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Research on the point contact mechanics model of sleeve structure with the contact process of the inner core concerned

Xiaofeng Zhao[1, 2, a], Bo Shen[1, 2, b,*], Kejian Ma[1,2, c]

[1]Space Structures Research Center, GuiZhou University, Guiyang 550003, China;

[2]Key Lab of Structure Engineering of GuiZhou Province, Guiyang 550025, China.

[a]xfzhao0923@163.com, [*b]gy-shenbo@sohu.com, [c]makejian2002@163.com

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Abstract. To study the sleeve structure under the axial pressure, taking the contact process of the inner core into account, the deformation process of point contact between the inner core and the flexible sleeve was studied theoretically. The second order differential equilibrium equations of a small deflection were adopted to deduce the formula of the physical quantities such as the deflection, moment, shear and contact reaction of the inner core and the sleeve. Experimental studies have been made with the results in the reference [14], and the following results can be obtained: 1). If use the mechanical model of the contact process of inner core, it can reflect the order process of the sleeve structure when point contact happen; 2). The mechanical model of the inner core calculated by using the computing model that concerns the contract process of inner core provides basis for the end restraint design of inner core and outer sleeve; 3). This mechanical model is compatible with real conditions and can make the sleeve be applied in engineering practice effectively.

1. Introduction

To manage instability that can easily happen in framed structure, related members are produced. Its application has made a difference in a great deal of engineering practices[1-3]. One of instability management devices is sleeved column member. If there is accidental load, sleeved column members could offset transient load impact and reduce structural dynamic response and further lighten overall damage by inner core buckling. Sleeved column has become popular with the advantages of stability and high load capacity[4-6].

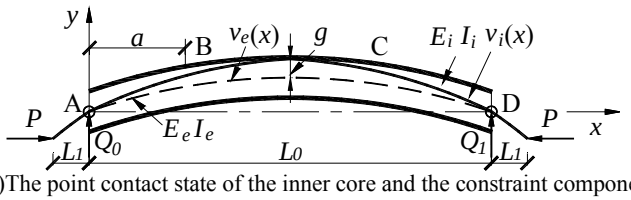
YIN-Zhan-zhong has axial compression test towards inner core of rigid sleeved column [7]. Sridhara B Na has theoretical research on point contact between flexible sleeved column and inner core through equilibrium equation, and deduces relevant calculation formulas on physical quantity [8]. Domokos G Chai H and Poteau A has theoretical research and experiment towards an elastic column under axial compression when outer sleeve is rigid [9-11]. Shen Bo, with the help of flexible linear contact calculation model, studies a calculation formulas of disturbance degree generated by rotating shaft, axial displacement, and contact internal force with flexible sleeved column constrained by hinges on both ends [12]. Shen Bo studies, as the increasing axial compression, the deformation process between inner core and flexible sleeved column when the contact shifts from linear contact to buckling linear contact [13]. Shen Bo studies sleeved column member under axial compression by theory and experiment [14]. But, it needs to point out that mechanical model in theoretical research is not perfectly matched with experiment outcome.

Based on previous researches, when sleeved column member is undergoing axial compression and its kernel buckling, kernel and outer sleeve in position of point contact are analyzed. In this paper, considering the impact of inner core extension of sleeved column member in theoretical analysis process, a mechanics model based on point contact between inner core and sleeved column is established. Then, transformation process of inner core that is under axial compression and bound by flexible sleeved column through small deflection theory and linear theory of elasticity is studied, deducing relevant formulas on inner core and some physical quantities including sleeved column deflection, bending moment, shearing force. Finally, to confirm the mechanical model by comparing

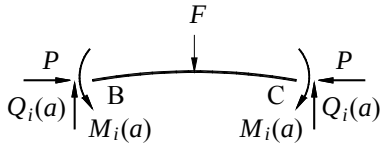
the outcome of deduced formula and the formula in bibliography [14].

2. The Mechanical Model That Takes The Extension Section of Inner Core Into Consideration and Parameter Definition

In order to facilitate the installation and the use of sleeved column, its inner core needs to extend out of the sleeve and the ends constraint by hinges. Besides, the inner core and the sleeve ends connect with caging device. The relevant mechanical model is shown in Figure 1, which presents the deformation of the inner core axis and the sleeve axis when the inner core and the outer sleeve are in point contact.



