

The Comparative Analysis of Fourier Transformation and Hilbert Huang Transformation Based on Dynamic EEG Data

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Keywords: Fourier transformation, Hilbert Huang transformation, dynamic EEG data, time-frequency characteristics

Abstract. The basic law and method of Fourier transformation and Hilbert Huang transformation is introduced. Taking the obtained dynamic EEG data under +Gz acceleration as an example, EEG change feature under +Gz acceleration is analyzed, and Fourier transformation spectrum graph and Hilbert Huang transformation spectrum graph are compared, then the time-frequency characteristics of the two methods are made analysis and comparison.

1. Introduction

Signal is the carrier of information, and can transmit the characteristic of the system. In the field of aeronautics and astronautics, in order to prevent pilots occurring air syncope in flying, many scholars have began to study on the monitoring and research of human physiological signals under +Gz acceleration[1][2]. In recent decades, all kinds of digital signal processing technology is widely used in analysis and feature extraction of EEG. At present, the signal processing methods mainly include Fast Fourier Transformation (FFT)analysis[3][4][5][6][7], Short Time Fourier Transformation (STFT) [8][9], Winger-Vill distribution[10][11], wavelet transformation[12][13][14][15], artificial neural network(ANN) [16][17], and the recent developing Hilbert Huang Transformation (HHT) analysis method[18][19][20][21][22][23][24][25][26]and so on. HHT is a kind of signal analysis method which can be applied not only for stationary signal but also for non-stationary signal, and is proposed by the United States scientists Huang N E and others in the year of 1996~1998 after making an in-depth studying for the concept of instantaneous frequency after wavelet transformation. The method is very suitable for non-stationary signal processing, and is considered to be a major breakthrough in the field of signal processing. This method has been favoured by the majority of researchers as soon as it appears, and has been rapidly applied in the research of various fields such as geophysics, vibration engineering, and medicine and so on. However, the study has not been seen that this method is applied to EEG and other

biomedical signals. In this study, firstly the EEG data under centrifuge +Gz acceleration is obtained, and the EEG data signal is analyzed by using FFT and HHT methods, so the FFT and HHT spectrum graphs of signal are obtained. The actual meaning of spectrum graph is analyzed, then the analysis results of two methods are made comparison and analysis, and the characteristics of them are studied. The advantages and disadvantages of HHT method in signal processing are analyzed and discussed, so the application prospect of HHT method in processing and analysis of dynamic EEG data is explored, and the application value of this method is discussed.

2. Method introduction [27]

2.1. The basic theory of FFT

It is convenient that $f(t)$ is expressed as a linear combination of a set of basic time functions in mathematics. FFT analysis is taking a series of sine functions to approximate the fitting signal, achieving the perfection of mathematical sense:

$$x(t) = \sum_{n=0}^{\infty} A_n \sin(nt + \theta) \quad (1)$$

So the positive transformation of FFT is as:

$$X(j\omega) = \int_{-\infty}^{+\infty} x(t)e^{-j\omega t} dt \quad (2)$$

The inverse transformation of FFT is as:

$$X(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(j\omega)e^{j\omega t} d\omega \quad (3)$$

Through positive transformation and inverse transformation of FFT, function can make mutual transformation in time domain and frequency domain.

2.2. The basic theory of HHT

Hilbert Huang transformation decomposes the signal into the form of a series of narrowband signal which can be processed by Hilbert transformation. Based on this idea, HHT includes two steps.

2.2.1. EMD

That is a complex signal is decomposed into a number of order IMF by using decomposition method of empirical mode. The specific step of EMD algorithm is as:

Initialization:

$$r_0(t) = x(t), i = 1 \quad (4)$$

Extracting IMF signal of the number i , that is C_i , making an initialization:

$$h_0(t) = r_i(t), k = 1 \quad (5)$$

Obtaining point sequence of the maximum value and point sequence of the minimum value of $h_{k-1}(t)$. Respectively fitting point sequence of the maximum value and point sequence of the minimum value of $h_{k-1}(t)$ by using **cube** spline interpolation, the lower and upper envelope curves of $h_{k-1}(t)$, $u_{k-1}(t)$ and $v_{k-1}(t)$ are obtained. The mean curve of the lower and upper envelope curves is calculated,

$$m_{k-1}(t) = (u_{k-1}(t) + v_{k-1}(t)) / 2 \quad (6)$$

$$\text{Calculating } h_k(t) = h_{k-1}(t) - m_{k-1}(t) \quad (7)$$

If $h_k(t)$ meets sifting stop standard:

$$SD_k = \frac{\sum_{t=0}^T |h_{k-1}(t) - h_k(t)|^2}{\sum_{t=0}^T h_{k-1}^2(t)} \quad (8)$$

Then $r_i(t) = r_{i-1}(t) - c_i(t)$, $c_i(t) = h_k(t)$, otherwise, $k = k + 1$, jumping to step b and continuing making screening.

