

# Improved Analytic Hierarchy in Water Diversion Scheme

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**Abstract.** This paper is to propose a water diversion scheme to alleviate the shortage of water resources. We select six main reservoirs around Beijing, consider each reservoir characteristic parameter and use Improved Analytic Hierarchy Process to calculate the weight which can fix the problem of subjectivity caused by conventional methods. And we regard the weight as reservoir water transfer ratio. In this model, we use relative proportion to calculate the scale. By this way, we avoid the influence of subjectivity. In the end, the water transfer ratio about different reservoirs is presented. Xidayang reservoir accounts for 21.34% of reservoir water supply, providing the largest amount of water.

## 1. Introduction

We list the main reservoirs around Beijing and use the Improved Analytic Hierarchy Process (IAPH) <sup>[1]</sup> to get our decision-making project of reservoir water supply distribution. We improve the traditional Analytic Hierarchy Process (AHP) <sup>[2][3]</sup> and use numerical ratio that have the same dimension to calculate the scale. Finally we work out the optimal allocation of water supply according to the relationship of each weight vector. This method avoids the subjectivity when selecting the scale.

## 2. Improved Analytic Hierarchy Process (IAHP)

### 2.1 Model Design

In order to get a intervention plan to improve Beijing’ s water situation, we adopt the Improved Analytic Hierarchy Process (IAHP), focus on the reservoir water diversion and select six main reservoirs around Beijing <sup>[4][5]</sup> to establish a model and get specific measures.

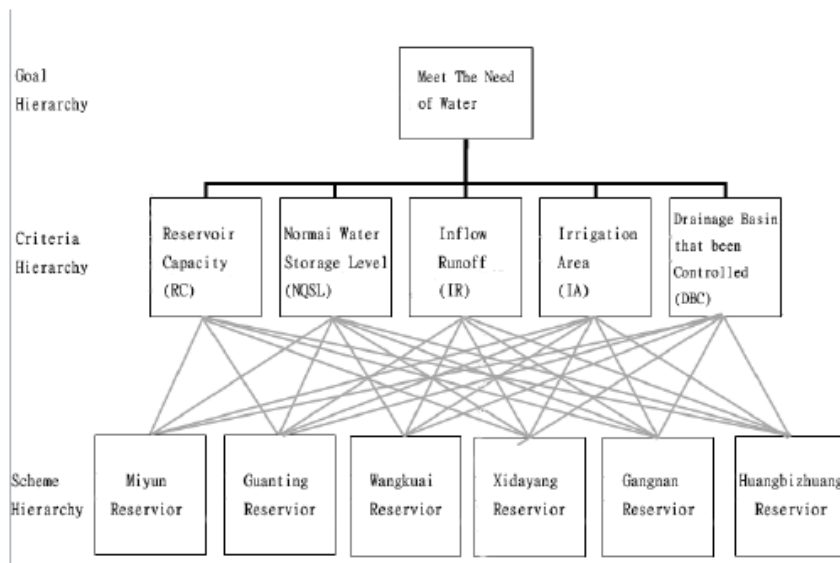


Figure 1: Hierarchical structure.

Table 1: Selection criteria of the scale

scale	implication
1	Factor i and j have equal value;
3	Factor i has a slightly higher value than j;
5	Factor i has a strongly higher value than j;
7	Factor i has a very strongly higher value than j;
9	Factor i has an absolutely higher value than j;
2,4,6,8	Intermediate scales between two adjacent judgments;
Reciprocals	Factor i has a lower value than j;

**2.2 Calculation**

**2.2.1 Set up the Weight Vector between Goal Hierarchy and Criteria Hierarchy**

According to the importance of these five indexes (not giving unnecessary details because o length of the article), we have the paired comparison matrix of goal hierarchy toward criteria hierarchy:

$$A = \begin{bmatrix} 1 & 4/5 & 4 & 4/5 & 4/3 \\ 5/4 & 1 & 5 & 1 & 5/3 \\ 1/4 & 1/5 & 1 & 1/5 & 1/3 \\ 5/4 & 1 & 5 & 1 & 5/3 \\ 3/4 & 3/5 & 3 & 3/5 & 1 \end{bmatrix} \tag{1}$$

Then we use MATLAB and obtain the maximal characteristic root of A:

$$\lambda_m = 5$$

The sum of characteristic roots N = 5. The corresponding normalized values of the weight vector is:

$$W_A = [0.2222 \quad 0.2778 \quad 0.0556 \quad 0.2778 \quad 0.1667]^T \tag{2}$$

**2.2.2 Calculation the Model**

When selecting the scale, we improved the traditional Analytic Hierarchy Process AHP. The traditional method determine the influence of each criterion through artificial judgement and it is inevitable to have a subjective influence on the result. We improve the traditional method to have a more objective selection of scale.