(a)The point contact state of the inner core and the constraint component



(b)The calculation model of the contact length of the inner core

Fig. 1 Flexible line contact between axial compression member and restrained member

When deducing the contact equation between the inner core and sleeved point, the symmetry of the deformation is not taken into account, and the graphs, deductions and examples of the dimensionless quantity are given. The following basic assumptions are used: the inner core has first and second orders mixed initial bending; the sleeve has no initial bending; the material is linear elasticity; the deformation is small; and the friction between the inner core and the sleeve is ignored.

In theoretical analysis, dimensionless form is used. The connotation of symbol is as follows: letters with a horizontal line above represent dimensional quantity, while letters without horizontal lines above represent that dimensionless quantity; the subscript i represents the corresponding physical quantity of inner core, and the subscript e indicates the corresponding physical quantity of the outer sleeve. As shown in Figure 2, $\overline{P}$ indicates the axial pressure, $\overline{x}$ the axial coordinates, $\overline{y}$ the horizontal coordinates, $\overline{L}_0$ the length of the outer sleeve, $\overline{L}_1$ the length of each side of the core extension of the sleeve, $\overline{f}$ midpoint deflection of inner core, $\overline{g}$ the net clearance between inner core and outer sleeve, $\overline{v}_{i0}(\overline{x})$ the initial bending of the inner core, $\overline{v}_i(\overline{x}),\overline{v}_e(\overline{x})$ respectively indicate the deflection of the inner core and the outer sleeve, $\overline{E}_i\overline{I}_i,\overline{E}_e\overline{I}_e$ respectively represent the bending stiffness of the inner core and outer sleeve. While other physical quantities follow the common usage, $\overline{M}_i(\overline{x})$ indicates inner core bending moment, $\overline{Q}_i(\overline{x})$ the shearing force of inner core, $\overline{Q}_0$, $\overline{Q}_1$ respectively indicate the reaction force of inner core and the outer sleeve point contacting $\overline{Q}_c$ resultant force of concentrated contact force of the inner core and the outer sleeve contact point, and $\overline{\delta}_M$ the axial displacement produced by blending of the inner core, as is shown in Figure.5 and 6. Let $\overline{P}_E=\pi^2\overline{E}_i\overline{I}_i/\overline{L}^2$, and define it as dimensionless quantity:

$$x=\frac{\overline{x}}{\overline{L}_0},a=\frac{\overline{a}}{\overline{L}_0},L_1=\frac{\overline{L}_1}{\overline{L}_0},\ f=\frac{\overline{f}}{\overline{L}_0},\ g=\frac{\overline{g}}{\overline{L}_0},\ p=\frac{\overline{P}}{\overline{P}_E},\eta=\pi\sqrt{p},v_{i0}(x)=\frac{\overline{v}_{i0}(\overline{x})}{\overline{L}_0} \tag{1}$$

$$v_i(x)=\frac{\overline{v}_i(\overline{x})}{\overline{L}_0},v_e(x)=\frac{\overline{v}_e(\overline{x})}{\overline{L}_0},\delta_M=\frac{\overline{\delta}_M}{\overline{L}_0},M_i(x)=\frac{\overline{L}_0}{\overline{E}_i\overline{I}_i}\overline{M}_i(\overline{x}),Q_i(x)=\frac{\overline{L}_0^{\,2}}{\overline{E}_i\overline{I}_i}\overline{Q}_i(\overline{x}) \tag{2}$$

$$Q_0=\frac{\overline{L}_0^{\,2}}{\overline{E}_i\overline{I}_i}\overline{Q}_0,Q_1=\frac{\overline{L}_0^{\,2}}{\overline{E}_i\overline{I}_i}\overline{Q}_1,Q_c=\frac{\overline{L}_0^{\,2}}{\overline{E}_i\overline{I}_i}\overline{Q}_c,r_i=\sqrt{\frac{\overline{E}_i\overline{I}_i}{\overline{E}_i\overline{I}_i+\overline{E}_e\overline{I}_e}},\beta=\frac{\overline{E}_i\overline{I}_i}{\overline{E}_e\overline{I}_e} \tag{3}$$

