Application of Vine Copula in Multi-market Dependence Research

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Abstract. This paper forecasted VaR for multi-market assets (serval kinds of energy assets, stock, gold and US Dollars) by using Vine-copula model. The findings of the research showed that the Vine Copula model displayed more flexibilities and efficiencies than the traditional Bivariate Copula model in characterizing dependencies between multi assets, and R-Vine Copula model proved the best accuracy compared with the C-Vine and D-Vine Copula model.

Introduction

Energy plays an important role in the national economy. With the oil crisis and market turmoil, the price of ri hment of a risk management model to capture the characteristics is very important for risk managers.

This study of interdependence between different markets is the focus for recent domestic and foreign scholars. Krehbiel and Adkins (2005) applied McNeil and Frey's two-stage method to construct model of futures market risk, and pointed out that the conditional extreme value theory has obvious advantages in the assessment of the energy market's price risk. But they couldn't make comparison between conditional extreme model and fat-tailed model (such as the distribution of student t) and make backward inspection for empirical results. Marimoutou, Raggad,and Trabelsi (2009) applied the same method to model VaR of oil futures market and conducted a backward test. It is pointed out that conditional extreme frame should be superior to conventional risk management methods. Jia Liu (2011) analyzed and discussed the conventional risk management, value ant risk, the expected loss and replaceable relevant risk management methods in the research of risk management on energy market. McNeil and Frey's two-stage method was used to analyze investment portfolio of energy futures and stock (S & P) and dollars, based on historical data.

The dependence of multi-market assets (including energy assets, stock, dollar and gold) will be studied in this research, in order to provide theoretical suggestions for policy makers and financial investors.

Research methods

Edge distribution model. The financial assets return rate data would generally show asymmetric volatility aggregation and leverage effect and features such as peak, fat tail and skewness. The application of AR (1)-GJR-GARCH (1, 1)-Skew-t model could effectively describe such features of return rate data of assets in stock types,

$$r_{it} = c_{i0} + c_{i1} r_{it-1} + \varepsilon_{it}, \qquad (1.1)$$

$$\varepsilon_{it} = \sigma_{it} z_{it} \qquad (1.2)$$

$$\sigma_{it}^2 = \mu_i + \alpha_i \varepsilon_{it-1}^2 + \beta_i \sigma_{it-1}^2 + \gamma_i \varepsilon_{it-1}^2 I_{it-1} \qquad (1.3)$$

$$z_{it} \sim i.i.d.ST(\eta, \lambda) \qquad (1.4)$$

In which, when $\varepsilon_{it-1} \geq 0$, $I_{it-1} = 0$; $\varepsilon_{it-1} \leq 0$, $I_{it-1} = 1$; r_{it} is the assets return rate at the t transaction date, β_{i0} is the mean value of self-regression of assets return rate, β_{i1} is the coefficient of r_{it-1}, means the impact of assets return rate at the transaction date on the return rate at the $(t-1)$ transaction date; $\varepsilon_{it} = \sigma_{it} z_{it}$ is the asset income residual at the t transaction date, in which z_{it} is the complied, independent standardized residual under same distribution; σ_{it}^2 is the conditional variance of ε_{it}; μ_i is the mean value of conditional variance regression; is γ_i leverage coefficient, used to measure impact of rate change on after-market return rate under different conditions; η and λ mean the degree of freedom and nonparametric parameters of Skew-t distribution respectively.

Vine Copula Model

Binary Copula function decomposition. Give an n-dimensional random vector $X = [x_1, x_2 \cdots, x_n]$, whose joint distribution probability density function could be expressed as follows subject to conditional probability distribution density function theory:

$$f(x_1, \cdots, x_n) = f_n(x_n) \cdot f(x_{n-1} | x_n) \cdot f(x_{n-2} | x_{n-1}, x_n) \cdots f(x_1 | x_2, ..., x_n) \tag{2.1}$$

According to Copula theory proposed by Sklar [], the joint distribution probability density function of the n-dimensional random vector could also be expressed by Copula density function and edge distribution density function, with following forms:

$$f(x_1, \cdots, x_n) = c_{1,2,...,n}(F_1(x_1), ..., F_n(x_n)) \cdot f_1(x_1) \cdots f_n(x_n) \tag{2.2}$$

In which $c_{1,2,...,n}(F_1(x_1), ..., F_n(x_n))$ is the n-dimensional Copula density function and $F_i(x_i)$ is the edge distribution density function.

