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Energy and Angular Momentum Conservation Analysis of Tornado

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Abstract. Tornado always exists like funnel shaped, In this paper, according to the basic principles of tornado to establish a physical model of tornado. According to this model, a tornado of angular velocity, density and the spin turn radius in air layer parameterized description. Based on this model, the relationship between the energy of tornado and the parameters of tornado swirling airflow is explored. The rotational motion of a tornado is also restricted by the conservation of its angular momentum, so the relationship between the density and the radius and the angular velocity of the rotating airflow layer is also demonstrated.

Introduction

A tornado is a strong wind in a very unstable weather conditions, resulting from the strong convection of the air with a high speed rotating funnel shaped column of strong wind vortex, with the center of the wind speed is particularly large, highly destructive characteristics. Therefore, a lot of researches have been done to study the eddy current ratio, which can indicate the complex structure of tornado. Therefore, this paper studies the dynamics of tornado.

Description of Tornado Physical Model

As the tornado is always a funnel shape [1], the general tornado is abstracted as an idealized tornado physical model (As shown in Figure 1), In the radius of $r_{\min} - r_{\max}$ (maximum radius at the height of the tornado [2] in a tornado swirling flow layer), as the tornado swirling flow zone in the area, are concentrated around the center of the tornado air layer rapidly rotating, the regional gas flow centrifugal force and centripetal force balance layer by the existence of two a tornado will make the air flow of the compression effect of the tornado flow density increases. Suppose an instantaneous rotation of the air layer of the tornado $r_{\min} - r_{\max}$ the density of ρ.

In a high place to choose a tornado for *h* tornado, the value is small enough, and made them within the scope of the *h* height, $r_{\min}$ and $r_{\max}$ all remain the same, the rotation of the tornado rho flow density and other physical conditions remain the same, so that a tornado can abstract for the rules of the hollow cylinder (shown in Figure 1), it will help to quantitative analysis of the tornado. All of the following discussion is based on the tornado model.

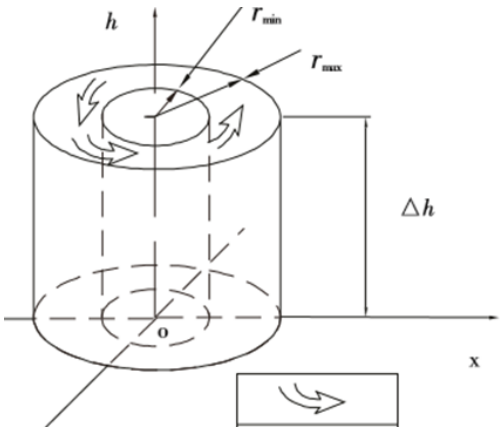

Figure 1 Physical model of tornado

Energy Analysis of Tornado

Select a tornado as part of the Δh and combined with the tornado model research and explanation. Because a tornado main rapidly rotating airflow layer and horizontal movement [3], as so Δh the height of the tornado air layer of the energy ($E_{\Delta h}$) about rotating airflow layer of rotational kinetic energy (E_0), horizontal motion (E_1), and the motion of the upward movement of the sum of the kinetic energy. That is:

$$E_{\Delta h} \approx E_0 + E_1 + E_2 \tag{1}$$

It is assumed that the horizontal velocity of the tornado is v_1, and the velocity component of the rotating air layer in the vertical direction is v_2, according to (1):

$$\begin{aligned} E_{\Delta h} \approx E_0 + E_1 + E_2 &= \sum_{r=r_{\min}}^{r_{\max}} \frac{1}{2} m v^2 + \sum_{r=r_{\min}}^{r_{\max}} \frac{1}{2} m v_1^{\ 2} + \sum_{r=r_{\min}}^{r_{\max}} \frac{1}{2} m v_2^{\ 2} = \sum_{r=r_{\min}}^{r_{\max}} \frac{1}{2} m v^2 + \sum_{r=r_{\min}}^{r_{\max}} \frac{1}{2} m (v_1^{\ 2} + v_2^{\ 2}) \\ &= \frac{1}{2} \sum_{r=r_{\min}}^{r_{\max}} (2\pi \cdot r\Delta h \Delta r \rho)(r\omega)^2 + \frac{1}{2} \sum_{r=r_{\min}}^{r_{\max}} (2\pi \cdot r\Delta h \Delta r \rho)(v_1^{\ 2} + v_2^{\ 2}) \\ &= \pi \Delta h \rho \omega^2 \int_{r_{\min}}^{r_{\max}} r^3 dr + \pi \Delta h \rho (v_1^{\ 2} + v_2^{\ 2}) \int_{r_{\min}}^{r_{\max}} r dr \\ &= \frac{1}{4} \pi \Delta h \rho \omega^2 (r_{\max}^4 - r_{\min}^4) + \frac{1}{2} \pi \Delta h \rho (v_1^{\ 2} + v_2^{\ 2})(r_{\max}^2 - r_{\min}^2) \end{aligned} \tag{2}$$

