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Adaptive Synchronization in Multi-coupled Dynamic Network

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Abstract. Synchronization of multi-coupled dynamical networks has received considerable attention recently. However, it is often difficult to estimate the coupling coefficients since we cannot get the exact boundaries of the variables for most chaotic systems. In this paper the network consisting of N ($N\geq3$) coupled FHN systems will synchronize by adding only one adaptive feedback gain equation without exactly estimating the coupling coefficients of the chaotic systems, which is very useful for future practical applications.

I. Introduction

Synchronization of interacting oscillators in biological systems has been widely studied over the last few years. Classical phenomena such as mutual synchronization, entrainment and chaotic synchronization are now observed in many biological experiments and numerical simulations [1-4].

The FitzHugh-Nagumo (FHN) model is a two-dimensional simplification of the widely known Hodgkin–Huxley model [5] describing the signal transmission across axons in neurobiology. In the past ten years, Most of the methods mentioned above are used to synchronize two or three coupled FHN chaotic systems. Synchronization of multi-coupled dynamical networks has received considerable attention recently. Deng et al. studied the relation between coupling strength and the dynamics of multi-coupled FHN system[6,7]. In these studies, an estimation of coupling coefficient should be given before designing some controllers. However, as is known, it is often difficult to estimate the coupling coefficients since we cannot get the exact boundaries of the variables for most chaotic systems[8]. To overcome these difficulties, in this paper, an effectively adaptive synchronization approach is proposed, based on the complex dynamic network model consisting of N ($N\geq3$) nonlinearly diffusively coupled FHN systems under external electrical stimulation. In fact, the network will synchronize by adding only one adaptive feedback gain equation without exactly estimating the coupling coefficients of the chaotic systems, which is very useful for future practical applications.

The rest of the paper is organized as follows. In Sect. 2, dynamics of nonlinear cable model for individual FHN neuron are investigated. Then, in Sect. 3, a network model of multi-coupled FHN neurons is presented. In Sect. 4, adaptive synchronization criterion for the complex dynamical network is proposed. The numerical simulations given in Sect. 5 demonstrate the effectiveness of the control method. Finally, conclusions are given in Sect. 6.

II. Dynamics of Individual Neuron

In Ref.9 and 10, based on the FitzHugh-Nagumo (FHN) simplification of the Hodgkin-Huxley model for active membranes[5], a nonlinear cable model was developed to study the response of cylindrical cells to external electric fields.The model equation is a coupled partial differential equation for the transmembrane voltage V along the nerve fibre and the recovery variable W of the form:

$$\begin{cases} \dot{x} = x(x-1)(1-rx) - y + I_0(t) \\ \dot{y} = bx \end{cases} \tag{1}$$

where x and y are membrane voltage V and recovery variable W rescaled by V_P , the peak of the active potential, respectively. $x = \frac{V}{V_P}, y = \frac{W}{V_P}$ .The variable r is $r = \frac{V_P}{V_T}$. Where V_T denotes the threshold membrane voltages.

In our previous literature[11], the dynamics of individual FHN neuron under external electrical stimulations are studied. With the variation of the stimulation and the initial condition of the neuron, the complex behaviors including periodicity, quasiperiodicity and chaos are revealed. When the parameters are $r = 10, b = 1, A = 0.1,$ and the frequency of the external stimulation $f = 129Hz,$ the individual neuron without coupling is chaotic.

III. Multi-Coupled FHN Network Model

Consider a multi-coupled dynamic network consisting of N ($N \geq 3$) nonlinearly coupled FHN systems with uncertain nonlinear diffusive couplings, which is described by:

$$\begin{cases} \dot{x}_1 = x_1(x_1-1)(1-rx_1) - y_1 + I_0 - d_1(x_1 - x_2) \\ \dot{y}_1 = bx_1 - d_2(y_1 - y_2) \\ \dot{x}_2 = x_2(x_2-1)(1-rx_2) - y_2 + I_0 - d_1(x_2 - x_3) \\ \dot{y}_2 = bx_2 - d_2(y_2 - y_3) \\ \vdots \\ \dot{x}_N = x_N(x_N-1)(1-rx_N) - y_N + I_0 - d_1(x_N - x_1) \\ \dot{y}_N = bx_N - d_2(y_N - y_1) \end{cases} \tag{2}$$

The network topology of the multi-coupled system is shown in FIG.1. As shown in Ref.12, such a ring is synchronized identically for sufficiently large values of d_i . In the following section, an adaptive feedback gain equation for d_i will proposed to achieve the synchronization.