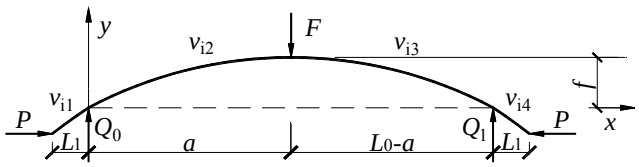
According to the definition of the dimensionless quantity and constitutive equation, geometric relation and equilibrium equation [15, 16] of the inner core micro-body, the following relation is derived:

$$\begin{cases} M_i(x) = -v_i''(x)\ ,\ Q_i(x) = v_i'''(x) + \eta^2 v_i'(x)\ ,\ q(x) = -\left[v_i^{IV}(x) + \eta^2 v_i''(x)\right] \\ \delta_M = \int_0^{0.5} \left[v_i'(x)\right]^2 dx \end{cases} \tag{4}$$

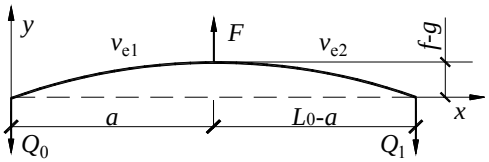
Thereinto, $(\bullet)'$ represents the first derivative with respect to x.

3. Basic Equation and Derivation of Point Contact

As the axial force increases from P to P_E, the inner core shaft undergoes a first order buckling mode, and its external surface comes to contact with the internal surface of the constraining member. Meanwhile, the peripheral restraining members do not bend or deform. As the increasing axial force P exceeds P_E, the deformation of relevant axial compression member continues to develop, and comes to point-contact with the internal wall of the constraining member, which then starts to bend, as shown in Figure 2. When the axial compression member and the constraining member are equal in curvature at the contact position, the state of point contact ends, and the axial force P at this time is the end load of the point contact.



(a)The force diagram of the point contact in the inner core



(b)The force diagram of the point contact in the sleeve

Fig. 2 Diagrams of load and point contact reactions between the inner core and the sleeve

According to the equilibrium equation of inner core, there is:

$$F_0 = \frac{\bar{Q}_0 \bar{L}_0}{\bar{L}_1 + \bar{L}_0 - \bar{a}},\ \bar{Q}_1 = \frac{\bar{a} - \bar{L}_1}{\bar{L}_1 + \bar{L}_0 - \bar{a}} \bar{Q}_0,\ \bar{Q}_c = \frac{\bar{L}_0}{\bar{L}_1 + \bar{L}_0 - \bar{a}} \bar{Q}_0 \tag{5}$$

According to the theory of small deflection, here comes the equilibrium differential equation of the inner core:

$$\begin{cases} \bar{E}_i \bar{I}_i \bar{V}_{i1}''(\bar{x}) + \bar{P}\bar{V}_{i1}(\bar{x}) = \bar{E}_i \bar{I}_i \bar{V}_{i0}''(\bar{x})\ , & (-\bar{L}_1 \le \bar{x} \le 0) \\ \bar{E}_i \bar{I}_i \bar{V}_{i2}''(\bar{x}) + \bar{P}\bar{V}_{i2}(\bar{x}) = \bar{Q}_0 \bar{x} + \bar{E}_i \bar{I}_i \bar{V}_{i0}''(\bar{x})\ , & (0 \le \bar{x} \le \bar{a}) \\ \bar{E}_i \bar{I}_i \bar{V}_{i3}''(\bar{x}) + \bar{P}\bar{V}_{i3}(\bar{x}) = \bar{Q}_1 (\bar{L}_0 - \bar{x}) + \bar{E}_i \bar{I}_i \bar{V}_{i0}''(\bar{x})\ , & (\bar{a} \le \bar{x} \le \bar{L}_0) \\ \bar{E}_i \bar{I}_i \bar{V}_{i4}''(\bar{x}) + \bar{P}\bar{V}_{i4}(\bar{x}) = \bar{Q}_0 \bar{x} + \bar{E}_i \bar{I}_i \bar{V}_{i0}''(\bar{x})\ , & (\bar{L}_0 \le \bar{x} \le 1 + \bar{L}_1) \end{cases} \tag{6}$$

