Type-2 Fuzzy VIKOR Method and Its Application in Facility Location Selection

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Abstract. This study presents a multi-criteria group decision making (MCGDM) approach named interval type-2 fuzzy VIKOR method, which aims to help new enterprises determine facility locations in a more reliable way. Initially, decision makers (DMs) describe the alternative locations within the context of linguistic variables. Moreover, this paper models the values of these variables with interval type-2 fuzzy sets (IT2FSs). Finally, a compromise solution which is the 'closet' one to the ideal solution can be obtained through computing by VIKOR method under IT2FSs environment. A numerical example is given to demonstrate the validity of the presented method.

1. Introduction

Recently, decelerating economic growth in China has been bringing more challenges for enterprises in developing. For new manufacturing enterprises, they are faced with more drastic competitions. As the key step at the beginning, the determination of facility location is very important that it can directly influence late development of the enterprise. Therefore, to survive in the fierce competition, an enterprise must choose the most suitable facility location for further operation and development.

Facility location selection (FLS) is a multi-criteria group decision making (MCGDM) problem. Researchers have done some new work in relative areas in recent several years. Kahraman, Shuo-Yan and Shen, Chun-Ying et al. [1-3] proposed type-1 fuzzy methods to evaluate and select facility locations. Lin et al. [4] presented an approach on facility location selection based on the generalized interval 2-tuple linguistic Shapley chi-square averaging operator. Wang et al. [5] solved the FLS problem by using fuzzy-connective-based aggregation networks. Kannan et al. [6] aimed at prioritizing alternative potential locations for manufacturing firms with respect to economic, environmental, and societal dimensions.

However, as the FLS is very important, it is necessary to pay more attention to choosing the right location to keep the enterprise competitive. The process of solving FLS contains two stages: decision makers collect descriptions of potential locations from different dimensions the first stage; model and compute this information and then get the proposed location in the second stage. Compared with real numbers, the linguistic variables have more freedom in the aspect of semantic expression. IT2FSs can cover more uncertain factors than type-1 fuzzy sets (T1FSs) in modeling linguistic variables. The proposed location given by above methods are optimal solutions. But, sometimes, they might not be the most suitable location for the development of the enterprise. The potential locations have advantages in different aspect, they don't have the uniform object to compare with. Thus, motivated by these, this study aims to propose an interval type-2 fuzzy VIKOR method which concerns on finding the compromise solution that is the 'closet' one to the ideal solution. Hence, it can be sure that the final location determined is the most suitable location.

The rest of paper is organized as follows. Section 2 reviews some basic concepts and definitions of IT2FSs and the VIKOR decision method. Section 3 proposes the VIKOR method under interval type-2 fuzzy environment. Section 4 includes a simple numerical example to verify the proposed method. Finally, conducts conclusions from study in section 5.

2. Preliminaries

2.1 The concepts of T2FSs and IT2FSs

Since originally proposed by Zadeh [7] in 1975, the type-2 fuzzy sets (T2FSs) were mainly developed by Mendel and al. The T2FSs are characterized by two membership functions that are primary membership function (PMF) and the second membership function (SMF).

Definition [8]. In the universe of discourse X, a T2FSs $\widetilde{\widetilde{A}}$ can be characterized by $\mu_{\widetilde{\widetilde{A}}}$, the type-2 membership function , expressed as follows:

$$\widetilde{\widetilde{A}} = \left\{ \left((x, u), \mu_{\widetilde{\widetilde{A}}}(x, u) \right) \middle| \forall x \in X, \forall u \in [0,1] \right\}, \tag{1}$$

where $0 \leqq \mu_{\widetilde{\widetilde{A}}}(x, u) \leqq 1$ for any admissible x and u.

In Eq.(1), when $\mu_{\widetilde{\widetilde{A}}}(x, u) = 1$, $\widetilde{\widetilde{A}}$ is an IT2FSs which can be regarded as a special case of T2FSs. For simplifing the computation, we take trapezoidal interval type-2 fuzzy sets (TrIT2FSs) $\widetilde{\widetilde{A}}$ to represent IT2FSs.

