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Option Pricing Analysis Based on Financial Mathematics Technology

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Abstract. With the rapid development of economy and society, financial industry in China has made considerable progress in recent years. The importance of financial options has also gradually been known to people. But according to the author's global survey, it is found that option pricing has always been a challenge for the development of financial industry in China. For this end, an option pricing analysis is conducted based on financial mathematics technology in this paper, in the hope of inspiring better development of financial market in China.

Introduction

Since the Nobel Prize in Economics was awarded to Robert Merton, to encourage his contribution to the field of financial mathematics in 1997, financial mathematics technology has developed rapidly in the last 2 decades. In this development, countless studies are reported on option pricing based on financial mathematics technology. To enable more people to gain a deep insight into option pricing under financial mathematics technology is also the reason why the author conducts the present study.

Overview of Option Pricing

To probe into option pricing based on financial mathematics technology, first we need to gain a deep insight into option pricing. Combined with related literature, it is not hard to find that options refer to the power to buy or sell in a given period in the future. They are a kind of derivative contracts, while option pricing refers to returns obtained by option holders by exercising rights, instead of buying shares directly.

For option pricing, it is necessary for us to understand three kinds of traditional option pricing models, i.e., partial differential equation model, martingale method model and binary tree model. Tab. 1 shows three models visually, from which we can see that none of the three traditional pricing models consider interest arbitrage. This deficiency shall arouse our attention [1].

Tab. 1 Three Traditional Option Pricing Models

Option Pricing Model	Characteristic	Deficiency
Partial Differential Equation Model	Based on the solution of partial differential equations;	Assuming that the financial market is an arbitrage-free, balanced and complete market;
Martingale Method Model	Equating an arbitrage-free market to having a unique equivalent martingale measure;	Assuming that the market is free from frictions and complete;
Binary Tree Model	A discrete version of partial differential equation model;	Assuming that the financial market is an arbitrage-free, balanced and complete market.

Option Pricing Model Based on Financial Mathematics Technology

Considering the deficiencies of the three traditional option pricing models, based on financial mathematics technology, our study attempted to improve traditional American lookback option pricing model using the least squares Monte Carlo (LSM) approach. Hopefully, our study would bring a new train of thought for research on option pricing based on financial mathematics technology.

2.1 Traditional American lookback option pricing model

Lookback options are a new kind of options, which is mainly used to satisfy over-the-counter (OTC) trade of products with special market demands, to realize the maximum price of shares in the validity of options. Among domestic and foreign studies on lookback options, European lookback options are the main research object. American lookback options have unfixed execution time and are time-consuming and entirely based on Brownian motion assumption, etc. These restrict the applications of American lookback options in the financial market in China. To reverse this situation, the author proposed to improve the American lookback option pricing model using LSM simulation [2].

2.2 Total least squares Monte Carlo simulation

Total LSM simulation is established on LSM simulation and partakes of both simple parameter estimation and remarkable application effect. Fig. 1 shows a comparison between applications of total LSM simulation and LSM simulation. From this figure, we can understand the superiority of LSM simulation intuitively [3].

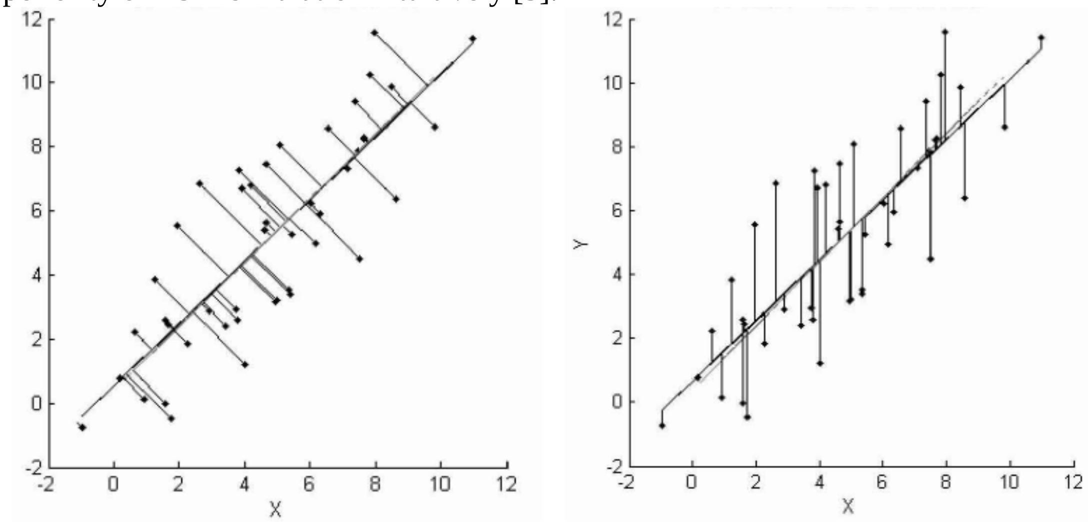

Fig. 1 Comparison between Regression Results of Stochastic Simulation of 45 Data Points