In a 2 dimensional case,

$$f(x_1, x_2) = c_{1,2}(F_1(x_1), F_2(x_2)) \cdot f_1(x_1) \cdot f_2(x_2) \tag{2.3}$$

$$f(x_1, x_2) = f_2(x_2) \cdot f(x_1 | x_2) \tag{2.4}$$

We can get from above two formulas that:

$$f(x_1 | x_2) = c_{1,2}(F_1(x_1), F_2(x_2)) \cdot f_1(x_1) \tag{2.5}$$

Expand it to 3 dimensional case, we can get:

$$f(x | v) = c_{x, v_k | v_{-k}}(F(x | v_{-k}), F(v_k | v_{-k})) \cdot f(x | v_{-k}) \tag{2.6}$$

In which, v is the n-1 dimensional vector after removing x, v_k is one of components, v_{-k} is the n-2 component of vector v after removing v_k, $c_{x, v_k | v_{-k}}(\cdot,\cdot)$ is the binary Copula density function. We can get the conditional distribution function $F(x | v)$ by following formula:

$$F(x | v) = \frac{\partial C_{x v_k | v_{-k}}(F(x | v_{-k}), F(v_k | v_{-k}))}{\partial F(v_k | v_{-k})} \tag{2.7}$$

Vine Copula Modelling. Considered that the multi-dimensional Copula density function could be decomposed into product of various types of binary Copula functions, Bedford and Cooke (2002) proposed a regular vine graphical modeling to decompose the multi-dimensional Copula density function, which include R-Vine, C-Vine and D-Vine. In which R-Vine model includes tree, node and edge, each tree consists of several nodes and such nodes are connected by edges. The node means variable or conditional variable, edge means the binary Copula function used to describe the dependence relation among variables. C-Vine and D-Vine are recognized as two special types of R-Vine models, in which C-Vine model applies star structure and D-Vine applies parallel structure as shown in Figure 1. This paper, based on AIC/BIC fitting inspection standards, selected the optimal binary Copula functions among the nodes of 3 kinds of Vine Copula models and the parameter estimation applied Maximum Likelihood Estimation (MLE).

Empirical results and analysis

Data selection and statistical analysis. This paper selected samples from three kinds of energy assets including West Taxes Intermediate (WTI), Natural Gas Spot Prices (NG) and Brent North Sea (Brent). Standard & Poor's 500(GSPC)，sport price of London Gold market(Gold), US Dollar Index(USDI) are also modelled in this research, in order to study the dependence and volatility risk of various financial assets.

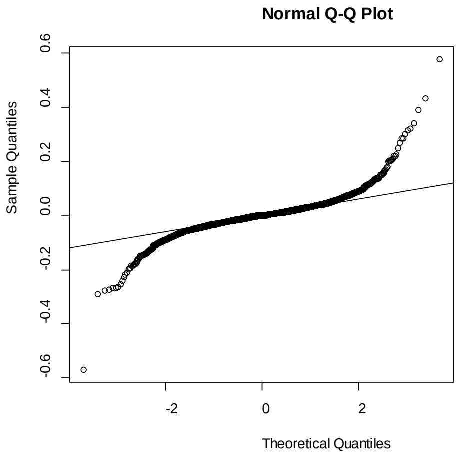
The data was specifically selected from May 8, 1998 to October 8, 2016, subject to closing price of each stock index every day and sourced from Energy Information Administration (EIA), London Bullion Market Association (LBMA), Yahoo financial channel and Inter Continental Exchange (ICE). As affected by holidays, the transaction time on stock market in each country was different from each other and after screening and matching of all data on above markets, we finally got the sample date of 4428 transaction dates and 4427 groups of logarithm return rate sample data according to their logarithmic return rate.

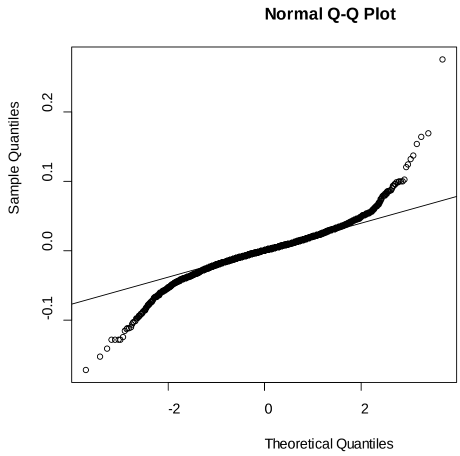
Table 1 listed Descriptive Statistical Results on Logarithmic Return Rate of the selected assets. From the average value, it can be seen that the average daily return on the remaining assets are positive in the past 20 years in addition to natural gas (NG). The results of standard deviation also confirm that other assets except the US dollar index showed strong volatility, and the volatility of energy assets was the most obvious, which shows the investment risk is the highest. The results of autoregressive conditional heteroscedasticity (ARCH LM) test show that all the return rates of the assets have obvious volatility aggregation effect under different lag orders.