$$\widetilde{\widetilde{A}} = \left(\widetilde{A}^+, \widetilde{A}^- \right) = \left(\left(a_1^+, a_2^+, a_3^+, a_4^+; h_{\widetilde{\widetilde{A}}}^+ \right), \left(a_1^-, a_2^-, a_3^-, a_4^-; h_{\widetilde{\widetilde{A}}}^- \right) \right),$$ shown in Fig 1.
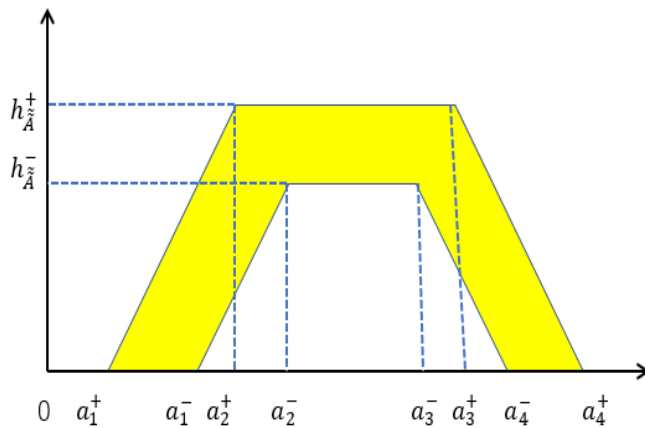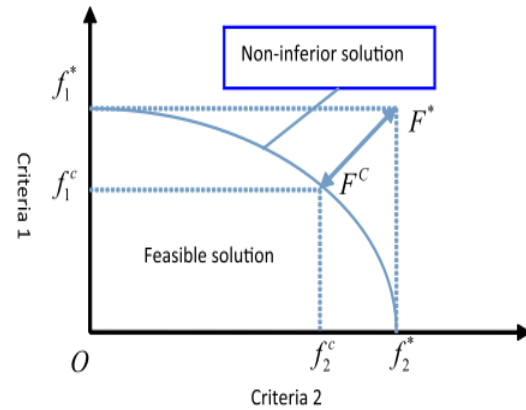

Fig. 1 A trapezoidal interval type-2 fuzzy set   Fig. 2 The compromise solution of VIKOR.

Definition 2 [9]. Let $\widetilde{\widetilde{A}} = \left(\widetilde{A}^+, \widetilde{A}^- \right)$ and $\widetilde{\widetilde{B}} = \left(\widetilde{B}^+, \widetilde{B}^- \right)$ be two TrIT2FSs, the arithmetic operations are defined as follows:

① Addition operation

$$\widetilde{\widetilde{A}} \oplus \widetilde{\widetilde{B}} = \left(\widetilde{A}^+, \widetilde{A}^- \right) \oplus \left(\widetilde{B}^+, \widetilde{B}^- \right)$$
$$= \left(\left(a_1^+ + b_1^+, a_2^+ + b_2^+, a_3^+ + b_3^+, a_4^+ + b_4^+; \min\left(h_{\widetilde{\widetilde{A}}}^+, h_{\widetilde{\widetilde{B}}}^+ \right) \right), \right.$$
$$\left. \left(a_1^- + b_1^-, a_2^- + b_2^-, a_3^- + b_3^-, a_4^- + b_4^-; \min\left(h_{\widetilde{\widetilde{A}}}^-, h_{\widetilde{\widetilde{B}}}^- \right) \right) \right). \tag{2}$$

② Multiplication by real number operation

$$k\widetilde{\widetilde{A}} = \begin{pmatrix} \left(ka_1^+, ka_2^+, ka_3^+, ka_4^+; \min\left(h_{\widetilde{\widetilde{A}}}^+, h_{\widetilde{\widetilde{B}}}^+ \right) \right), \\ \left(ka_1^-, ka_2^-, ka_3^-, ka_4^-; \min\left(h_{\widetilde{\widetilde{A}}}^-, h_{\widetilde{\widetilde{B}}}^- \right) \right) \end{pmatrix}. \tag{3}$$

Definition 3 [10]. Distance between $\widetilde{\widetilde{A}}$ and $\widetilde{\widetilde{B}}$

$$R_d\left(\widetilde{\widetilde{A}}, \widetilde{\widetilde{1}} \right) = \frac{1}{2h_{\widetilde{\widetilde{A}}}^- h_{\widetilde{\widetilde{A}}}^+} h_{\widetilde{\widetilde{A}}}^- (a_4^+ - a_3^+ - a_4^- + a_3^-)$$

$$- \frac{1}{2h_{\widetilde{\widetilde{A}}}^- h_{\widetilde{\widetilde{A}}}^+} h_{\widetilde{\widetilde{A}}}^+ (0.5(a_2^- - a_1^- - a_2^+ + a_1^+) - (a_4^- - a_3^- - a_2^- + a_1^-)) \tag{4}$$

$$+ 1 - a_4^- - 0.5(a_1^- - a_1^+ + a_4^+ - a_4^-).$$

Where $\widetilde{\widetilde{1}} = ((1,1,1,1; 1), (1,1,1,1; 1))$.