Relevant boundary constraint condition of the inner core and the continuity condition of deformation are as follows:

$$\begin{cases} \bar{V}_{i1}(0) = 0\ ,\ \bar{V}_{i3}(\bar{L}_0) = 0\ ,\ \bar{V}_{i1}(0) = \bar{V}_{i2}(0)\ ,\ \bar{V}_{i1}'(0) = \bar{V}_{i2}'(0) \\ \bar{V}_{i2}(0) = \bar{V}_{i3}(0)\ ,\ \bar{V}_{i2}'(\bar{a}) = \bar{V}_{i3}'(\bar{a})\ ,\ \bar{V}_{i3}(\bar{L}_0) = \bar{V}_{i4}(\bar{L}_0)\ ,\ \bar{V}_{i3}'(\bar{L}_0) = \bar{V}_{i4}'(\bar{L}_0) \end{cases} \tag{7}$$

According to the theory of small deflection, there is the differential equilibrium equation of the sleeve:

$$\begin{cases} \bar{E}_i \bar{I}_i \bar{V}_{e1}''(\bar{x}) = -\bar{Q}_0 \bar{x}\ , & (0 \le \bar{x} \le \bar{a}) \\ \bar{E}_i \bar{I}_i \bar{V}_{e2}''(\bar{x}) = -\bar{Q}_1 (\bar{L}_0 - \bar{x})\ , & (\bar{a} \le \bar{x} \le \bar{L}_0) \end{cases} \tag{8}$$

Relevant boundary constraint condition of the sleeve and the continuity condition of deformation are as follows:

$$\begin{cases}\bar{V}_{e1}(0)=0\ ,\ \bar{V}'_{e1}(\bar{L}_0)=0\\ \bar{V}_{e1}(\bar{a})=\bar{V}_{e2}(\bar{a})\ ,\ \bar{V}'_{e1}(\bar{a})=\bar{V}'_{e2}(\bar{a})\end{cases} \tag{9}$$

The deformation coordinating conditions of the inner core and sleeve are as follows:

$$\begin{cases}\bar{V}_{i1}(\bar{a})-\delta_g=\bar{V}_{e1}(\bar{a})\\ \bar{V}'_{i1}(\bar{a})=\bar{V}'_{e1}(\bar{a})\end{cases} \tag{10}$$

Based on equations (1) to (3), the equations (5) to (6) could be simplified to corresponding equilibrium differential equations of dimensionless inner core; with the continuity condition being the constraint condition and according to the equilibrium differential equations of inner core, there are:

$$\begin{cases}V_{i1}(x)=C(1)\cos(2x\eta)+C(2)\sin(2x\eta)+g(x)\ , & (-L_1\le x\le 0)\\ V_{i2}(x)=C(3)\cos(2x\eta)+C(4)\sin(2x\eta)+\dfrac{xQ_0}{4\eta^2}+g(x)\ , & (0\le x\le a)\\ V_{i3}(x)=C(5)\cos(2x\eta)+C(6)\sin(2x\eta)+\dfrac{(1-x)Q_1}{4\eta^2}+g(x) & (a\le x\le 1)\\ V_{i4}(x)=C(7)\cos(2x\eta)+C(8)\sin(2x\eta)+g(x)\ , & (1\le x\le 1+L_1)\end{cases} \tag{11}$$

Where $g(x)=\pi^2(\dfrac{d_1\sin(\pi x)}{\pi^2-4\eta^2}+\dfrac{d_2\sin(2\pi x)}{\pi^2-\eta^2})$

The corresponding coefficients C (1) to C (8) of inner core are:

$$C(1)=0\ ,\ C(3)=0 \tag{12}$$

$$C(2)=\frac{Q_0}{8(-1+a-L_1)\eta^3}\{-1+a-L_1+\csc(2\eta)[2L_1\eta\cos(2(-1+\mathrm{a})\eta)+\sin(2\eta-2a\eta)]\} \tag{13}$$

$$C(4)=\frac{Q_0}{8(-1+a-L_1)\eta^3}\{\cot(2\eta)[2L_1\eta\cos(2(-1+\mathrm{a})\eta)+\sin(2\eta-2a\eta)]\} \tag{14}$$

$$C(5)=\frac{Q_0}{8(-1+a-L_1)\eta^3}[-2L_1\eta\cos(2\mathrm{a}\,\eta)+\sin(2a\eta)] \tag{15}$$