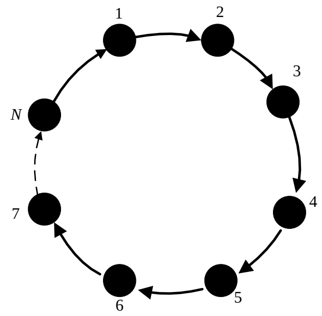


FIG.1 Network topology of multi-coupled FHN system

IV. Adaptive Synchronization of *Multi*-Coupled Fhn Network

Let us define the synchronizing errors of the system as $e_k = x_i(t) - x_j(t)$ or $y_i(t) - y_j(t)(i, j = 1,2,3,\cdots,N; i \neq j)$, then,

$$e_1 = x_1 - x_2,\ \ e_3 = x_2 - x_3,\ \ e_5 = x_3 - x_4, \cdots,\ \ e_{2N-1} = x_N - x_1,$$
$$e_2 = y_1 - y_2,\ \ e_4 = y_2 - y_3,\ \ e_6 = y_3 - y_4, \cdots,\ \ e_{2N} = y_N - y_1.$$

Where $e_{2N-1} = -e_1 - e_3 - \cdots - e_{2N-3},\ \ e_{2N} = -e_2 - e_4 - \cdots - e_{2N-2}.$

The problem of synchronization between the coupled systems can be translated into a problem of how to realize the asymptotical stabilization of the system (2) at the origin. The goal is to design the adaptive feedback gain equation d_i, such that $\lim_{t\to\infty}\|e(t)\| = 0.$

Here, we propose the feedback gain as follow:

$$\begin{cases} \dot{d}_1 = k_1(e_1^2 + e_3^2 + \cdots + e_{2N-5}^2 + 2e_{2N-3}^2 - e_1e_3 - e_3e_5 - \cdots - e_{2N-7}e_{2N-5} + \\ e_1e_{2N-3} + e_3e_{2N-3} + \cdots + e_{2N-7}e_{2N-3}) \\ \dot{d}_2 = k_2(e_2^2 + e_4^2 + \cdots + e_{2N-4}^2 + 2e_{2N-2}^2 - e_2e_4 - e_4e_6 - \cdots - e_{2N-6}e_{2N-4} + \\ e_2e_{2N-2} + e_4e_{2N-2} + \cdots + e_{2N-6}e_{2N-2}) \end{cases} \qquad (3)$$
$$(k_1, k_2 > 0)$$

where $k_i > 0$ is the coefficient representing the coupling strength of the network.

Consider the coupled system with N FHN neurons, which is described by Eq. (2) in section 3. Similarly, we can have a block matrix $P(d_1^*,\ d_2^*) = (p_{i,j})_{2(N-1)\times 2(N-1)}$. The diagonal elements of P are N-1 two-order symmetrical matrices. The front N-2 matrices are： $\begin{bmatrix} -\Delta + d_1^* & \frac{1-b}{2} \\ \frac{1-b}{2} & d_2^* \end{bmatrix}$ And the last one is ：

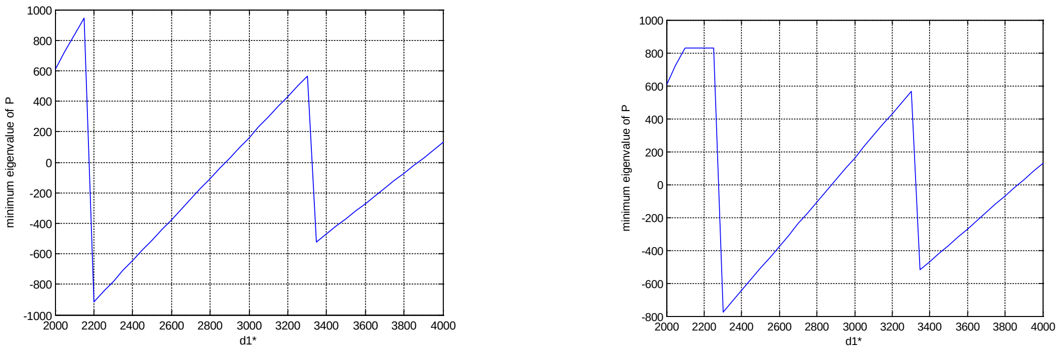
$\begin{bmatrix} -\Delta + 2d_1^* & \frac{1-b}{2} \\ \frac{1-b}{2} & 2d_2^* \end{bmatrix}$ The other non-zero elements of the matrix $P(d_1^*,\ d_2^*)$ are .

$p_{2N-3,2N-7} = p_{2N-7,2N-3} = \frac{d_1^*}{2}$, $p_{2N-5,2N-7} = p_{2N-7,2N-5} = -\frac{d_1^*}{2}$, $p_{2N-2,2N-6} = p_{2N-6,2N-2} = \frac{d_2^*}{2}$, $p_{2N-4,2N-6} = p_{2N-6,2N-4} = -\frac{d_2^*}{2}$. If there exist constants d_1^*, d_2^* such that the matrix $P(d_1^*, d_2^*)$ is positive definite, then synchronization of the multi-coupled FHN chaotic systems is achieved by adding the adaptive feedback control.