Table 1 Descriptive Statistical Results on Logarithmic Return Rate①

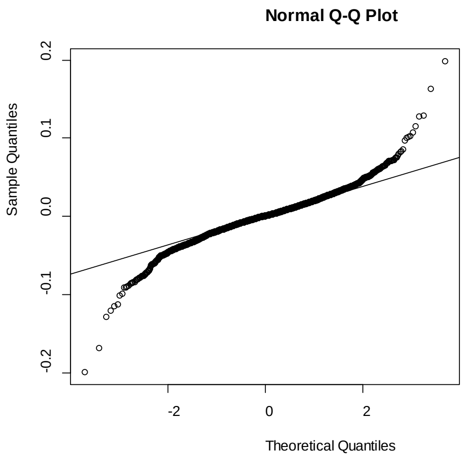
	Mean Value	Standard skewness	Skewness	Peakedness	J-B	LM(5)	LM(10)	LM(15)
NG	-0.0001	0.0481	0.4892	22.7932	7.25E+04*	541.2*	580.1*	602.5*
GSPC	0.0002	0.0252	0.1096	10.5194	1.05E+04*	753.4*	1074.3*	1150.1*
WTI	0.0002	0.0235	-0.1099	8.5654	5.73E+03*	264.6*	280.8*	326.3*
Brent	0.0002	0.0120	-0.2387	10.1729	9.60E+03*	94.2*	115.2*	279.*
Gold	0.0003	0.0113	-0.1599	8.8519	6.37E+03*	145.3*	169.4*	192.1*
USDI	0.0000	0.0030	-0.0933	6.8721	2.79E+03*	270.6*	387.5*	456.7*

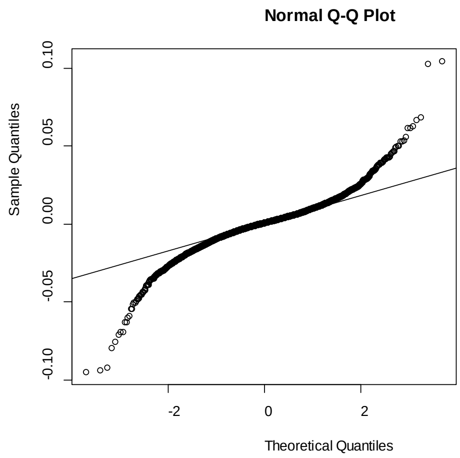
Figure 1showed the QQ curve of six assets return rate. The results of skewness and peak showed that each stock index showed the non-normal features such as asymmetry, peak and fat tail, which has been verified by extremely significant J-B test results.