Calculating the residual signal:

$$r_i(t) = r_{i-1}(t) - c_i(t) \quad (9)$$

If the number of extremum value point of $r_i(t)$ is greater than 2, then jumping to $i = i + 1$, and jumping to the previous step, that is, obtaining point sequence of the maximum value and point sequence of the minimum value of $h_{k-1}(t)$, otherwise, the decomposition finishes, $r_i(t)$ is the residual amount of signal.

The reconstruction of the original signal is as:

$$x(t) = \sum_{i=1}^n c_i(t) + r_n(t) \quad (10)$$

2.2.2. Hilbert transformation

Making Hilbert transformation for IMF, the instantaneous frequency and amplitude of each IMF with time variation are obtained. Then, the three-dimensional spectrum distribution of "time-frequency-amplitude" is obtained, which is called Hilbert spectrum, moreover, the marginal spectrum of signal can be further obtained. Assuming a signal is $x(t)$, its time-frequency spectrum can be expressed as:

$$H(w, t) = \text{Re} \sum_{i=1}^n a_i(t) e^{j \int w_i(t) dt} \quad (11)$$

The marginal spectrum of signal can be obtained by that $H(w, t)$ making an integrating over time, that is:

$$h(w) = \int_0^T H(w, t) dw \quad (12)$$

3. Comparison and analysis of dynamic EEG data

3.1. Data acquisition

The experimental data is acquired from 5 healthy volunteers. The subjects are all healthy, no human centrifuge contraindications. The main equipment is domestic, new type human centrifuge, and subjects are exposed under +Gz acceleration according to the set curve. The set curve starts from 1G, reaches the maximum G value at a certain growth rate, lasting for 10-15 seconds, and then drops to 1G at a certain growth rate. The maximum G value starts from 2.5G, increasing at the rate

of 0.5G/r, until the subject gets to the endurance end. The doctor real-time monitors ear pulse, ECG and other physiological signals, and comprehensively judges if the subject gets to the endurance end, according to the level state of ear pulse, combining with the subjective narration and the expression of the subject. EEG data when running, and before and after running is recorded by using a portable EEG recorder. Electrode signal of 16 leads is recorded in accordance with the electrodes positions of 10-20 system of international EEG standard electrodes installation method. All electrodes take the electrode at ipsilateral earlobe (A1 or A2) as the reference electrode [28]. Subject is worn of different types of fasten mesh cap, and each electrode is affixed special medical tape to prevent becoming loose in running. In experiment the subject is required to try to keep the natural state. Measuring method of single electrode lead is selected. 16 leads signal of EEG goes to amplify box after going to EEG interference box to remove interferes with the sampling rate of 128Hz, then the signal is recorded onto the internal SD card with the specific format. Data can be used for subsequent analysis and processing after format conversion.

3.2. The comparison and analysis of the dynamic EEG data under +Gz acceleration

EEG data of each lead of 16 lead, at quiet state that is when 1G and the EEG data under +Gz acceleration are separately selected, then EEG data of 10s is selected .The data is analyzed by using Fourier transformation and Hilbert Huang transformation, FFT spectrum graph and HHT spectrum graph of Fp1 lead when 1G and 3G are made comparison and analysis. The following Figure1 and Figure2 are the frequency spectrum graphs of FFT and HHT analysis of Fp1 lead at quiet state that is when 1G, and the following Figure3 and Figure4 are the frequency spectrum graphs of FFT and HHT analysis of Fp1 lead when 3G. In figure, the abscissa is data points number which 10s time is corresponding to, and the ordinate is frequency. The energy of frequency time point corresponding to is expressed through the relative amount of cold and warm colours of corresponding coordinate points, colour tone warmer, energy scale higher. Dial gauge is seen on the colour scale in the right side of figures.

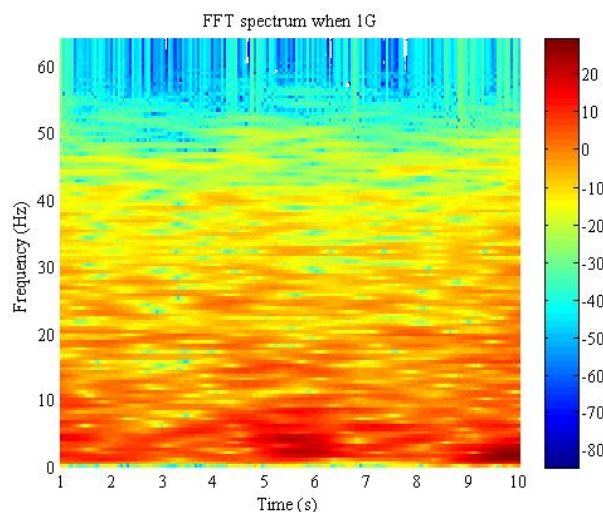


Figure 1 FFT spectrum of Fp1 lead when 1G.