2.3 American lookback option pricing using total LSM simulation

With more insight into total LSM simulation, we can apply this approach to price American lookback options. For this end, we needed to obtain a formula, $S_t = S_o + \int_o^t X_s(\mu ds + \sigma dw_s) + \sum_{j=1}^{N_t} X_{\bar{\tau}_j} U_j$. This formula showed continuous price change of underlying assets and price leap of underlying assets visually. According to this formula, we can price American lookback options specifically.

Tab. 2 shows American lookback option pricing using total LSM simulation visually. From this table, it is not hard to find that

Tab. 2 American Lookback Option Pricing Steps

American Lookback Option Pricing Step	Specific Content
Step 1	Using the indicators of stock value and time, the continuous price change of assets was simulated with $S_o + \int_o^t X_s(\mu ds + \sigma dw_s)$. The price leap of assets was simulated with $\sum_{j=}^{N_t} X_{\tau_j^-} U_j$.
Step 2	The continued holding value of options was calculated, using the formula $Y_p = a_1 X_1(S_t) + a_2 X_2(S_t) + a_3 X_3(S_t)\, p = 1,2,3..., M$.
Step 3	The value on the maturity date T was identified as $\max_{0\le i\le K}\{S_{ti}^p\} - S_T^p$. Using total LSM simulation, the regression equation $Y_p = a_1 X_1(S_t) + a_2 X_2(S_t) + a_3 X_3(S_t)$ was solved. In doing this, we can obtain the continued holding value of options;
Step 4	Combined with the equation $Y_p = a_1 X_1(S_t) + a_2 X_2(S_t) + a_3 X_3(S_t)$, coefficients a1, a2 and a3 were determined.
Step 5	The option value was obtained, i.e., $V = \frac{1}{M}\sum_{p=1}^{M} \exp(-r\tau_1^p) V_p^e(\tau_1^p)$

2.4 Numerical analysis

Combined with the above statement about American lookback option pricing using total LSM simulation, we needed to compare the results of option value obtained from this statement with results of tree graph construction. This comparison must be supported by constructing a binary tree graph. Considering this comparison actually required valuating American lookback options of bonus-free shares, we needed to consider that the upward trend of Y was consistent with the downward trend of shares. From this, we can obtain the calculation results of American lookback option pricing by constructing a binary tree graph i.e.,

$$f_{i,t} = \max\{Y - 1, e^{-r\Delta t}[(1-p) f_{i+1,j+1} d + p f_{i+1,j} u]\}$$

Given that the term of American lookback put options was not more than 1 year, the author set the volatility σ of underlying assets and risk-free rate r as constants. From this, the author obtained the price of American lookback put options under binary tree approach, as shown in Tab. 3. According to this table, we conducted a further analysis and obtained the price of American lookback options under total LSM simulation [5].

Tab. 3 The Price of American Lookback Put Options Under Binary Tree Approach

Time Periods	1,000	100,000	50,000	200,000	500,000	1,000,000
Option Price	15.86	16.12	16.19	16.21	16.23	16.23
Run Time	0.04s	4.36s	1m43s	33m	3h13m	8h24m

Tab. 4 The Price of American Lookback Put Options Under Total LSM Simulation

Option Price	16.2398	16.1977	16.1828	16.2361	16.3218
Run Time	12m33s	12m36s	12m31s	12m32s	12m33s

From Tabs 3 and 4, it is not hard to find that the calculation of price of American lookback put options under total LSM simulation outweighed the calculation of price of American lookback put options under binary tree approach. The difference between their errors was not large, which further verified the superiority of calculation of price of American lookback put options under total LSM simulation.

Conclusion

In the present study on option pricing based on financial mathematics technology, the author expounds on relevant concepts of option pricing. Based on financial mathematics technology, using LSM approach, traditional American lookback option pricing model is improved. The better improvement of traditional American lookback option pricing model proves the superiority of applying financial mathematics technology to option pricing. Hopefully, this knowledge can draw the attention of relevant professional staff, to further develop the field of option pricing in China.

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