

Double closed loop controller of wheeled mobile robot for trajectory tracking based on back-stepping and Lyapunov method

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Keywords: Wheeled mobile robot, Trajectory tracking, Back-stepping, Lyapunov, Kinematics model, Dynamics model

Abstract: A new trajectory tracking controller for a wheeled mobile robot based on back-stepping and Lyapunov approach is presented. The proposed controller is developed based on the kinematic model and dynamics model of wheeled mobile robot. Firstly, the kinematic controller is designed by using back-stepping control method. Secondly, according to the dynamic model of multi input static coupling, the virtual control is used to decouple the dynamic model, and then the dynamic controller is designed by using the Lyapunov method. The output of the kinematic controller is used as the input reference of the dynamics controller. The simulation results show that the designed control scheme has a better dynamic performance and steady state performance, and has a certain practical value in tracking the trajectory of wheeled mobile robots system.

1. Introduction

Wheeled mobile robot has the advantage of light deadweight, big carrying, simple structure, relative convenience to drive and control, flexible mobility, high work efficiency, and is widely applied in industry, agriculture, anti-terrorism explosion-proof, family and space exploration and other fields. Robot has strong coupling, time-varying and nonlinear dynamic characteristics, so control is very complicated. In recent years, the motion control of wheeled mobile robot has been the research hotspot in the field of control. Some scholars have used nonlinear system control method in the control of wheeled mobile robot, such as the sliding mode variable structure control[1- 2], adaptive control[3-4] and back-stepping control[5-6]. At the same time, the different path tracking control strategies are also proposed in [7]. Among them, back-stepping control design method to deal with the problem of nonlinear system control has better control effect. Based on the back-stepping design method, reference [5-6] et al. solve the trajectory tracking problem of wheeled mobile robot, but only for the kinematic model, but for the actual value of the dynamic model is rarely considered.

In this paper, a double closed loop controller is designed based on the kinematics model and dynamics model of wheeled robot. Firstly, the back-stepping control law is designed based on the kinematics model, and the output of the back-stepping control law is used as the input reference of the inner loop dynamics model. Secondly, the dynamic model is decoupled according to the virtual control transformation, and then the control law is designed for the two first order differential equation of after decoupling by Lyapunov method. Finally, the simulation is carried out in

matlab/simulink, and the results show that the control method designed in this paper has good tracking performance.

2. System Controller Design

The independent dual rear wheel differential drive mobile robot controls the speed and direction of the robot by different speeds of the two rear wheels. Two models are used, one is kinematics model, which is used to solve the problem of speed and position control, and the other is dynamic model, which is used to solve the control problem between speed and input force.

2.1. Kinematic Control Law Design

Assuming the dual rear wheel independent drive rigid mobile robot moving in the plane, and assuming that the absolute coordinates of the OXY fixed in the plane, then dual rear wheel drive mobile robot is shown in figure 1.

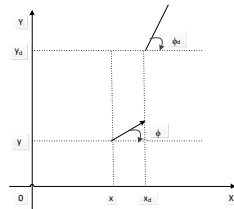


Figure 1 Position coordinate of mobile robot.

The state of the robot is represented by the position and direction angle ϕ of midpoint M in coordinate system. ϕ is the angle between the forward direction and the X axis, then the kinematics model of the robot is as follows:

$$\begin{cases} \dot{x} = v \cos \phi \\ \dot{y} = v \sin \phi \\ \dot{\phi} = \omega \end{cases} \quad (1)$$

where, v and ω are respectively the robot's linear speed and angular speed.

First step: The virtual input γ is introduced, according to the Eq. (1), has

$$\begin{cases} \dot{x} = v \cos \gamma \\ \dot{y} = v \sin \gamma \end{cases} \quad (2)$$

(x_d, y_d) is position setting value, (x_e, y_e) is position tracking error, where, x_e is equal to $x_d - x$, y_e is equal to $y_d - y$, take the Lyapunov function as follows:

$$V_1 = \frac{1}{2} x_e^2 + \frac{1}{2} y_e^2 \quad (3)$$

By Eq.(2), has

$$\dot{V}_1 = x_e \dot{x}_e + y_e \dot{y}_e = x_e(\dot{x}_d - v \cos \gamma) + y_e(\dot{y}_d - v \sin \gamma) \quad (4)$$

Make $\dot{x}_d + k_1 x_e$ equal to m_1 , $\dot{y}_d + k_2 y_e$ equal to m_2 , the linear velocity control law is designed as follows:

$$v = \sqrt{m_1^2 + m_2^2} \quad (5)$$

$$\gamma = \arctan \frac{m_2}{m_1} = \arctan \frac{\dot{y}_d + k_2 y_e}{\dot{x}_d + k_1 x_e}$$

then,

$$\dot{V}_1 = -k_1 x_e^2 - k_2 y_e^2 < 0 \quad (6)$$

Second step: Make e equal to $\gamma - \phi$, define the Lyapunov function as

$$V_2 = V_1 + \frac{1}{2} e^2 \quad (7)$$

then,

$$\dot{V}_2 = \dot{V}_1 + e \dot{e} = -k_1 x_e^2 - k_2 y_e^2 + e(\dot{\gamma} - \dot{\omega}) \quad (8)$$

The angular velocity control law is designed as follows:

$$\omega = \dot{\gamma} + k_3 e \quad (9)$$

then,

$$\dot{V}_2 = -k_1 x_e^2 - k_2 y_e^2 - k_3 e^2 < 0 \quad (10)$$

According to the Lyapunov stability theory, the system (1) is asymptotically stable under the control law Eq.(5) and Eq.(9).