Firstly, we do proportional calculation to the values with the same dimension, then calculate the relative proportion as the scale. The specific calculation method is as follows:

Set  $p_{ij}$  as the index value j of scheme i;  $\sigma_{ij}$  as the proportion of the criterion value j of scheme i for the criterion value j of all schemes. That is:

$$\delta_{ij} = \frac{p_{ij}}{\sum_{i=1}^6 p_{ij}} \tag{4}$$

$\eta_{ij}$  means the relative proportion of scheme i toward scheme i - 1.

$$\eta_{ij} = \begin{cases} 1 & i = 1 \\ \frac{\delta_{i-1j} - \delta_{ij}}{\delta_{ij}} * 10 & i \geq 2 \end{cases} \tag{5}$$

Calculate the criterion value accounts for the proportion of the total criterion value of six scheme hierarchies and determine the weight matrix.

	reservoir capacity ( a hundred million m <sup>3</sup> )		normal water storage level(m)	
	first-hand data	proportion	first-hand data	proportion
Miyun Reservoir	40	0.3479	157.5	0.1214
Guanting reservoir,	21.9	0.1905	479	0.3692
Wanguai reservoir	13.89	0.1208	200.4	0.1545
West Ocean reservoir	11.37	0.0989	140.5	0.1083
Gangnan reservoir	15.71	0.1366	200.0	0.1542
Huangbizhuang reservoir	12.10	0.1052	120	0.0925
total	114.97	1	1297.4	1

inflow runoff(a hundred million m <sup>3</sup> )		Irrigation area(million mu)		drainage basin that been controlled(k m <sup>2</sup> )	
first-hand data	proportion	first-hand data	proportion	first-hand data	proportion
9.5	0.1442	200	0.2789	15776	0.1431
3.0	0.0455	150	0.2092	47000	0.4262
7.5	0.1138	140	0.1953	3770	0.0342
7.8	0.1184	50	0.0697	4420	0.0401
16.6	0.2519	20	0.0279	15900	0.1442
21.5	0.3263	157	0.2190	23400	0.2122
65.9	1	717	1	110268	1

Figure2: Schemes and indexes

Illustrated by the example of reservoir capacity, we can work out the scale according to the proportion:

For Miyun Reservoir account for the largest share, we take its scale for 1.Then we calculate the relative value of other reservoirs toward Miyun Reservoir:

Guanting Reservoir:(0.3479 - 0.1905)/0.3479 × 10 = 4.524, take scale for 4.

Wanguai Reservoir:(0.1366- 0.1208)/0.1366 × 10 = 1.157, take scale for 1.

West Ocean Reservoir:(0.1052- 0.0989)/0.1052 × 10 = 0.060, take scale for 1.

Gangnan Reservoir:(0.1905 - 0.1366)/0.1905 × 10 = 2.829, take scale for 3.

Huangbizhuang Reservoir:(0.1208- 0.1052)/0.1208 × 10 = 1.291, take scale for1.

So we have the paired comparison matrix of reservoir-capacity criterion toward scheme hierarchy:

$$B_1 = \begin{bmatrix} 1 & 1/4 & 1 & 1 & 1/3 & 1 \\ 4 & 1 & 4 & 4 & 4/3 & 4 \\ 1 & 1/4 & 1 & 1 & 1/3 & 1 \\ 1 & 1/4 & 1 & 1 & 1/3 & 1 \\ 3 & 3/4 & 3 & 3 & 1 & 1 \\ 1 & 1/4 & 1 & 1 & 1 & 1 \end{bmatrix} \tag{6}$$

Then we use MATLAB to obtain the maximal characteristic root of  $B_1$  :

$$\lambda_{1m} = 6.1414.$$

The sum of characteristic roots N = 6.The corresponding normalized values of the weight vector is:

$$W_{B_1} = [0.0921 \ 0.3682 \ 0.0921 \ 0.0921 \ 0.2378 \ 0.1179]^T \tag{7}$$

The same procedure may be easily adapted to comparison matrix of other criterion.

Analysis their consistency index CI:

$$CI = \frac{\lambda_m - n}{n - 1} \tag{8}$$

Compare with the random consistency index RI and we get consistency rate CR:

$$CR = \frac{CI}{RI} \tag{9}$$

When  $CR \leq 0.1$ , we can think that the consistency of A is within the allowable range and the normalized feature vector can be used as the weight vector.

The following data are satisfied with the consistency check and five groups of paired comparison matrix are consistency matrix, shows the accuracy of the relationship between criterion hierarchy and scheme hierarchy.

Table 2: Table for consistency check

	RC	NWSL	IR	IA	DBC
n					
CI	0.0283				
RI	1.24	1.24	1.24	1.24	1.24
CR	0.0228				

### 2.2.3 Combination Weight Vector

$$W = W_B * W_A \tag{10}$$

And we have:  $W = [0.1063 \quad 0.1514 \quad 0.1884 \quad 0.2134 \quad 0.2087 \quad 0.1318]^T$

### 3. Conclusion

According to our model, we got the optimal water diversion scheme of water supply for Beijing area, and the distribution of each reservoir water supply is shown in Figure 3.

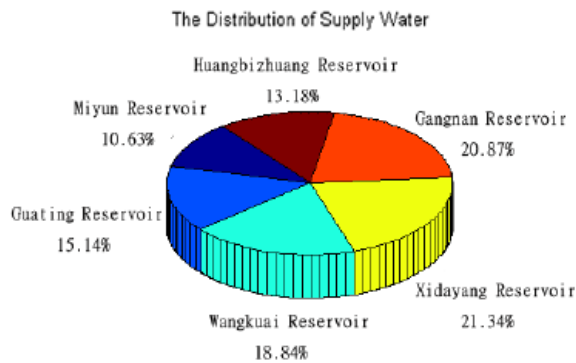


Figure 3: The distribution of each reservoir water supply.

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