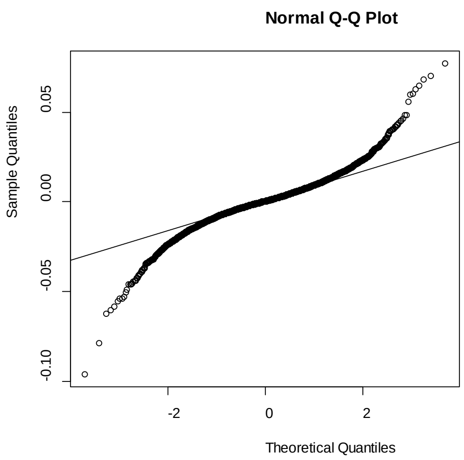


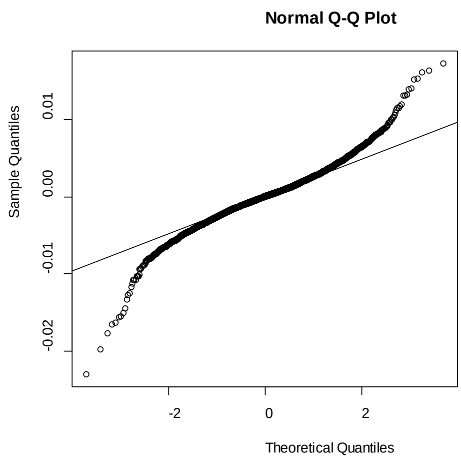
(a) NG (b) WTI





(c) Brent (d) GSPC





(e) Gold (f) USDI

Figure 1, Autocorrelation Chart of Daily Return

Parameter Estimation of Edge Distribution Model. According to the non-normal features such as skewness, peak and fat tail and the fluctuation aggregation effect showed by logarithmic return rate of each asset, AR(1)-GJR-GARCH(1,1)-Skew-t model was used to describe them. According to the non-normal features such as skewness, peak and fat tail and the fluctuation aggregation effect showed by logarithmic return rate of each stock index, AR(1)-GJR-GARCH(1,1)-Skew-t model was used to describe them. In which first order self-regression model (AR(1)) could effectively eliminate the self-regression features of data and the first order asymmetric generalized autoregressive conditional heteroskedasticity model GJR-GARCH(1,1) could effectively simulate the features such as skewness, peak and fat tail of standardized residuals. Measurement and analysis in this study was mainly based on R and Matlab software, with model parameter estimation subject to maximum like hood method and parameter estimation results are shown in Table 2.

Table 2 Estimation of Edge Distribution Parameters and Test Results

	c_0	c_1	μ	α	β	γ	η	λ	L-BQ(5)
NG	-4.81E-05	1.96E-02	6.10E-05	1.60E-01	0.82	0.00	1.00	5.31	0.445
GSPC	2.87E-04	-5.91E-02	1.77E-06	1.84E-07	0.91	0.16	0.86	7.05	0.546
WTI	3.29E-04	-2.65E-02	3.28E-06	2.32E-02	0.94	0.03	0.93	5.63	0.874
Brent	5.78E-05	2.48E-02	9.88E-07	1.40E-02	0.97	0.04	0.95	5.92	0.603
Gold	2.35E-04	-1.78E-02	1.41E-06	7.26E-02	0.93	-0.04	0.98	3.91	0.516
USDI	1.54E-05	3.17E-02	5.56E-08	5.03E-02	0.95	-0.01	1.01	6.33	0.437

Parameter Estimation of Vine Copula. This paper, built the modeling of 6 kinds of financial markets selected by using of 3 kinds of Vine Copula models, including R-Vine, C-Vine and D-Vine, each model has 15 groups of interrelations required estimation. The upper Triangle part in Figure 3 listed the matrix scatter plot of each two stock markets and the lower Triangle part listed the corresponding binary standard normal distribution contour map. From the figure it could be seen that the dependence among oil markets (WTI and Brent) and other markets is significant.

Vine-Copula Model fitting effect test. In order to compare the fitting effect of 3 different Vine Copula models, we applied AIC/BIC fitting test standards and Vuong tests to determine the merits of fitting. Table 3 listed the AIC/BIC fitting value of each model, we can see from the AIC/BIC fitting test principle that R-Vine model fitting has the best effect, followed by that of C-Vine model and D-Vine model, with no significant difference.

Table 3 AIC/BIC Values of Vine Copula Model

	AIC	BIC
R-Vine	-3747.7	-3619.8
C-Vine	-3702.1	-3548.4
D-Vine	-3735.4	-3606.5

Multi-Asset Portfolio VaR Forecast and Robustness Test. After getting the edge distribution of 6 financial markets and describing the Vine Copula function of their dependence structure, Monte Carlo model was used to calculate the VaR of portfolio. Firstly, apply Monte Carlo simulation to produce the random number of the 6 kinds of markets that complied with the 3 Vine Copula models, use the inverse function of previously fitted empirical cumulative distribution function to obtain the related standardized residual sequence and the daily return rate of each stock market. After giving the investment weight coefficient of each market, the predicted value of VaR of portfolio could be obtained by using of empirical fractile.

In order to verify the prediction effect of VaR of portfolio, we divided all data samples selected into estimated samples and predicted samples, in which estimated samples are collected from May 7, 1998 to June 8, 2016, used to estimate model parameters, predicted samples are collected from June 7, 2016 to October 8, 2016, used to verify the VaR prediction effect.

We applied prediction method beyond samples of sliding time windows to make rolling prediction on portfolio VaR at 300 transaction dates beyond the estimated samples. The confidence level was selected as 95%, 97.5 % and 99% respectively, considering 2 groups of random investment weight coefficients (see Table 4) to test the stability of portfolio VaR prediction effect.

In consecutive 300 transaction dates, under confidence level of 95%, 97.5 % and 99%, the failure number was 3, 8 and 13 respectively, showed good prediction effect.

Table 4 Random Investment Weight Coefficient of Portfolio

No	1	2	3	4	5	6
Weight Coefficient 1	1/6	1/6	1/6	1/6	1/6	1/6
Weight Coefficient 2	0.153	0.267	0.218	0.110	0.014	0.234
Weight Coefficient 3	0.104	0.272	0.067	0.081	0.212	0.260

Table 4 listed the results of failure number of VaR prediction and prediction effect validity inspection (Kupiec LR) on 3 Vine Copula models within 300 dates of portfolio using 2 groups of random investment weight coefficients, which showed that:

（1）All of VaR prediction results by using of 3 Vine Copula models passed the Kupiec LR inspection, which verified the validity of Vine Copula models on VaR prediction of measured multi-asset portfolio.