$$C(6)=\frac{Q_0\cot(2\eta)}{8(-1+a-L_1)\eta^3}[2L_1\eta\cos(2\mathrm{a}\,\eta)-\sin(2a\eta)] \tag{16}$$

$$C(7)=\frac{Q_0}{8(-1+a-L_1)\eta^3}[-2L_1\eta\cos(2a\eta)+(-a+L_1)\sin(2\eta)+\sin(2a\eta)] \tag{17}$$

$$C(8)=-\frac{Q_0\cot(2\eta)}{8(-1+a-L_1)\eta^3}[-2L_1\eta\cos(2a\eta)+(-a+L_1)\sin(2\eta)+\sin(2a\eta)] \tag{18}$$

Similarly, based on equations (1) to (3), equations (7) to (8) could be simplified into corresponding equilibrium differential equations of dimensionless sleeve; with the continuity condition being the constraint condition and according to the equilibrium differential equations of the sleeve, there are:

$$\begin{cases}V_{e1}(x)=-\dfrac{1}{6}\beta Q_0x^3+C(9)x+C(10)\ , & (0\le x\le a)\\ V_{e2}(x)=\dfrac{1}{6}\beta Q_1x^3-\dfrac{1}{2}\beta Q_1x^2+C(11)x+C(12)\ , & (a\le x\le 1)\end{cases} \tag{19}$$

Corresponding sleeve coefficients C (9) to C (12) are:

$$\begin{cases}C(9)=-\dfrac{\beta Q_0[a^3-2L_1-3a^2(1+L_1)+a(2+6L_1)]}{6(-1+a-L_1)}\ ,\ C(10)=0\\ C(11)=-\dfrac{\beta Q_0[2a+a^3-2L_1-3a^2L_1)]}{6(-1+a-L_1)}\ ,\ C(12)=\dfrac{\beta Q_0a^2(a-3L_1)}{6(-1+a-L_1)}\end{cases} \tag{7}$$

When the inner core contacts with the sleeve at the midpoint, it meets the dimensionless deformation compatibility conditions of the inner core and the sleeve, so by putting $V_{i1}(x)$, $V_{e1}(x)$ into equation (9): $V_{i1}(\frac{1}{2})-\delta_g=V_{e1}(\frac{1}{2})$, equation Q_0 could be got after simplification:

$$Q_0=\Big[24(-1+a-L_1)\eta^3\big(d_1\pi^2(-\pi^2+\eta^2)\sin(a\pi)+(\pi-2\eta)(\pi+2\eta)$$
$$\big(\delta_g(\pi-\eta)(\pi+\eta)-d_2\pi^2\sin(2a\pi)\big)\Big]/\Big\{\big(\pi^4-5\pi^2\eta^2+4\eta^4\big)\Big[6a(-1+a-L_1)\eta+$$
$$8(-1+a)a\big(a(-1+a-2L_1)+L_1\big)\beta\eta^3+3\csc(2\eta)\sin(2a\eta)\times$$
$$\big(2L_1\eta\cos\big[2(-1+a)\eta\big]+\sin(2\eta-2a\eta)\big)\Big]\Big\} \tag{8}$$

Based on equation (4), other physical quantities of the inner core could be got (when $-L_1\le x\le 1+L_1$):

$$\begin{cases} Q_{i1}=0\ ,\ (-L_1\le x\le 0)\ ;\ Q_{i2}=Q_0\ ,\ (0\le x\le a) \\ Q_{i3}=-Q_1\ ,\ (a\le x\le 1)\ ;\ Q_{i4}=0\ ,\ (1\le x\le 1+L_1) \end{cases} \tag{9}$$

$$\begin{cases} M_{i1}=4\eta^2\{C(1)\cos(2x\eta)+C(2)\sin(2x\eta)+g(x)\}\ ,\ (-L_1\le x\le 0)\ ; \\ M_{i2}=4\eta^2\{C(3)\cos(2x\eta)+C(4)\sin(2x\eta)+g(x)\}\ ,\ (0\le x\le a) \\ M_{i3}=4\eta^2\{C(5)\cos(2x\eta)+C(6)\sin(2x\eta)+g(x)\}\ ,\ (a\le x\le 1) \\ M_{i4}=4\eta^2\{C(7)\cos(2x\eta)+C(8)\sin(2x\eta)+g(x)\}\ ,\ (1\le x\le 1+a) \end{cases} \tag{10}$$

