# Optimizing the design of highway toll plaza 

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#### Abstract

We consider the problem how to optimize the design of highway toll plaza when determined the number of the tollbooths .In order to reasonably arrange the size and shape of the toll plaza on the high-speed road, including fan-in area and fan-out area as we considered,tosatisfy certain throughput (number of vehicles per hour passing the point where the end of the plaza joins the outgoing traffic lanes), and consider the cost and security to achieve the optimal value.By applying the Cellular automata theory we simulate the lane changing phenomenon in the process of car driving, which is a practical social appearance, to meet the need of security and extended this conclusion to maintain the size of toll plaza.


## Introduction

As we all know, the unreasonable design of toll station may cause chaos when the abundant number of cars come into it, which not only reduce the efficiency of it but also hassomepotential safety hazard. Thus, a key way is how to design the toll plaza rather than change the tollbooths to efficiency improve the unsuitable express waybarrier charging. In this paper, we established our model based on the existing situation when the number of tollbooths are determined to meet the optimal design of toll plaza. Therefore, the maximum throughput can be determined and considering about the security based on the drivers' lane changing desire to obtain the optimal design of the toll plaza,in order to take different situation into consideration we divide this model into two parts including fan-out area and fan-in area, and we also use the computer simulation method to simulate the situation in highway.

## Fan-out model

We establish our model based on the desire of lane change.
In the toll lane, the driver usually change to their most desirable lane, there are two factors that affect the chances for the driver to change lanes.

## Subjective lane change desire

Drivers desire to change lanes subjective, we set desire of the drivers driving from the $i$-th toll station to the $j$-th lane is

$$
w_{i j}(i \in(1, L), j \in(1, M))
$$

## The length of the queue

The distance differs from the $i$-th lane exit to the $j$-th toll station. The closer the distance, the more likely drivers are willing to go to the toll station. In the case where the distance from the running lane to the toll station in the vertical direction is uncertain, we can divide the importance by the distance between each lane and each toll station approximately according to the symmetry.

We assume that there are three driving lanes and eight toll stations, we simply mark the traffic lane from left to right as A, B, C, and toll stations by the forward direction of the same are marked as $1-8$. The distance from B to the 1-th, 8 -th two toll stations is the farthest, while from B to two toll lanes, 4 -th, 5 -th is of the nearest distance. The probability of changing the lane probability from the $i$-th lane to the $j$-th toll station forms a probability matrix $w_{i j}$.

Table 1: The probability of changing the road

|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| A | 0.111 | 0.148 | 0.185 | 0.158 | 0.148 | 0.111 | 0.074 | 0.037 |
| B | 0.05 | 0.1 | 0.15 | 0.2 | 0.2 | 0.15 | 0.1 | 0.05 |
| C | 0.037 | 0.074 | 0.11 | 0.148 | 0.185 | 0.185 | 0.148 | 0.111 |

To sum up, the driver changes from the $i$-th lane to the $j$-th toll station with the probability

$$
P_{i j}=w_{i j} /\left(l_{j}+1\right)
$$

In the above formula, when the queue length of $l_{i}$ is 0 , the queue length has no negative influence on the driver's decision-making upon whether to change to the $j$-th lane. However, the aspiration of changing the lane depends only on the subjective desire of the driver.

## Security guarantee

The shape of the fan-in area is subject to the safety of the vehicle traveling in this area, the number of accidents is

$$
s=\beta M\left(P_{a_{1}, b_{1}, c} \times P_{a_{2}, b_{2}, c}\right)
$$

At time $C$, both cars enter the fan-shaped area from the driving lane at the meanwhile. If $a_{1}<a_{2}$ and $b_{1}>b_{2}$, then the two vehicles cross each other at a certain point, whereas the crossing does not necessarily lead to a traffic accident, thus we introduce a cross- probability of accident $\beta$. $P_{a, b, c}$ stands for the probability that at $t$ moment cars will travel from the $a$-th lane to the $b$-th toll station. The total traffic flow multiplied by the total probability of occurrence of cross, multiplied by the probability of occurrence of traffic accidents, is the total frequency $S$ of traffic accidents occurred.