2.2. Dynamic Control Law Design

According to the principle of moment balance, for car body, has

$$I_m \dot{\omega} = F_r d - F_l d \quad (11)$$

where, I_m is the turn inertia around the center of gravity of the robot, F_r and F_l are respectively the driving forces of the left wheel and right wheel, d is the distance between the center of gravity of the robot and the left wheel and right wheel.

According to Newton's law, has

$$m_c \dot{v} = F_r + F_l \quad (12)$$

where, m_c is the car body weight.

According to the principle of moment balance, for the left wheel and right wheel, respectively have

$$\begin{cases} I_n \dot{\omega}_l + c\omega_l = k\mu_l - RF_l \\ I_n \dot{\omega}_r + c\omega_r = k\mu_r - RF_r \end{cases} \quad (13)$$

where, I_n is the turn inertia of wheel, ω_l and ω_r are rotation angular velocity of wheels, c is viscous friction coefficient, k is gain, μ_l and μ_r are control inputs. R is wheel radius.

By Eq.(13), has

$$I_n R(\dot{\omega}_l + \dot{\omega}_r) + cR(\omega_l + \omega_r) = kR(\mu_l + \mu_r) - R^2(F_l + F_r) \quad (14)$$

According to the principle of mobile robot, has

$$\begin{aligned} v &= \frac{v_r + v_l}{2} \\ d\omega &= \frac{v_r - v_l}{2} \end{aligned} \quad (15)$$

and

$$\begin{aligned} v_r &= R\omega_r \\ v_l &= R\omega_l \end{aligned} \quad (16)$$

By Eq. (12), (14), (15) and (16), has

$$\dot{v} = -\frac{2c}{m_c R^2 + 2I_n} v + \frac{kR}{m_c R^2 + 2I_n} (\mu_r + \mu_l) \quad (17)$$

By Eq. (13), has

$$I_n R(\dot{\omega}_l - \dot{\omega}_r) + cR(\omega_l - \omega_r) = kR(\mu_l - \mu_r) - R^2(F_l - F_r) \quad (18)$$

By Eq.(11), (15), (16) and (18), has

$$\dot{\omega} = -\frac{2cd^2}{I_m R^2 + 2I_n d^2} \omega + \frac{kRd}{I_m R^2 + 2I_n d^2} (\mu_r - \mu_l) \quad (19)$$

The dynamic model of wheeled mobile robot is composed of formula (17) and formula (19), has

$$\begin{cases} \dot{v} = a_1 v + b_1 (\mu_r + \mu_l) \\ \dot{\omega} = a_2 \omega + b_2 (\mu_r - \mu_l) \end{cases} \quad (20)$$

where, $a_1 = -\frac{2c}{(m_c R^2 + 2I_n)}$, $a_2 = -\frac{2cd^2}{I_m R^2 + 2I_n d^2}$, $b_1 = \frac{kR}{m_c R^2 + 2I_n}$, $b_2 = \frac{kRd}{I_m R^2 + 2I_n d^2}$.

It can be seen that the dynamic model of wheeled mobile robot has coupling part. Now, we introduce the virtual control quantity u_1 and u_2 , and make u_r equal to $u_1 - u_2$, u_l equal to u_2 , then Eq. (20) becomes

$$\dot{v} = a_1 v + b_1 u_1 \quad (21)$$

$$\dot{\omega} = a_2 \omega + b_2 u_1 - 2b_2 u_2 \quad (22)$$

v_d and ω_d are the output of kinematic controller. Linear velocity error is v_e . Angular velocity error is ω_e . v_e is equal to $v_d - v$. ω_e is equal to $\omega_d - \omega$.

For Eq. (21), define the Lyapunov function as

$$V_3 = \frac{1}{2} v_e^2 \quad (23)$$

then,

$$\dot{V}_3 = v_e \dot{v}_e = v_e (\dot{v}_d - a_1 v - b_1 u_1) \quad (24)$$

The control law u_1 is designed as follows:

$$u_1 = \frac{1}{b_1} (k_4 v_e + \dot{v}_d - a_1 v) \quad (25)$$

then,

$$\dot{V}_3 = -k_4 v_e^2 < 0 \quad (26)$$

For Eq. (22), define the Lyapunov function as

$$V_4 = \frac{1}{2} \omega_e^2 \quad (27)$$

then,

$$\dot{V}_4 = \omega_e \dot{\omega}_e = \omega_e (\dot{\omega}_d - a_2 \omega - b_2 u_1 + 2b_2 u_2) \quad (28)$$

The control law u_2 is designed as follows:

$$u_2 = \frac{1}{2b_2} (-\dot{\omega}_d + a_2 \omega + b_2 u_1 - k_5 \omega_e) \quad (29)$$

then,

$$\dot{V}_4 = -k_5 \omega_e^2 < 0 \quad (30)$$

According to the Lyapunov stability theory, the system (21) and (22) are asymptotically stable under the control law (25) and (29).

