

# Model of Strategy of Relative Stability and Flow for Kariba Dam

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**Abstract.** If tourism benefits, plutonomy benefits and generated hydropower are incorporated in the mathematical model to be studied, it is bound to make the model very complex and difficult to solve. To solve this problem, the key role of dam system in flood control, water and sediment regulation should be considered, then the problems would be simplified, and the accuracy of the main project optimization planning could be ensured. At the same time, in order to overcome the limitations of nonlinear programming in the research of small and medium-sized dams, the model of relative stability and flow strategy of new dam system is established based on the principle of system dynamics.

## 1. Introduction

The Kariba Dam is a double curvature concrete arch dam in the Kariba Gorge of the Zambezi River basin, which is built to generate hydropower basically. Institute of Risk Management of South Africa warned that the dam is in dire need of maintenance. The dam forms Lake Kariba which is one of the largest reservoirs in the world, holding 185 cubic kilometers of water.

There are lots of previous researches on managing Zambezi River, which can facilitate our comprehension of the problem.

For example, Ashok Swain and Patrik Stålgren researched existing circumstances of Zambezi River basin. The team of Philip H. Brown modeled the costs and benefits of dam construction from a multidisciplinary perspective.

But all these attempts failed. So we need to build a more scientific and available scheme to solve the problems above.

## 2. Calculation of the basic characteristics of the dam

(1) Dam height and storage capacity

$$V = 825 \times \left(\frac{H}{2}\right)^{2.535} \quad (2.1)$$

(2) Dam height and area of sediment

$$S = 330H^{2.55} \quad (2.2)$$

(3) Dam height, crest length and earthwork volume of dam body

$$W = 1.37H^{2.035}b^{0.838} \quad (2.3)$$

## 3. Calculation of spillway flood discharge and flood modulation

### 3.1 Calculation of cost of the spillway

The flow through the spillway is the core of calculation of spillway project cost. It is necessary to consider the discharge flow of the upstream dam and the total discharge flow in a certain period in calculating the maximum flow of each dam spillway, and then use the Gaoqieliin Formula to calculate the maximum flow of spillway of the dam. The cost of the spillway project can then be calculated from the regression equation between flow and engineering costs. [7] Calculation formula of spillway's maximum flow is an empirical formula:

$$W_y = (2.422Q^{0.4316})L \quad (3.1)$$

Where:

$W_y$ : Spillway works (or construction costs)

$Q$ : Maximum flow of spillway

$L$ : Length of spillway

### 3.2 Calculation of flood modulation for dam system

When the upstream dam is flooded, the downstream dam's spillway flow is calculated as follows:

$$Q = (q_p + q_u) \left(1 - \frac{V''}{W_p + W_u}\right) \quad (3.2)$$

$q_p$ : Peak flow with frequency  $p$  in interval, can be found through historical data at the same period ( $\text{m}^3/\text{s}$ )

$q_u$ : Maximum flow of spillway of adjacent upstream dam ( $\text{m}^3/\text{s}$ )

$W_p$ : The total flood volume with frequency  $p$  in interval area ( $10^4 \text{ m}^3$ )

$W_u$ : Before calculating the maximum spillway discharge of the dam, the total flood discharge of the adjacent upstream dam spillways ( $10^4 \text{ m}^3$ )

$V''$ : The detention capacity of dam which is one of the decision variables of optimization planning ( $10^4 \text{ m}^3$ )

## 4. Establishment of model of strategy of relative stability and flow

Description of the objective function: Objective function of multi-objective optimization is designed for the purpose of minimizing cost-benefit and maximizing safety factor. It should be noted that, unlike the objective function of the "Dam Number Determination" model described above, only the calculations of spillway and peak shaving are carried out in the strategy model without account for the full cost. When the objective function reaches the minimum value, it means that the value of the corresponding decision variable is the best.

Objective function:

$$\begin{cases} \max I_p = \frac{Wp_i}{S(h + h_c)} \\ \min f(X_i) = \sum_{i=1}^n \{d_v \alpha_i X_i^{\beta_i} + d_{yi} \delta \left[ (q_{pi} + q_{ui}) \left(1 - \frac{X_{n+i}}{W_{pi} + W_{ui}}\right) \right]^{\varepsilon_i} \\ \times L_{yi} + d_{yq} (U_i + 2h_{ki}) L_{yi} \} - \sum_{i=1}^n T_s \beta R \psi_i X_i^{\varphi_i} \\ X_i \geq 0 \\ V_i - X_i - X_{n+i} \geq 0 \\ \frac{A_i W_{X_i}}{\gamma} - 30 \geq 0 \end{cases}$$

Where:

(1)  $n$  —The number of dam sites

(2)  $x_i$  —Decision variables, marking the mud storage capacity of the  $i$ th dam site,  $i = 1, 2, \dots, n$

(3)  $x_{n+i}$  —Decision variables, marking flood storage capacity of the  $i$ th dam site,  $i = 1, 2, \dots, n$

(4)  $d_b$  —Unit price of earthwork volume of dam body

(5)  $dy_t$  —Unit price of earthwork volume of spillway

(6)  $dy_q$  —Unit price of unit of spillway

(7)  $\alpha_i$  —The  $i$ -th dam according to the requirements of the dam volume engineering coefficient

(8)  $\beta_i$  —The  $i$ -th dam according to the storage capacity for dam engineering quantity index

(9)  $\delta_i$  —The  $i$ -th dam calculates the coefficient of the spillway earthwork according to the spillway flow

- (10)  $\varepsilon_i$  —The  $i$ -th dam calculates the index of spillway earthwork according to the spillway flow  
 (11)  $\varphi_i$  —The coefficient of the dam area required by the  $i$ -th dam according to the silt storage  
 (12)  $\psi_i$  —The  $i$ -th dam is based on the index of dam area required by the silo

## 5. Restrictions

Non - negative constraints: The reservoir capacity of the dam cannot be negative, that is,  $x_i \geq 0$ , ( $i = 1, 2, \dots, 2n$ ),  $n$  is the number of dam sites, the dam on each dam has a detention and detention capacity of flood detention capacity, so the type  $i$  in the maximum value of  $2n$ .

Terrain constraint: Topography limits the growth of dam height, the maximum dam height corresponding storage capacity is the maximum storage capacity of terrain constraints, so the actual storage capacity of the dam should be less than the maximum, that is,  $V_i - x_i - x_{n+i} \geq 0$ ,  $i = 1, 2, \dots, n$ .

Sedimentation time constraints:  $A_i M_{xi/\gamma} - 30 \geq 0$ .

## 6. Conclusion

We fully considered the three options of their own characteristics. We build different computational models for each option separately, and get their respective costs and benefits. In the detailed analysis of Option 3, we considered many factors and consulted the sufficient data. Accordingly, there is sufficient basis for the proposed model and the establishment of the dam system.

## References

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