Study on the Influence of Flow on Water Temperature

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Abstract. Our goal is to establish a model that makes the optimal strategy to keep a stable temperature and consume minimum amount of water. In specific, we are required to determine the degree to which are mentioned in the problem affect our model. In addition, we also need to consider the situation where bubble bath additive is used in the tub. We need to solve the following tasks. We divide the problem into two-phases. In the first phase, we imply the equations of heat quantity, and we generate a mathematical model to obtain convex equation of minimum flow. In this part, we assume that there was no heat transfer process between the human body and the external environment. After some theoretic derivation, we obtain a nonlinear relation. In the second phase, we establish an optimization model to optimize the total number of water. To simplification we defined an angle between the human body and the bottom of bathtub to describe the action extent of bather. Eventually, we find out the optimal value of the size of the bathtub and the angle. Since the coefficient value of the exchanging heat between air and water approximately equal to 0, there is no heat exchange in this part. After calculation, we can come to a conclusion that addition of bubble bath agent will not affect our model's results.

1. Introduction

We are required to develop mathematical models and we face mainly three problems: In order to keep a suitable temperature for people and save water, we require to make the best strategy. Ensure the degree to which the shape and volume of the tub, the shape and volume of the person in the bathtub, and the motions made by the person in the bathtub affects our model. Discuss the influence on our model after adding a bubble bath additive to the tub.

2. Our Models

From the thermodynamics mechanism, we know the quantity of heat that system absorb from hot water getting into the bathtub:

$$q_x = q_f \tag{1}$$

$$q_x = \rho q_v c(T_3 - T_2) \tag{2}$$

$$q_f = q_1 + q_2 \tag{3}$$

$$q_1 = \frac{A(T_2 - T_1)}{\frac{1}{\alpha_1} + \frac{d}{\lambda} + \frac{1}{\alpha_2}} \tag{4}$$

$$q_2 = \frac{A(T_2 - T_1)}{\frac{1}{\alpha_3}} \tag{5}$$

$$\mathrm{q_V} = \frac{\frac{\mathrm{A(T_2 - T_1)}}{\frac{1}{\alpha_1} + \frac{\mathrm{d}}{\lambda} + \frac{1}{\alpha_2}} + \frac{\mathrm{A_1(T_2 - T_1)}}{\frac{1}{\alpha_3}}}{\rho \mathrm{c(T_3 - T_2)}} \tag{6}$$

We assume that human finish bathing in one hour. So we can calculate the total hot water flow during this time:

$$\mathrm{Q_L} = 3600 \times \mathrm{q_V} \tag{7}$$

Eventually, we can obtain a specific connection between the hot water flow unit time $\mathrm{q_V}$ and the other parameters, such as the surface area of bathtub, the density of water, the specific heat capacity of water and the convective heat transfer coefficient of various substances.

2.1 Parameter Optimization Model

The bathing water will be drained away frequently.We take the total water flow as two parts containing the volume of the water in tub(V1-V2) and the water flow that partially escape to the bathtub QS. We can get:

$$Q = V_1 - V_2 + Q_S \tag{8}$$

$$Q_s = Q_L \tag{9}$$

$$Q = V_1 - V_2 + Q_L \tag{10}$$

Based on the Wissler Model [1], we also divide the human body into sections and each segment is approximated as a cylinder. However, what's different is that we treat abdomen and chest as a segment.

To make the volume maximum we take the maximum value of the reference value.

Finaly,we get $V_{2\max} = 0.090653\mathrm{m}^3$ (11)

$$V_1 = abh \tag{12}$$

$$h = l_1 \times \sin\alpha \tag{13}$$

$$V_1 = abl_1 \times \sin\alpha \tag{14}$$

$$Q = abl_1 \times \sin\alpha - 0.090563 + Q_L \tag{15}$$

$$Q_{S\min} = Q_{L\min} \tag{16}$$

Then we discuss how to make total input of hot water Q_L to the minimum

in a given time, from the Eq.(7) we can conclude that when q_v is the minimum, Q_L is the minimum too.