3. Experimental results and analysis

$x_d = \sin(t)$, $y_d = \cos(t)$. The initial position of the controlled object is [0.5 0.5 0]. The parameters of kinematics controller and dynamics Controller are selected for $k_1 = 9$, $k_2 = 9$, $k_3 = 48$, $k_4 = 9$, $k_5 = 9$. The amplitudes of the control input u_r and u_l are limited to 9. Simulation has been carried out in MATLAB/Simulink. The simulation results are shown in Fig.(2) to (8).

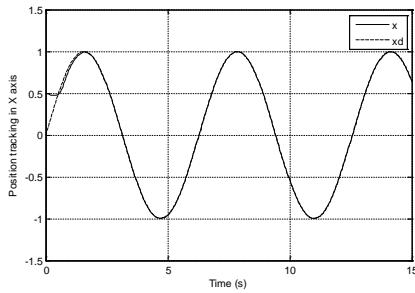


Figure 2 Position tracking in X axis

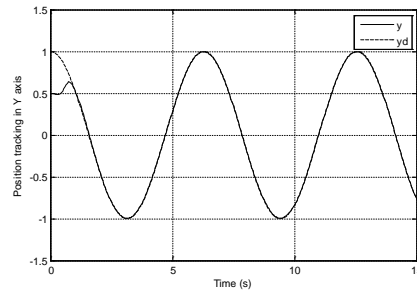


Figure 3 Position tracking in Y axis

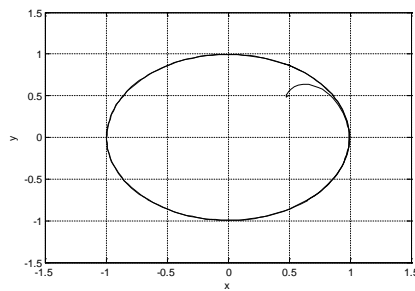


Figure 4 Position tracking curve

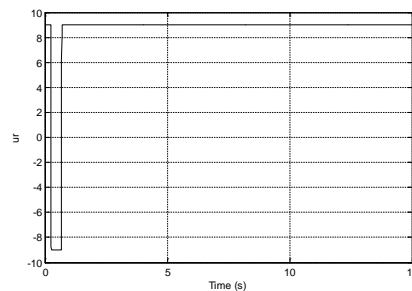


Figure 5 Control input ur

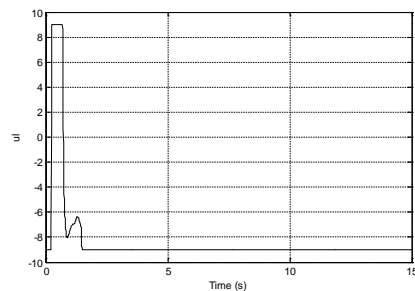


Figure 6 Control input ul

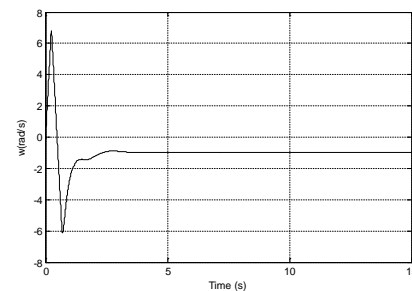


Figure 7 Angular speed curve

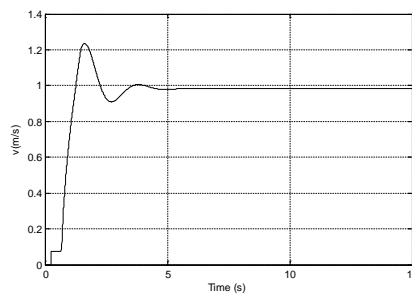


Figure 8 Linear speed curve

Fig.(2), (3) and (4) are the position tracking curve, it can be seen from the charts, despite the initial error, but under the action of control law, trajectory tracking is still achieved, after 4 seconds, the tracking error converges to a small interval. Fig. (5) and Fig.(6) are the control signals which are limited. Fig. (7) and Fig.(8) are the angular velocity and linear velocity curves.

4. Conclusions

Aiming at the trajectory tracking problem of wheeled mobile robot, a double closed loop control based on kinematics model and dynamics model is proposed in this paper. The outer loop is a kinematic controller, and the back-stepping control method is adopted. The inner loop is a dynamic control, and the Lyapunov method is adopted. The simulation results show that the designed control law can ensure the accuracy and stability of trajectory tracking.

Acknowledgment

The work is financially supported by the plan project of Education Department of Jilin Province of China (2015LY502L13)

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