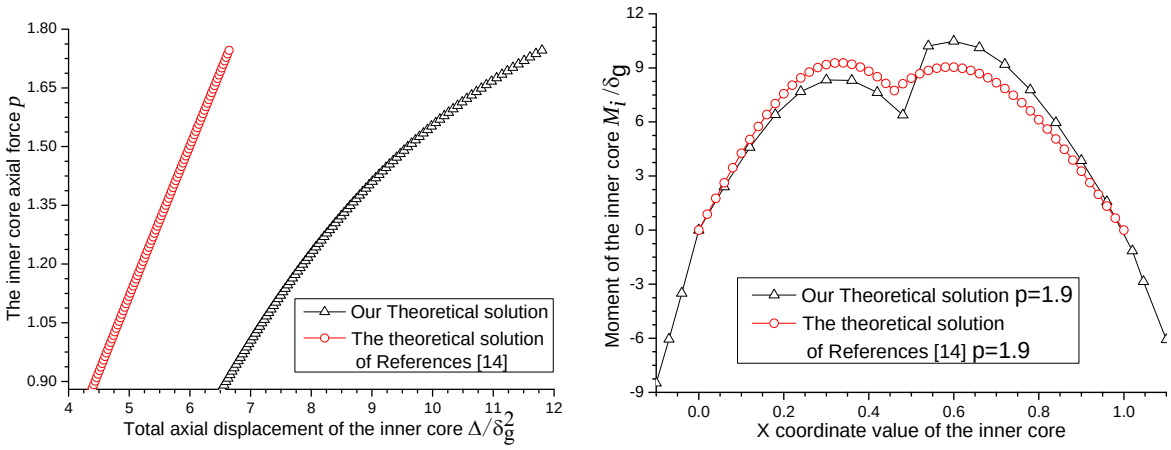
Meanwhile, as shown in free-body diagrams of the inner core and the outer sleeve, the curvature of the inner core and the sleeve is the same at contact point B, i.e., $\bar{V}''_{i1}(\bar{a})=\bar{V}''_{e1}(\bar{a})$, then after putting equations (10) and (18) into that, the equation a could be got after simplification and organization :

$$-4\eta^2C(4)\sin(2a\eta)+\pi^2\Big[-\frac{\pi^2d_1\sin(a\pi)}{\pi^2-4\eta^2}-\frac{4\pi^2d_2\sin(2a\pi)}{\pi^2-4\eta^2}\Big]=-\beta Q_1(a-1) \tag{11}$$

4. Examples and Results

4.1 Example 1

In this example, the bending rigidity ratio of the outer sleeve to inner core shaft is small. The outer sleeve and the inner core shaft are in the shape of round steel. Inner core: diameter $\bar{D}_i=60\text{mm}$, wall thickness $\bar{t}_i=5\text{mm}$. Sleeve; diameter $\bar{D}_e=152\text{mm}$, wall thickness $\bar{t}_e=5\text{mm}$; casing length $\bar{L}_0=2\text{m}$, length of extending section of inner core $\bar{L}_1=0.2\text{m}$, the gap between inner core and outer casing $\bar{\delta}_g=41\text{mm}$, the elastic modulus of inner core and outer casing are both $\bar{E}_i=\bar{E}_e=2.06\times10^5\text{N/mm}^2$; the slenderness ratio of inner core $\lambda_i=102.43$, the slenderness ration of sleeve $\lambda_e=38.46$. The corresponding dimensionless quantity is $\delta_g=0.021$, $r_i=0.224$, $\beta=0.053$. The axial displacement of the axial pressure bending of axial compression sleeve and the bending moment of inner core are shown in Figure 3.