$$R_d\left(\widetilde{\widetilde{A}}, \widetilde{\widetilde{B}} \right) = \left| R_d\left(\widetilde{\widetilde{A}}, \widetilde{\widetilde{1}} \right) - R_d\left(\widetilde{\widetilde{B}}, \widetilde{\widetilde{1}} \right) \right|. \tag{5}$$

Definition 4. [11] Ranking value of $\widetilde{\widetilde{A}}$

$$\text{DTraIT}\left(\widetilde{\widetilde{A}}\right) = \frac{1}{8}(a_4^+ - a_1^+) + \left(h_{\widetilde{\widetilde{A}}}^+ \cdot a_2^+ - a_1^+\right) + \left(h_{\widetilde{\widetilde{A}}}^+ \cdot a_3^+ - a_1^+\right)$$
$$+ \frac{1}{8}(a_4^- - a_1^-) + (h_{\widetilde{\widetilde{A}}}^- \cdot a_2^- - a_1^-) + (h_{\widetilde{\widetilde{A}}}^- \cdot a_3^- - a_1^-) + \frac{1}{2}(a_1^+ + a_1^-). \tag{6}$$

2.2 The VIKOR method

The VIKOR method was proposed by Opricovic [12] and is considered as an efficient tool in finding the compromise solutions when solving MCGDM problems in which a set of conflicting criteria included. Fig. 2 shows that the compromise solution F_c is a feasible solution that is the "closest" one to the ideal solution.

3. The proposed fuzzy VIKOR method

3.1 The application of linguistic variables

Compared with real numbers, words have more space to depict the emotional and semantic expression. This study chooses TrIT2FSs which contains ten parameters to model linguistic variables. Table 1 shows the interval typer-2 fuzzy scales of linguistic variables.

Table 1. Linguistic variables and interval typer-2 fuzzy scales

Linguistic variables	Trapezoidal interval type-2 fuzzy scales
Very Poor (VP)	((0,0,0,0.1;1),(0,0,0,0.5;0.9))
Poor (P)	((0, 0.1, 0.2,0.3;1),(0.05, 0.1, 0.15,0.2;0.9))
Medium Poor (MP)	((0.1, 0.3, 0.4,0.5;1),(0.2, 0.3, 0.35,0.4;0.9))
Medium (M)	((0.3, 0.5, 0.6,0.7;1),(0.4, 0.5, 0.55,0.6;0.9))
Medium Good (MG)	((0.5, 0.7, 0.8,0.9;1),(0.6, 0.7, 0.75,0.8;0.9))
Good (G)	((0.7, 0.9, 0.95,1;1),(0.8, 0.85, 0.9,0.95;0.9))
Very Good (VG)	((0.9.1.1.1;1),(0.95,1,1,1;0.9))

3.2 The fuzzy VIKOR method

Considering the decision makers E_s $(s = 1,2,\dots,m)$ with respect to weight w_S and the criterion C_i $(i = 1,2,\dots,N)$ with respect to weight w_i, this study aims to select the most suitable facility location with the proposed method. The key steps of the fuzzy VIKOR method is shown as follows.

Step 1. Construct and aggregate the decision matrix

For each alternative location A_k $(k = 1,2,\dots,n)$, each decision maker E_s give their evaluation with linguistic variables in all criteria dimensions. The fuzzy decision matrix $\widetilde{\widetilde{D}}_{E_s}$ constructed by decision makers can be expressed as Table 2:

Table 2. The fuzzy decision matrix

Criteria	$\widetilde{\widetilde{D}}_{E_1}$			$\widetilde{\widetilde{D}}_{E_2}$			$\cdots$	$\widetilde{\widetilde{D}}_{E_m}$		
	A_1	$\cdots$	A_n	A_1	$\cdots$	A_n	$\cdots$	A_1	$\cdots$	A_n
C_1	$\widetilde{\widetilde{d}}_1^{11}$	$\cdots$	$\widetilde{\widetilde{d}}_1^{n1}$	$\widetilde{\widetilde{d}}_1^{12}$	$\cdots$	$\widetilde{\widetilde{d}}_1^{n2}$	$\cdots$	$\widetilde{\widetilde{d}}_1^{1m}$	$\cdots$	$\widetilde{\widetilde{d}}_1^{nm}$
C_2	$\widetilde{\widetilde{d}}_2^{11}$	$\cdots$	$\widetilde{\widetilde{d}}_2^{n1}$	$\widetilde{\widetilde{d}}_2^{12}$	$\cdots$	$\widetilde{\widetilde{d}}_2^{n2}$	$\cdots$	$\widetilde{\widetilde{d}}_2^{1m}$	$\cdots$	$\widetilde{\widetilde{d}}_2^{nm}$
$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$
C_N	$\widetilde{\widetilde{d}}_N^{11}$	$\cdots$	$\widetilde{\widetilde{d}}_N^{n1}$	$\widetilde{\widetilde{d}}_N^{12}$	$\cdots$	$\widetilde{\widetilde{d}}_N^{n2}$	$\cdots$	$\widetilde{\widetilde{d}}_N^{1m}$	$\cdots$	$\widetilde{\widetilde{d}}_N^{nm}$

