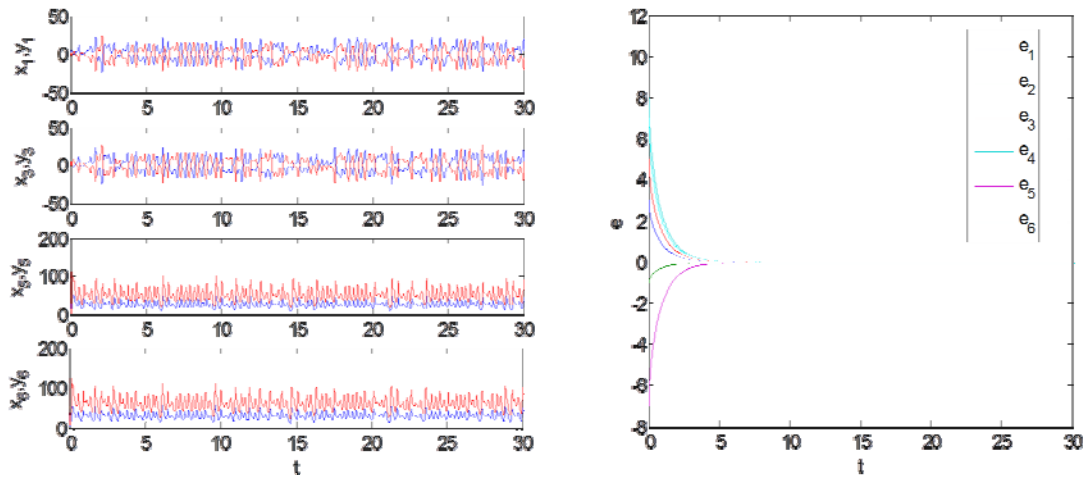


Fig.2. State variables and synchronization errors of drive system and response system for scale factors  $m_1 = -1, m_2 = -1, m_3 = 2, m_4 = 2$

### Conclusions

In this paper, dynamical behavior of complex system is studied. Based on the stability theory of nonlinear system and control method, the scheme of synchronzaiton of the fractional-order hyper-chaotic complex system is designed. Numerical simulations are used to demonstrate the effectiveness and feasibility of the proposed scheme.

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