That is:

$$E_{\Delta h} \approx \left[\frac{1}{2} \omega^2 (r_{\max}^2 + r_{\min}^2) + (v_1^{\ 2} + v_2^{\ 2}) \right] \frac{1}{2} \pi \Delta h \rho (r_{\max}^2 - r_{\min}^2) \tag{3}$$

The tornado in the location unit height of energy E is:

$$E \approx \frac{E_{\Delta h}}{\Delta h} = \left[\frac{1}{2} \omega^2 (r_{\max}^2 + r_{\min}^2) + (v_1^{\ 2} + v_2^{\ 2}) \right] \frac{1}{2} \pi \rho (r_{\max}^2 - r_{\min}^2) \tag{4}$$

And as for Δh fully is small, so the tornado in the location unit height of energy is a tornado in the height of the energy. When a tornado is a part of a highly $r_{\max}, r_{\min}, \rho, \omega$ and v_1, v_2 roughly determined, can be used to estimate the height of this formula with the energy of a tornado. In general, the larger the position with the energy of a tornado, the stronger the its destructive power.

Conservation of Angular Momentum of Tornado

Calculation of Angular Momentum of Tornado Center. Angular momentum [4-5] is a universal law in physics. For a stable tornado, the angular momentum in a certain position is conserved. In order to facilitate the analysis, the angular momentum of the center position of a tornado is selected as the research object.

Figure 2 shows the rotation velocity of airflow direction, combined with figure 1 indicates that the velocity vector v air layer rotating around the center axis direction of horizontal tornado of radial r is always perpendicular rotating airflow layer is always around the center axis of the tornado circular motion, according to the expression of angular momentum $L = r \times mv$.

The angular momentum of the swirling flow layer $(r_{\min} - r_{\max})$ at the height of the tornado is obtained

$$L_{\Delta h} = \sum_{r=r_{\min}}^{r_{\max}} r \times mv = \sum_{r=r_{\min}}^{r_{\max}} r(2\pi \cdot r\Delta h \Delta r \rho)(r\omega) = 2\pi \Delta h \rho \omega \int_{r_{\min}}^{r_{\max}} r^3 dr = \frac{1}{2} \pi \Delta h \rho \omega (r_{\max}^4 - r_{\min}^4) \tag{5}$$

Then, the angular momentum L of the tornado in the position unit height is:

$$L = \frac{L_{\Delta h}}{\Delta h} = \frac{1}{2} \pi \rho \omega (r_{\max}^4 - r_{\min}^4) \tag{6}$$