Each $\widetilde{\widetilde{d}}_i^{ks}$ in the fuzzy decision matrix $\widetilde{\widetilde{D}}_{E_s}$ is an IT2FS. Before the next step, fuzzy decision matrix constructed by different DMs need to be aggregated. The weighted averages method is used in this process. In the aggregated fuzzy decision matrix, $\widetilde{\widetilde{d}}_i^k = \sum_{s=1}^{m}(w_s * \widetilde{\widetilde{d}}_i^{ks})$.

Step 2. Select the best value and the worst value of each criterion

With respect to each criterion C_i, we choose one of the $\widetilde{\widetilde{d}}_i^k$ $(k = 1,2,\dots,n)$ whose ranking value is the biggest as the best value $\widetilde{\widetilde{f}}_i^*$ and one of the $\widetilde{\widetilde{d}}_i^k$ $(k = 1,2,\dots,n)$ whose ranking value is the smallest as the worst value $\widetilde{\widetilde{f}}_i^-$.

Step 3. Calculate the values of S_k, R_k and Q_k with the following relations:

$$S_k = \sum_{i=1}^{N}(w_i * d\left(\tilde{\tilde{f}}_i^*, \tilde{\tilde{d}}_i^k\right)/d\left(\tilde{\tilde{f}}_i^*, \tilde{\tilde{f}}_i^-\right)), \tag{7}$$

$$R_k = \max_i\left\{w_i * d\left(\tilde{\tilde{f}}_i^*, \tilde{\tilde{d}}_i^k\right)/d\left(\tilde{\tilde{f}}_i^*, \tilde{\tilde{f}}_i^-\right)\right\}, \tag{8}$$

$$Q_k = \lambda\frac{S_k - S_k^-}{S_k^* - S_k^-} + (1-\lambda)\frac{R_k - R_k^-}{R_k^* - R_k^-}. \tag{9}$$

In Eq. (9), $S_k^* = \max_k\{S_k\}$, $S_k^- = \min_k\{S_k\}$, $R_k^* = \max_k\{R_k\}$, $R_k^- = \min_k\{R_k\}$ and the parameter λ means the weight of the strategy of the maximum group utility. Considering the generality in using, assume that the value of λ is set to 0.5.

Step 4. Propose the alternative location

Rank the Q_k, S_k and R_k respectively from lower to higher. Before the alternative supplier A^1 with first position in the Q ranking proposed as the compromise solution, there are two conditions to satisfy.

Condition 1. Acceptable advantage:

$$Q(A^2) - Q(A^1) \geq \frac{1}{n-1} \tag{10}$$

where A^2 means the alternative location with the second position in the Q ranking list, and n is the number of alternative suppliers.

Condition 2. Acceptable stability:

The alternative A^1 must also be the best in the S and R ranking list. If either condition is not satisfied, then go to the additional step to get the compromises solution:

When Condition 2 is not satisfied, then both A^1 and A^2 are compromise solutions.

When Condition 1 is not satisfied, the maximus value of N need to be explored with following relationship:

$$Q(A^N) - Q(A^1) < \frac{1}{n-1}. \tag{11}$$

Step 5. End.

4. Numerical example

In this section, we will identify the proposed method step by step.

The location must be located where maximum distribution and the minimum cost of production can be obtained. And then, other factors are also important such as room for expansion and safe living conditions for plant operation as well as the surrounding community. There are various criteria that must be considered while selecting a suitable location for an enterprise, but here, we have considered the following six attributes studied in [13], shown in Table 3.

Table 3 The criteria of FLS

Criteria	C_1	C_2	C_3	C_4	C_5	C_6
	Skilled workers	Transport facilities	Investment cost	Material resources	Climate	Expansion possibility

Assume that the decision-making group contains three decision makers E_1, E_2 and E_3 with the value of weight 0.5, 0.3 and 0.2, respectively. And the weight of C_i (i = 1,2, ... ,6) is 0.2, 0.15, 0.25, 0.18, 0.1 and 0.12, respectively. They need to choose one location as the facility location from five potential alternative locations A_k (k = 1,2, ... ,5).