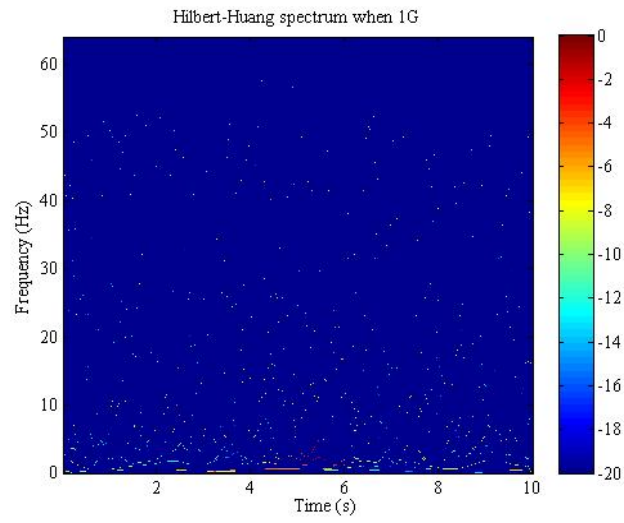


Figure 2 HHT spectrum of Fp1 lead when 1G.

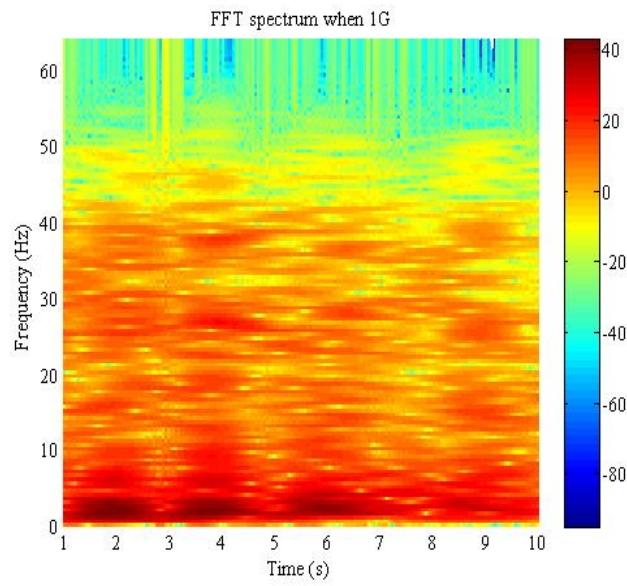


Figure 3 FFT spectrum of Fp1 lead when 3G.

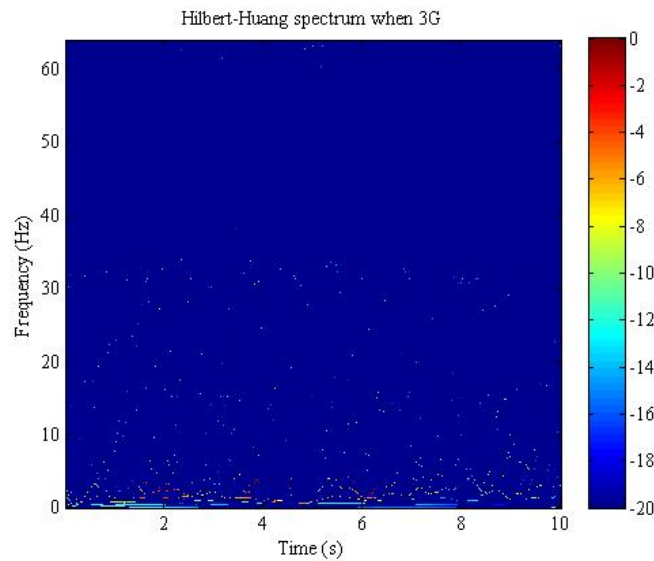


Figure 4 HHT spectrum of Fp1 lead when 3G.

