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# Analysis of water temperature in bathtub under radiation and convection heat transfer

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**Abstract:** In this article, we analyze two conditions of changing of water temperature with time: reducing of water temperature naturally and pouring hot water into the bathtub. According to heat-transfer principles, there are three ways for heat to reduce: convection effects, radiation effects and thermal conduction in bathtub. Based on the law of conservation of energy conversion and use, given the influence of different conditions, we get the function of water temperature with time.

**Keywords:** heat transfer, convection, radiation, temperature

## 1. Introduction

The model describes the changes of water temperature with time. Bathing process involves two conditions: reducing of water temperature naturally and pouring hot water into the bathtub. The authors derive relationship between total reducing heat and temperature. Thus, we can get the relationship between temperature and time. Similarly, given heat reducing condition, the authors propose the partial differential equation to describe the heat and temperature when pouring hot water into the bathtub, and derive the relationship between temperature and time.

## 2. Changes of water temperature with time

### 2.1 Reducing of water temperature naturally

**Step1: Convection effects and radiation effects**

According to the law of conservation of energy conversion and use, when the quality of m in the object contained the heat is

$$Q = CMT \tag{1}$$

We can get the water temperature $T_w$ in a time, $Q_1$ radiation heat energy and convection energy $Q_2$, we can create the following model:

$$M_r C_r T_n + M_w C_w T_w = M_p C_p T_w + M_w C_w T_w + Q_1 + Q_2 \tag{2}$$

We only consider the natural cooling water, so we have radiant heat loss rate as follow:

$$\frac{dQ_1}{dt} = -\varepsilon\sigma A\left(T^4 - T_a^{\ 4}\right) \tag{3}$$

Similarly, we can get convection heat loss rate as follow:

$$\frac{dQ_2}{dt} = -hA\left(T - T_a\right) \tag{4}$$

**Step2: Thermal conduction in bathtub**

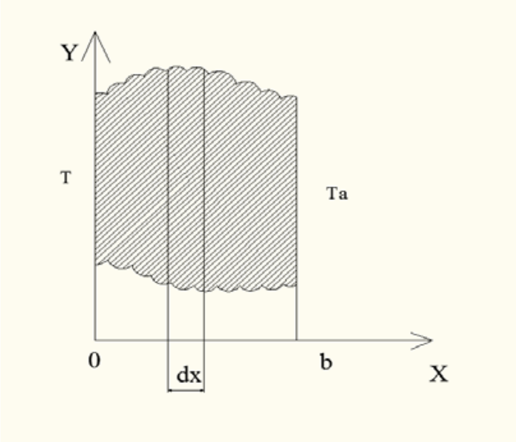


**Fig.1.Sectional view of bathtub**

$$Q_x = Q_{x+dx} + S\rho C_p \frac{\partial t}{\partial \theta} dx \tag{5}$$

For the stable temperature field, we have

$$\frac{dt}{dx} = 0 \tag{6}$$

$$Q_x = Q_{x+dx} = Q = const \tag{7}$$

$$Q = -\lambda_t S \frac{dt}{dx} \tag{8}$$

$$t = T, x = 0; t = T_a, x = b \tag{9}$$

The $T$ represents the temperature of one side of the tub wall near the water and the $T_a$ represents the temperature of the other side of the tub wall near the air.

$$\int_0^b Qdx = -\int_T^{T_a} \lambda_t S dt \tag{10}$$

We can assume the value of $\lambda_t$ is constant.

$$Q_3 = \frac{\lambda_t}{b} S\left(T - T_a\right) = \frac{T - T_a}{\frac{b}{\lambda_t S}} \tag{11}$$

$$\frac{dQ_3}{dt} = -\frac{\lambda_t S}{b}\frac{dT}{dt} \tag{12}$$

**Step3: Changes of water temperature with time**

$$Q_t = C\rho\left(V_W - V_P\right)T \tag{13}$$

The temperature of human body is constant. Thus, we get the follow equation to describe the rate of the heat energy that body absorbed:

$$\frac{dQ_4}{dt} = -CkM_p\left(T - T_p\right) \tag{14}$$

We consider the function of temperature as a continuous variable, and the water is naturally cooled, so we can get the follow equation by Eq. 13:

$$C\left(V_w - V_p\right)\rho\frac{dT}{dt} = \frac{dQ_1}{dt} + \frac{dQ_2}{dt} + \frac{dQ_3}{dt} + \frac{dQ_4}{dt} \tag{15}$$

$$C\left(V_w - V_p\right)\rho\frac{dT}{dt} = -\varepsilon\sigma A\left(T^4 - T_a^{\ 4}\right) - hA\left(T - T_a\right) - \frac{\lambda S}{b}\frac{dT}{dt} - CkM_p\left(T - T_p\right) \tag{16}$$