Step 1. Three decision makers construct their own fuzzy decision matrix with rich experience in relative areas. Table 4 shows the decision matrix in detail.

Table 4. The fuzzy decision matrix

C	$\widetilde{\widetilde{D}}_{E_1}$					$\widetilde{\widetilde{D}}_{E_2}$					$\widetilde{\widetilde{D}}_{E_3}$				
	A_1	A_2	A_3	A_4	A_5	A_1	A_2	A_3	A_4	A_5	A_1	A_2	A_3	A_4	A_5
C_1	M	MG	G	P	M	M	M	MG	MP	MG	M	G	VG	M	M
C_2	MP	G	MG	M	MG	MP	P	G	M	G	M	G	MG	MP	MG
C_3	M	P	G	MP	M	G	M	MP	G	MP	MG	M	MP	G	MP
C_4	MG	M	M	G	MP	MP	G	MG	M	M	MG	P	G	P	M
C_5	G	MP	P	MG	M	VP	G	M	P	MP	VP	M	P	M	MP
C_6	P	G	M	M	MP	G	MP	MG	MP	MG	MP	MG	M	MG	P

Step 2. Defuzzy the aggregated decision matrix by Eq. (6), we can obtain the best value and the worst value though comparison among them, shown in Table 5.

Table 5. The best value and the worst value

Criteria	The best value $\tilde{\tilde{f}}_i^*$	The worst value $\tilde{\tilde{f}}_i^-$
C_1	((0.6, 0.8, 0.88, 0.95; 1),(0.7, 0.78, 0.82, 0.88; 0.9)	((0.05, 0.2, 0.3, 0.4;1),(0.12, 0.2, 0.25, 0.3;0.9)
C_2	((0.6, 0.8, 0.88, 0.95; 1),(0.7, 0.78, 0.82, 0.88; 0.9))	((0.1, 0.3, 0.4, 0.5;1),(0.2, 0.3, 0.35, 0.4;0.9))
C_3	((0.5, 0.7, 0.78, 0.85; 1),(0.6, 0.68, 0.72, 0.78; 0.9))	((0.15, 0.3, 0.4, 0.5;1),(0.22, 0.3, 0.35, 0.4;0.9))
C_4	((0.5, 0.7, 0.8, 0.9; 1),(0.6, 0.7, 0.75, 0.8; 0.9))	((0.4, 0.6, 0.7, 0.8; 1),(0.5, 0.6, 0.65, 0.7;0.9))
C_5	((0.4, 0.6, 0.68, 0.75; 1),(0.5, 0.58, 0.62, 0.68; 0.9))	((0.15, 0.3, 0.4, 0.5;1),(0.22, 0.3, 0.35, 0.4;0.9))
C_6	((0.4, 0.6, 0.7, 0.8; 1),(0.5, 0.6, 0.65, 0.7; 0.9))	((0.2, 0.4, 0.5, 0.6;1),(0.3, 0.4, 0.45, 0.5;0.9))

Step 3. Calculate the values of S_k, R_k and Q_k by using Eqs. (7-9), values are presented in Table 6:

Table 6 Values of S_k, R_k and Q_k

Alternative locations	S_k	R_k	Q_k
A_1	0.78	0.36	0.68
A_2	0.5	0.25	0.25
A_3	0.35	0.18	0
A_4	0.6	0.2	0.28
A_5	0.85	0.54	1

Step 4. Propose the compromise solution

Rank the alternative locations, sorting by the values S_k, R_k and Q_k in ascending order. The results are three ranking lists as follows: $S_3 < S_2 < S_4 < S_1 < S_5$; $R_3 < R_4 < R_2 < R_1 < R_5$; $Q_3 < Q_2 < Q_4 < Q_1 < Q_5$. $Q(A^2) - Q(A^1) = 0.25 \geq \frac{1}{5-1}$. It is apparently that proposing A_3 as the compromise solution satisfies two conditions.

Step 5. End.

5. Conclusions

This study introduced an interval type-2 fuzzy VIKOR method which aims to find the most suitable facility location for enterprises. The linguistic variables can depict the semantic expressions in a better way. Furthermore, model the linguistic variables with IT2FSs can capture more uncertain factors in decision making. The compromise solution given by this method is the 'closet' one to the ideal solution. And, the numerical example identified the validity of the proposed method. The result shows this method is efficient and can be used in solving MCGDM problems.

Acknowledgments

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