Then we have the following theorem.

Theorem. *If there exists a constant* d^* *such that the* $P(d^*)$ *is positive definite, then, by adding the adaptive feedback gain equation* (3), *the coupled system* (2) *will synchronize; that is,* $e_i(t) \to 0 (t \to \infty), i = 1,2,\cdots,2N$. *for any initial values* $(x_i(0), y_i(0)), d_i(0), (i = 1,2,\cdots,N)$.

FIG.2 shows the relation between the minimum eigenvalue $\lambda_{\min}(P)$ of the matrix P and the value of d_1^* when N=5 and 6. It can be seen from it that $\lambda_{\min}(P) > 0$ for some values of d^*.For N=5, $\lambda_{\min}(P) = 834.2540$ when $d_1^* = 2100$, $d_2^* = 3000$ .For N=6, $\lambda_{\min}(P) = 829.9135$ when $d_1^* = 2200$, $d_2^* = 2000$. Therefore, if d_1^*, d_2^* exists, for which the matrix P is positive definite, guaranteeing adaptive synchronization of the coupled system.



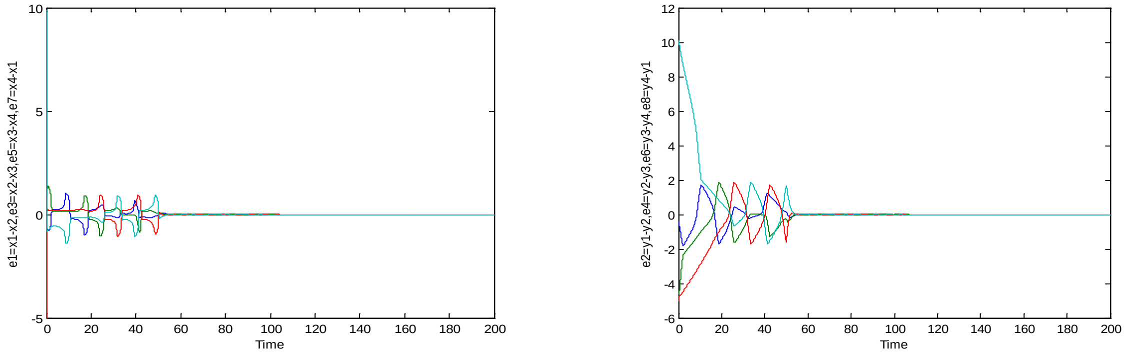
(a). $N = 5$, $d_2^* = 3000$ (b). $N = 6$, $d_2^* = 2000$

FIG.2 the relation between the minimum eigenvalue $\lambda_{\min}(P)$ of the matrix P and the value of d_1^* .

V. Numerical Simulations

In this section, numerical simulations are carried out to observe the synchronization of the multi-coupled FHN neuronal systems.

The parameters are chosen as $r = 10, b = 1, A = 0.1$, $f = 129Hz$, $k_1 = k_2 = 0.01$.And the initial values are: $x_1(0) = y_1(0) = 0.1$, $x_2(0) = y_2(0) = 0.5$, $x_3(0) = y_3(0) = 5$, $x_4(0) = y_4(0) = 10$, $d_1(0) = d_2(0) = 1$, for system (2)(N=4). FIG.3 shows time evolution curves of the synchronization errors of coupled system (2) (N=4). We switch on the coupling controller at time $t = 50ms$. As shown in FIG. 3, before the control is implemented, the individual neurons exhibit their own chaotic dynamical behaviors and are not synchronized. After the adaptive controller is applied, the errors converge to a small region around zero rapidly and then the synchronization is obtained.



(a) (b)

FIG.3 Time evolution curves of the synchronization errors of coupled system (*N*=4).

(a) $e_1 = x_1 - x_2$, $e_3 = x_2 - x_3$, $e_5 = x_3 - x_4$, $e_7 = x_4 - x_1$

(b) $e_2 = y_1 - y_2$, $e_4 = y_2 - y_3$, $e_6 = y_3 - y_4$, $e_8 = y_4 - y_1$

VI. Conclusions

This paper investigates the synchronization of a general multi-coupled FHN neurons network with uncertain coupling coefficients. An adaptive feedback controller for such network is designed. The network will synchronize by adding only one adaptive feedback gain equation for any initial values of the system without exactly estimating the coupling coefficients of the chaotic systems, which is important to practical applications. Furthermore, the effectiveness of these synchronization criteria has been demonstrated by numerical simulations.

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