## Determination of queue length

The length of the vehicle added per unit of time minus the length of the vehicle passing through the toll station per unit of time, i.e. the length of the queue. Since the number of toll stations is determined, the throughput of the fan-out area is also determined. Therefore, as the number of vehicles reaches the threshold, the queue length of each toll lane will eventually stabilize over time.

## Fan-in model

Model establishment based on Cellular automata. Based on Cellular automata model[4], in our model, the car enters the lane from the toll plaza. From safety considerations, each car has a lane change probability, can be obtained in a certain throughput to accommodate the case of the square shape and minimum area, the establishment of the square size, throughput and safety as a function of the relationship, can be quantified as a relationship between the seven variables.

Sets the function

$$
x_{i j}^{t}\left\{n_{i}^{t}, s_{i, j}^{t}, v_{i, j}^{t}, a_{i, j}^{t}, d_{i, j}^{t}, \widehat{d}_{i, j}^{t}, p_{i, j}^{t}\right\}
$$

Where,


Figure 1: Fan-out model flow chart
$x_{i j}^{t}$ represents seven variables, the first subscript is the vehicle, the second is the lane, and the superscript is the time. Which is defined as the function of the $i$-th vehicle in the $j$-th lane at time t .

The schematic diagram of the model is as follows:


Figure 2: Schematic diagram of Cellular automata
At the moment $t$, when the car is on the road $n_{i}$, its speed is $v_{i, j}$, while the acceleration is $a_{i, j}$. The probability for vehicles to change to the adjacent lane is $p_{i, j}$. Besides, the distance away from the toll station is $s_{i j}$, while the distance to the right front is $d_{i, j}$, and the distance between the left front vehicle is $\hat{d}_{i, j-1}$. Similarly, the distance between the right front vehicle is $\hat{d}_{i, j+1}$. Furthermore, the lane changing desire $p_{i, j}$ determines the location $n_{i}$ of the car.

The change in acceleration is related to the distance of the front vehicle

$$
a_{i, j}^{t+1}=\mu \frac{d_{i, j}^{t}-d_{v}}{d_{v}}
$$

Where $d_{v}$ is the safety distance of the vehicle and $\mu$ is the safety factor. The acceleration is inversely proportional to the distance from the safety distance, the smaller the distance safety distance, the smaller the acceleration.

Define safety distance

$$
d_{v}=1.1 v_{i, j}^{t}+0.054\left(v_{i, j}^{t}\right)^{2}
$$

Which is a function of speed, the specific formula available empirical formula.
Distance change

$$
\begin{gathered}
d_{i j}^{t}=s_{i, j}^{t}-s_{i+1, j}^{t} \\
\hat{d}_{i, j}^{t}=s_{i, j \pm 1}^{t}-s_{i+1, j}^{t}
\end{gathered}
$$

Represents the distance between the $i$-th vehicle and the $(i+1)$-th vehicle at timet, including the forward direction and the forward direction.

In the unit time, the position of the car changes

$$
s_{i, j}^{t+1}=s_{i, j}^{t}+v_{i, j}^{t} d t
$$

In the unit time, the speed of the vehicle changes

$$
v_{i, j}^{t+1}=v_{i, j}^{t}+a_{i, j}^{t} d t
$$

When we establish the function of lane change, we assume that lane change is only related to the distance from the vehicle ahead.

$$
p_{i, j}^{t}=\frac{\alpha d_{i, j}^{t}}{\alpha d_{i, j}^{t}+\beta \hat{d}_{i, j}^{t}}
$$

$d_{i, j}^{t}$ is the distance from the front of the vehicle, $\hat{d}_{i, j}^{t}$ is the distance to the vehicle in front of the around vehicle, and the probability of the constant path in the course of travel is higher. The distance between the front and the periphery is weighted $\alpha$ and $\beta$.

## Conclusion

Cellular automata system are important to optimize the design of toll plaza. In this paper, we establish the model in toll plaza in lane changing desire to simulate the true situation, to optimize the design of highway toll plaza when determined the number of the tollbooths.


Figure 3: Schematic diagram of Cellular automata

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