Therefore, we only need to study the condition of minimum value of A In the equation(6):

$$A = 2h(a + b) \tag{17}$$

$$A_1 = ab \tag{18}$$

Where A stands for the vertical area of the bathtub,A_1 stands for the upper horizontal area of bathtub, a stands for bath width, b stands for bath length.

Then we get the mathematical expression:

$$Q = abl_1 \times \sin\alpha - 0.090563 + \frac{\frac{2l_1 \times \sin\alpha(a+b)(T_2-T_1)}{\frac{1}{\alpha_1}+\frac{d}{\lambda}+\frac{1}{\alpha_2}} + \frac{ab(T_2-T_1)}{\frac{1}{\alpha_3}}}{\rho c(T_3-T_2)} \quad (19).$$

$$q1 = 281.63\ \mathrm{m}^3 \tag{20}$$

$$q2 = 1260\ \mathrm{m}^3 \tag{21}$$

$q_v = 7.3446 \times 10^{-6} \times (a + b) * \sin\alpha + 4.8246 \times 10^{-5} \times ab$(22)

$Q_{\min} = 0.0264(a + b) \times \sin\alpha + (0.1737 + 0.52 \times \sin\alpha)ab - 0.090653$ (23)

$$\text{s.t.} \quad \begin{cases} 0.45 \le a \le 0.9 \\ 1.4 \le b \le 1.7 \\ \pi/6 \le \alpha \le \pi/3 \end{cases}$$

We get the final conclusion:

$$Q_{\min} = 0.207\ \mathrm{m}^3 a = 0.45\ \mathrm{m} b = 1.4\ \mathrm{m} \alpha = \pi/6 V_1 = 0.33\ \ \mathrm{m}^3$$

3. AHP Model

We use the analytic hierarchy process to research the contribution degree. After discussion and ananlysis, we know that the main factors which influence the total volume of the water are the volume of tub, the volume of person and the motions of the person in tub.

Layer O :The total volume of the water

Layer C: The shape and volume of the bathtub. The motions of the person

Layer p: Length, width, Length, Circumference, Angle

According to Saaty and others, the maximum eigenvalue of the square matrix is used as the weight vector.

Consistency index CI=(ε -n)/(n-1)

Take $\omega_1^{(3)}\omega_2^{(3)}\omega_3^{(3)}$ as the column vector matrix $W^{(3)}=[\omega_1^{(3)}\omega_2^{(3)}\omega_3^{(3)}]$

The combination weight vector of the first layer is the third layer:

$$\omega^{(3)}=W^{(3)}\omega^{(2)}=[1.0277\ 0.3426\ 0.2684\ 0.3814\ 0.8481]^T$$

Consistency check:

The numerical value of the random consistency index RI is shown in the follow Combinatorial consistency test:

Second layer consistency index and random consistency index:

$CI^{(2)}=CI^{(2)}\omega^{(1)}=(\varepsilon-2)/2=0$

$RI^{(2)}=0.58$

$CI^{(3)}=[CI_1^{(3)}CI_2^{(3)}CI_3^{(3)}]\omega^{(2)}=-0.111$

$RI^{(3)}=[RI_1^{(3)}RI_2^{(3)}RI_3^{(3)}]\omega^{(2)}=-1.5686$

$CR^{(2)}=CI^{(2)}/RI^{(2)}=0$

$CR^{(3)}=CI^{(3)}/RI^{(3)}=0.0707$

$CR^{*}=CR^{(2)}+CR^{(3)}=0.0707<0.1$

So we can know that the combination of consistency test, ω(3)can be used as the basis for the final decision.

4. Conclusion

In the task 1, using the model built, we find the relationship between the total water flow and the factors that affect it. And we calculate the value of the minimum flow. In task 2, the model we developed could distinguish principle factors and secondary factors of the total water flow. In task 3, considering the human bathing with a bubble bath additive, our final outcome will not change. So we come to a conclusion that adding bubble bath additive will not affect our results of models . But it will change the direct heat exchange between air and water.

References

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