It can be seen from the graphs, the information of EEG change under +Gz acceleration expressed by the two methods is basically the same, namely the energy level of low-frequency part of EEG increases significantly, and the energy level of high-frequency part of EEG decreases, showing that, the energy of low-frequency of EEG increases, and the energy of high-frequency of EEG decreases, when under +Gz acceleration compared to EEG at static state. For delta (0~3Hz), theta (3~8Hz), alpha (8~13Hz), beta (13~20Hz), the energy of each frequency band has increasing of different degrees, and the energy of delta and theta frequency bands increases significantly. Compared to EEG at quiet state when under +Gz acceleration, the points number distribution in HHT spectrum graph changes from a large area of scattering to concentrating to low frequency part, especially the points number concentrating to delta and theta frequency band is more, decrypting that energy transfers from high frequency to low frequency, basically according with the information of energy change shown by FFT. The two methods both can very well express change characteristics of EEG under +Gz acceleration, but these two methods have their advantages and disadvantages respectively. Although FFT has the advantages of fast operation speed, easy to use and so on, inevitably, this method also has its limitation. Firstly, the treatment effect of FFT for non stationary, nonlinear signal is not satisfying. While EEG signal, as a complex biological signal, has large nonlinear characteristic and stochastic characteristic, so for such a signal, the result of FFT tends to be discounted. Furthermore, according to the Heisenberg uncertainty theorem, the product of time width and frequency band width of signal should be a constant which is associated with sampling rate. In other words, when it is needed to improve the accuracy of time domain, the accuracy of frequency domain often is needed to be sacrificed. In turn, that's the way. If a segment of EEG data, if time resolution ratio can not satisfy analysis demand, that is signal needs to be subdivided for more time short paragraphs to make concrete analysis, time window has to be made thinning, then the result is that, although time precision doubles, frequency resolution ratio becomes 2 times of the original, not achieving the original precision. For EEG signal, resolution ratio of frequency can not be too low, at least accurately being 0.5 Hz, so when resolution ratio of frequency is required, time window length is likely to appear too large, at this time FFT may be very difficult to meet the requirement. While HHT is an energy calculation method of instantaneous frequency, having a unique advantage for the processing effect of nonlinear and non-stationary signals. This method does not exist with time window partition and translation steps, while single making

analysis for time sequence of small segment, so well giving consideration to the requirement of time resolution ratio and frequency resolution ratio. That is to say, as long as it meets the Shannon theorem, (that is sampling theorem), this method can improve time frequency resolution ratio as much as possible, under the situation of without sacrificing resolution ratio of frequency domain, which is just in line with the urgent demand for high resolution ratio of time frequency domain in this experiment, and which is also the biggest deficiency of FFT at the same time. Basis function of HHT method has adaptability, more suitable for the analysis and processing of non-stationary signal. HHT method can not only separate low frequency and high frequency part in transient signal by EMD decomposition, but also display distribution of signal energy in time frequency domain through HHT spectrum, and can help more clearly understand the distribution situation of low, high frequency part in signal, more in-depth understanding the essence of signal. While the got HHT spectrum after HHT analysis only includes IMF component decomposed by the original signal, not like that FFT spectrum includes various of component waves frequency range of which from 0 to infinite. But comparing to FFT algorithm, the biggest shortcoming of HHT algorithm is comparatively consuming computer internal memory and operation time, that is relatively to say, calculation is lower, and marginal effect and energy lose led by making the smallest unit of frequency domain analysis [shrank](#). Transformation can better distinguish characteristic of signal, and this property can be used to deal with problems which are existed in actual application. In addition, HHT can adopt some exclusive forms, such as instantaneous energy spectrum, time frequency spectrum and so on, to represent new characteristic, meaning being clear, expression being intuitive, and it may become a new analysis method of dynamic EEG data, at least being a powerful supplement for the existing methods. There are scholars who raise that HHT further development needs proving rationality of envelope, validate of EMD resolution theory, inhibition of end effect in boundary treatment and other problems. HHT adopts base quantity and function with local characteristic, providing possibility for revealing natural characteristic of non stationary signal, and theory improving will also be a long process. HHT method still needs to be further studied and improved.

4. Conclusions

This study introduces the basic law and method of FFT and HHT. The spectrum graphs at quiet state that is when 1G and under 3G acceleration are made out and analyzed by using FFT and HHT methods. Then EEG change feature under +Gz acceleration is analyzed. The results of two kinds of algorithms are made comparison, and the respective advantages and disadvantages are analyzed. The analysis result of HHT is basically consistent with that of FFT, and distribution and change of high, low frequency retains basically the identical tendency. HHT has the characteristics of self-adaptive, and obtaining a high time frequency resolution ratio at the same time and so on, so corresponding to FFT, it is more suitable for analysis and processing of non stationary signal, meeting the urgent demand for high resolution ratio of time frequency which FFT can not meet in this experiment. But HHT algorithm comparatively more consumes computer internal memory and operation time, and is easy to produce marginal effect and energy lose led by making the smallest unit of frequency domain analysis [shrank](#) and so on. On the whole, characteristic of recognition signal of HHT is more clear and intuitive than that of FFT. HHT can adopt some exclusive forms to represent new characteristic, and it may become a new analysis method of dynamic EEG data, at least being a powerful supplement for the existing methods. HHT method and theory still needs to be further improved and studied.

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