A study on how to keep the temperature of the water when bathing

Yajie Xu

School of North China Electric Power University, Baoding 071003, China

Yajie_Ncepu@163.com

Abstract: This paper mainly focuses on how to maintain the water temperature close to the initial temperature while bathing. Using law of energy conservation, we establish differential equations to figure out the function between water temperature and time. After adding 40^oC hot water to the tub, the water temperature would reach the comfortable temperature after about 160 seconds. The total water consumption is approximately $3.60m^3$. In our sensitivity, we analysis several factors that mainly influence the results separately.

Keywords: Energy conservation, differential equations, water consumption,

1. Introduction

A bathtub's valve is connected to water supply lines, and the drain connects to the home's drain line. Bathtub drains have two legs, one to the main drain opening and the other to the overflow drain opening. With a whirlpool tub, electrical power is needed to operate the pump[1]. And best bathtubs are made from enameled cast iron.

In our previous research, we find that if the temperature of the bathtub water is 40^oC, without any measures, it will approximately decrease to 27^oC after half an hour. The bathtub we design aims at two goals: to keep the temperature of the bathtub water close to the initial temperature, to save water as far as possible.

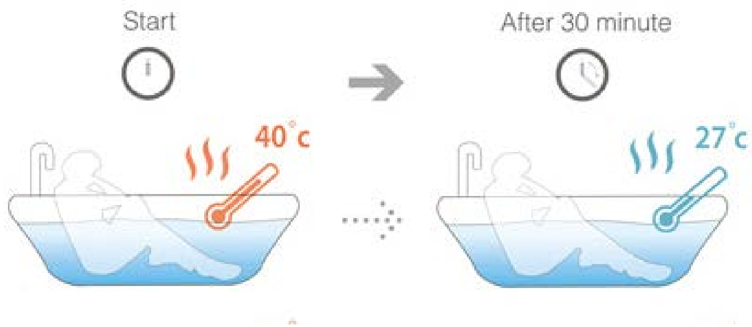

Figure 1. The temperature change in the bathtub[1]

2. Model Theory

2.1. Basic Model

Step1. Model Analysis: We design a bathtub filled with 35^oC water (just doesn't overflow) at beginning, afterwards, a single faucet continuously adds 40^oC hot water into the bathtub to rise the water temperature to a comfortable temp (38^oC~42^oC). We assume that the average time for a person to take a bath is half an hour.

The increase of the heat quantity (Q) of the tub system depends on two factors: The input heat quantity (Q_{in}), and the output heat quantity (Q_{out}), and they satisfy this function as: $Q = Q_{in} - Q_{out}$

Step2. The input heat quantity (Q_{in})**:** It comes from the added hot water, and we can compute this function as: $Q_{in} = f_1 \cdot \rho \cdot c \cdot T_1$

Step3. The output heat quantity (Q_{out})**:** It has eight main sources: the heat taken away by the Spilled water(Q_1), the outer wall of the bathtub while convecting with the air(Q_2), the outer wall of the bathtub in the heat conduction with the air(Q_3), the outer wall of the bathtub while radiating(Q_4), the surface water in the air convection(Q_5), the surface water in the heat conduction with the air(Q_6), the surface water in the radiation(Q_7), and the human body in the water(Q_8). To keep the temperature as close as possible to the initial temperature, we could set a equation to relate them:

$$Q_{out} = Q_1 + Q_2 + Q_3 + Q_4 + Q_5 + Q_6 + Q_7 + Q_8$$

Then, we use a series of functions to separately calculate these variables above:

$$Q_1 = f_2 \cdot \rho \cdot c \cdot T\ ;\ Q_2 = H_1 \cdot S_1 \cdot (T_3 - T_2)\ ;\ Q_3 = \frac{S_1 \cdot \lambda \cdot (T - T_3)}{d}\quad Q_4 = \varepsilon \cdot \delta_1 \cdot S_1 \cdot (T_3^{\,4} - T_2^{\,4})\ ;$$

$$Q_5 = H_2 \cdot S_2 \cdot (T - T_2)\ ;\ Q_6 = c \cdot m \cdot \Delta T\quad Q_7 = \varepsilon \cdot \delta_2 \cdot S_2 \cdot (T^{\,4} - T_2^{\,4})\ ;\quad Q_8 = \mu_n$$

Where: (Explanation of the functions)

	Q_3:	Fourier law
	Q_6:	Energy formula
	Q_{in}, Q_1:	Joule law
	Q_2, Q_5 :	Newtonian formula
Step4.	Q_4, Q_7 :	Stefan-Boltzmann law[4]