## 2.2 Pouring hot water into the bathtub

When water temperature reduce to 37 degrees, hot water is poured into bathtub by the faucet.

$$C\left(V_w - V_p\right)\rho\frac{dT'}{dt} = \frac{dQ_1}{dt} + \frac{dQ_2}{dt} + \frac{dQ_3}{dt} + \frac{dQ_4}{dt} + \frac{dQ_5}{dt} \tag{17}$$

In the Eq.17, $Q_5$ represents the heat energy of the hot water from faucet, $T'$ represents the temperature of the water after pouring hot water.

And we can get the heat energy of hot water unit of time[3]:

$$\frac{dQ_5}{dt} = \left(T_0 - T'\right)vC\rho \tag{18}$$

# 3.Calculation and Results

We assume the optimum water temperature is 40 degrees and open the faucet to pour hot water when water temperature reduce to 37 degrees. And we assume the person showers for 30 minutes. According to the criterion of general bathtub, we assume the volume of bathtub is 300 liters.

$$k = 0.01, M_p = 70kg, V_p = 0.07m^3, M_w = 230kg, V_w = 0.23m^3, v = 1m/s$$

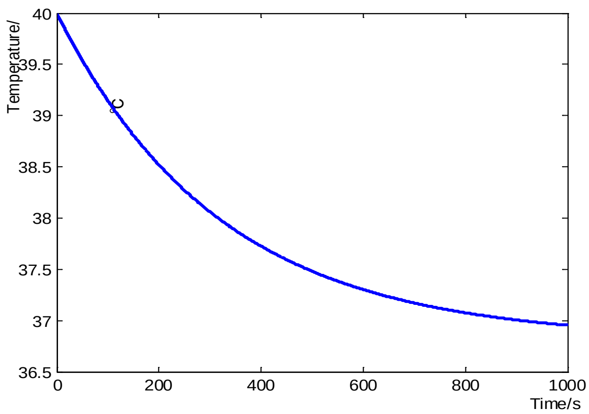


**Fig.2.Curve of temperature with time without hot water**

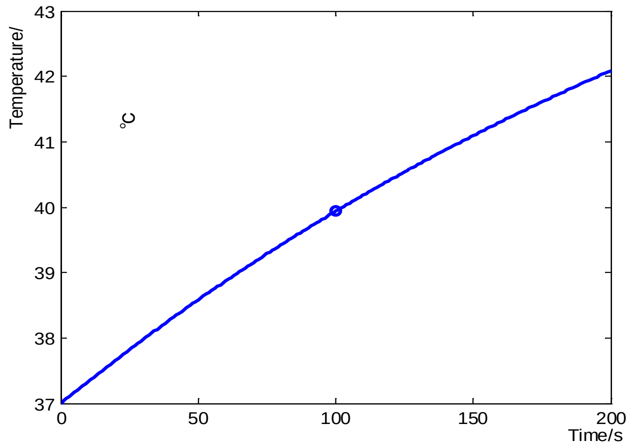


**Fig.3.Curve of temperature with time with pouring hot water**

Fig.2 shows that water temperature rises with time. And water temperature is reheated to 40 degrees from 37 degrees after opening the faucet for 100 seconds.

## 4. Conclusions

From the Fig.1, we can find the rate of reducing of temperature with time reduces gradually. And time that temperature decreases from 40 to 37 needs is about 1000 seconds.

Fig.2 shows that water temperature rises with time. And water temperature is reheated to 40 degrees from 37 degrees after opening the faucet for 100 seconds.

## References

[1] Kochin, N. E., Il'ja, A. K., & Roze, N. V. (1964). *Theoretical hydromechanics*. Interscience.

[2] Abe, K., Kondoh, T., & Nagano, Y. (1994). A new turbulence model for predicting fluid flow and heat transfer in separating and reattaching flows—I. Flow field calculations. *International journal of heat and mass transfer*, *37*(1), 139-151.

[3] Rogers, G. F., & Mayhew, Y. R. (1980). *Engineering Thermodynamics Work & Heat Transfer*. Longman Group Limited.