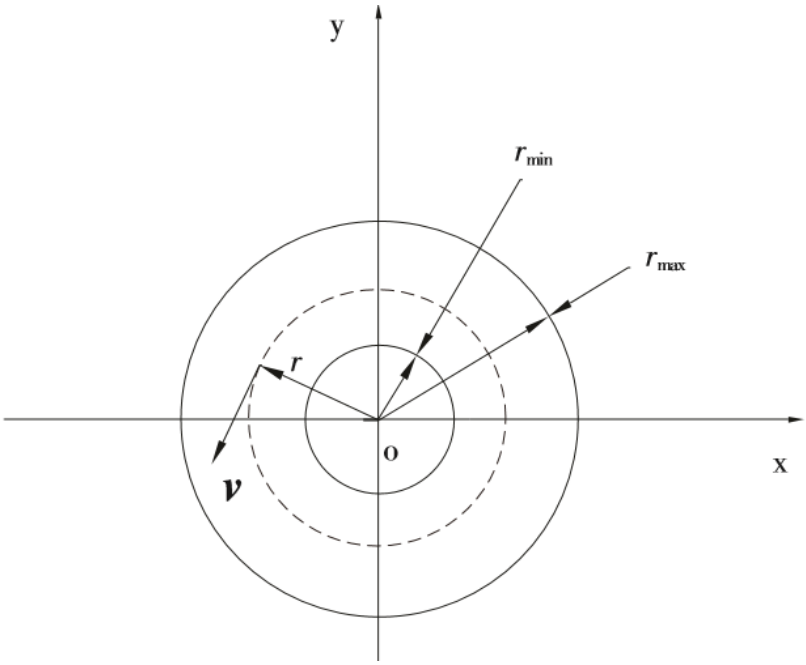

Figure 2 Rotational velocity direction

And because Δh is sufficiently small, so a tornado in angular momentum L is the position of the unit height of a tornado in the height position of the angular momentum. Due to the conservation of angular momentum, this can explain the tornado in the ground rotation of the bottom of the tornado, "body" is more subtle phenomenon.

The Influence of Angular Momentum Conservation on the Rotational Flow of Tornado. Assume that the height of the center position of the angular momentum of the tornado at any time t_1 is L_1, after the time Δt, the height of the center position in the tornado $t_1 + \Delta t$ time for angular momentum L_2, according to the angular momentum conservation $L_1 = L_2$.

After time Δt, when the tornado spinning air layer density and radius change and angular velocity is constant, we can know from $L_1 = L_2$:

$$\frac{1}{2}\pi\rho_1\omega(r_{1\max}^4 - r_{1\min}^4) = \frac{1}{2}\pi\rho_2\omega(r_{2\max}^4 - r_{2\min}^4) \tag{7}$$

That is:

$$\frac{\rho_2}{\rho_1} = \frac{r_{1\max}^4 - r_{1\min}^4}{r_{2\max}^4 - r_{2\min}^4} \tag{8}$$

But because $r_{1\max}^4 >> r_{1\min}^4, r_{2\max}^4 >> r_{2\min}^4$

So

$$\frac{\rho_2}{\rho_1} = \frac{r_{1\max}^4 - r_{1\min}^4}{r_{2\max}^4 - r_{2\min}^4} = \frac{r_{1\max}^4}{r_{2\max}^4} \tag{9}$$

When the tornado touches the ground, it will dust, debris and other places on the ground inside the tornado inhalation, resulting in increased density of the bottom end. According to the formula (9), when the angular velocity is constant, the radius of the rotating [6] air flow layer will decrease when the density of the height increases. That is when a tornado in a moment the position of high quality rotation of the air layer is increased from m_1 to m_2 (ρ_1 to ρ_2 increase), the tornado in the position of rotary flow height radius also decreased from $r_{1\max}$ to $r_{2\max}$.

After time Δt, when the tornado spinning air layer density and angular velocity change and radius is constant, we can know from $L_1 = L_2$:

$$\frac{1}{2}\pi\rho_1\omega_1(r_{1\max}^4 - r_{1\min}^4) = \frac{1}{2}\pi\rho_2\omega_2(r_{2\max}^4 - r_{2\min}^4) \tag{10}$$

That is:

$$\frac{\rho_2}{\rho_1} = \frac{\omega_1}{\omega_2} \tag{11}$$

By formula (11) shows that when the tornado swirling flow layer through the Δt and the density increases, the radius is constant, is inversely proportional to the angular velocity and density. For example, when a tornado touches the ground, the density of the bottom of the rotating air layer increases, the radius of the same case, the angular velocity will be reduced, which will distort the tornado, even in a short period of time to collapse.

Conclusion and Prospect

Based on the analysis of the energy of the tornado, the mathematical expression of the energy of the tornado is put forward, which provides a theoretical basis for the accurate determination of the energy of the tornado.

According to the conservation of angular momentum of the tornado was analyzed, found in the rotating angular velocity of airflow layer under the same side of the four rotating airflow layer density and the maximum radius is inversely proportional to the rotation of the tornado was on the ground at the bottom of the "body" "natural phenomena explained. When the radius is constant, the density of the rotating air layer is inversely proportional to its angular velocity, which explains the cause of the distortion.

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