Deleting unimportant variables to simplify calculation: Amount the eight variables, four variables (Q_2, Q_4, Q_6, Q_7) can be ignored to predigest calculation for their small influence to the model result. After that, we combine the functions and the simplification to obtain a general equation:

$$Q = Q_1 + Q_3 + Q_5 + Q_8$$

Then, we could a further function[5] as:

$$\beta = \left(\frac{S_1 \cdot \lambda}{d} + H_2 \cdot S_2\right)$$

$$Q = c \cdot m \cdot \frac{dT}{dt} = f \cdot \rho \cdot c \cdot (T_1 - T) - \beta \cdot k \cdot (T - T_2) - \mu_n$$

$$T_0 = 35^{o} C$$

Step5. Result: Using MATLAB, we worked out the function and the function image (as shown in **Figure 4**), and the function[2] of the water temperature (T) changing with the time (t) show as:

$$T = \frac{-(A_2 - e^{A_1 \cdot t} \cdot (35 \cdot A_1 + A_2))}{A_1}$$

Variable value: $A_1 = -0.0116$ $A_2 = 0.447$ $T_1 = 40^{o} C$

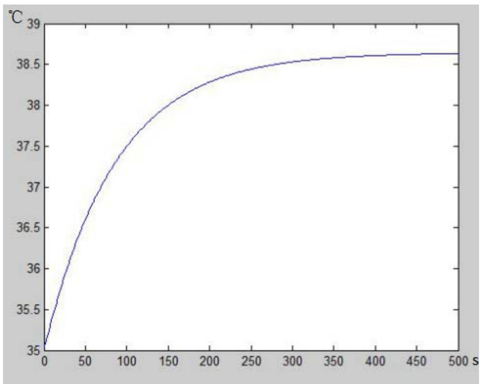

Figure 2. Water temperature changing with the time

According to **figure 2**, the water temperature in the bathtub will gradually rise to the comfort temperature in 5 minutes. In addition, the temperature will infinitely approach $40^{o}C$ later.

Assuming that the average time for a person to take a bath is half an hour. Because the water flow (f) is stable, we could figure out the total water consumption[3] (F) as:

$$F = f \cdot t = 0.5 \cdot 3600 \cdot 0.002 = 3.60 m^3$$

So, the the total water consumption is $3.60m^3$ under the the flow rate of the water is $0.002\ m^3/s$.

3. .Sensitivity

Based on the model above, we figure out the extent depends upon the shape and volume of the tub, the shape/volume/temperature of the person in the bathtub, and the motions made by the person in the bathtub.

Step 1. The size and shape of the bathtub:

Table 1. Different baths' size and shape

Bathtub type	Bath volume (m^3)	Bath surface area (m^2)	Bath oral area (m^2)	Water consumption (m^3)
Baby bathtub	0.09	1.5	0.3	3.118
Adult bathtub	0.18	4.5	1.3	3.216
Double bathtub	0.35	8.0	6.5	3.550

According to the model, we separately calculate the water consumption, and picture the image of water temperature (*T*) changing with time (*t*) (as shown in **Figure 3**), and the flow rate of water (*f*) along with time (*t*) (as shown in **Figure 4**).

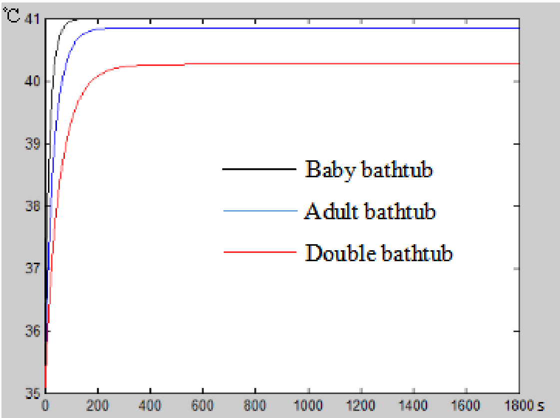

Figure 3. Water temperature

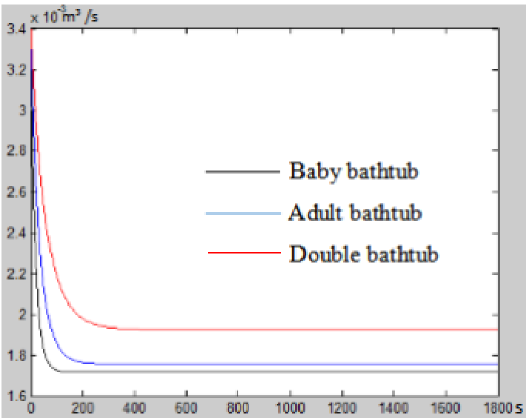

Figure 4. Flow rate of water

From the images,we could draw the conclusion that lager baths contributes to more water consumption and poorer heat preservation effect. This factor affect the strategy a lot.

4. Conclusions

we obtain the result that we should add 40°C water to keep the temperature to the initial value, and keep the water flowing at a speed of 0.002m^3/s. Then, the water temperature rises to approximately 39°C after 160 seconds, and the infinitely approach 40°C. But we don't focus on the water consumption, resulting in 3.60m^3 water consumption in half an hour.

Considering the shape and the volume of the bathtub and the person's size, a more suitable bathtub for users contributes to less water consumption and better heat preservation effect. As for the person's motions, flow of the bathwater would increase during the stirs made by the person. Finally, heat in the water would quicker lose to the air, resulting in about 0.3m^3 more water consumption.

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