（2）The overall accuracy of VaR prediction by R-Vine Copula model is superior to that by C-Vine and D-Vine Copula model, and has more accurate predictive ability on market risk of multi-asset portfolio; the prediction accuracy of C-Vine and D-Vine was much close and has no significant difference.

（3）For simulation efficiency, since as compared with C-Vine and D-Vine Copula models, the parameter estimation and simulation of R-Vine Copula model was more complicated, thus it would take more time in parameter estimation and simulation, which was about twice than the C-Vine and D-Vine Copula models.

Conclusion

This paper uses AR (1) -GJR-GARCH (1,1) -Skew-t modeling method which can measure the leverage effect to fit with individual financial asset returns, to estimate the conditional volatility of financial data and to introduce the extreme value theory EVT so as to better measure the extreme risk of marginal distribution of financial assets. It uses 3 different types of Vine Copula model to effectively describe the dependence structure of 6 types of assets, and it combines with the Monte Carlo simulation method to carry on outside sample forecast and robustness test to portfolio VaR7. The empirical results show that: the Vine Copula model is more effective and flexible than the traditional multivariate Copula model to describe a variety of asset dependencies. Portfolio risk management usually involves multidimensional financial assets, and uses GJR-GARCH-EVT-Vine Copula model for investment portfolio for modeling, which can well describe the relationship between different assets and can accurately have risk prediction on the market portfolio, thus providing a more effective choice for risk management. Considering the interdependence between financial markets will change with time, the introduction of time-varying binary Copula function has become a subsequent research direction in the right time.

The empirical research results show that there is relevance among the price of gold, stock and energy which have been paid much attention by investors. From the study of the historical data, it can be known that the dollar price fluctuations will affect the price change of energy and gold. The relationship between them can be explained by the following aspects: firstly, the dollar is regarded as the international energy assets, securities assets and currency price of generally gold choosing, and the gold price fluctuations will have a direct impact on other asset prices in investment portfolio. When the U.S. Dollars get appreciation, stocks, gold and energy and other commodity prices which are valuated by dollars will fall. Investors may invest more funds to the dollar, thus making the energy, gold and the stock price be affected. The changes of investment product demand mainly cause the financial asset price changes. Secondly, during the economic downturn, many countries would purchase bonds or increase the issuance of currency to achieve moderate inflation in order to stimulate its economy. Currency devaluation or revaluation are likely to affect various types of asset prices in investment portfolio. At the same time, energy, the major industrial raw materials, has great influence in a country's economy, production and life. The rise of energy prices will cause currency devaluation and inflation. The increasing demand of investors will further push up the price of gold. In addition, the war and the turmoil situation in some areas will also make investors exchange funds to gold products so as to avoid risk, which cause the price of gold rise in a certain extent. And when the energy producing countries have turmoils, the energy production will reduce, transport will disrupt, energy supply will fall sharply and the price will be pushed up.

The model used in this paper can accurately predict the risk of different types of asset portfolios in multi-markets, and it proves the validity and robustness of the model in the prediction of multi-market portfolio risk value. According to the results obtained by the use of three kinds of Vine Copula model for selected multi-market assets modeling, it can be known that the interdependence relationship is obvious between the low-end and high-end of asset portfolio. So investors in financial markets should pay attention to the tail extreme dependence while carrying on asset portfolio. The study of interdependence between the extreme benefits and extreme loss of multiple assets is helpful for asset managers to allocate the proportion of investment and avoid the extreme loss effectively.

This study can not only provide some help for risk managers of financial market to make policy, but also provide a new idea for the design of financial derivatives hedging strategy and the measurement of multi-market interdependence. But in reality, the non-linear relation in financial markets may fluctuate with the change of external conditions. On the basis of portfolio selection, managers should optimize the proportion of investment in order to reduce the investment risk and should also consider the interdependence structures among markets will change over time. In view of this, the timely introduction of time-varying Copula method and dynamic research and characterization about the interdependence of financial time series in the empirical analysis will be the direction of further study.

① *represents J-B test or ARCH LM test at 5% significance level（Lag order is in brackets）

② N1 and N2, respectively, indicate the number of failures and the number of theoretical prediction failures in VaR; P indicates failure rate of Kupiec LR test.

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