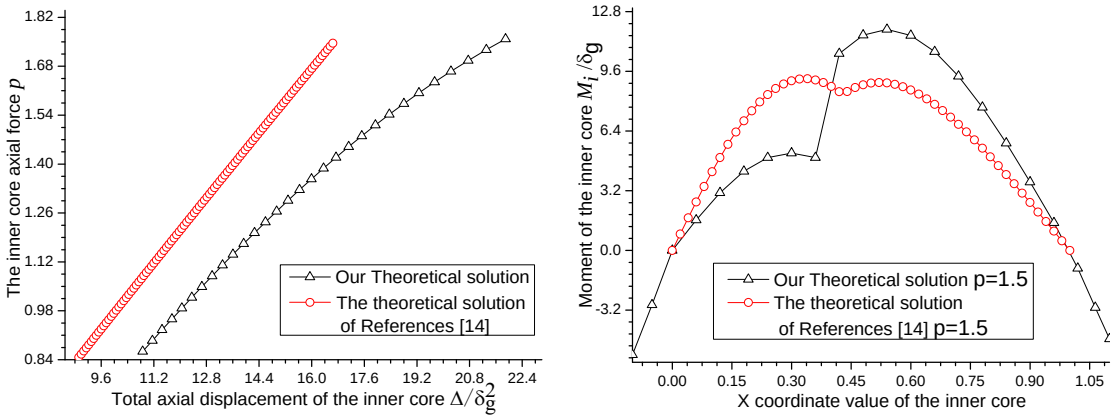


(a) The axial force of the inner core—total axial displacement (b) moment of the inner core

Fig. 3 Comparison of the theoretical results of two mechanical models of example 1

4.2 Example 2

In this example, the bending rigidity ratio of the outer sleeve to inner core shaft is small. The outer sleeve and the inner core shaft are in the shape of round steel. Inner core: diameter $\bar{D}_i = 60\text{mm}$, wall thickness $\bar{t}_i = 5\text{mm}$. Sleeve; diameter $\bar{D}_e = 114\text{mm}$, wall thickness $\bar{t}_e = 5\text{mm}$; casing length $\bar{L}_0 = 2\text{m}$, length of extending section of inner core $\bar{L}_1 = 0.2\text{m}$, the gap between inner core and outer casing $\bar{\delta}_g = 22\text{mm}$, the elastic modulus of inner core and outer casing are both $\bar{E}_i = \bar{E}_e = 2.06\times10^5\text{N/mm}^2$; the slenderness ratio of inner core $\lambda_i = 102.43$, the slenderness ration of sleeve $\lambda_e = 51.84$. The corresponding dimensionless quantity is $\delta_g = 0.011$, $r_i = 0.338$, $\beta = 0.129$. The axial displacement of the axial pressure bending of axial compression sleeve and the bending moment of inner core are shown in Figure 4.

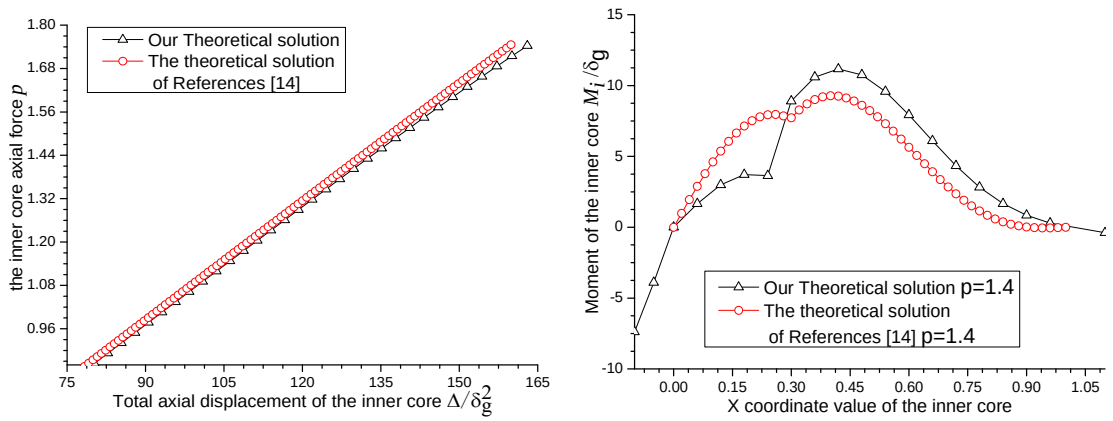


(a) The axial force of the inner core—total axial displacement (b) moment of the inner core

Fig. 4 Comparison of the theoretical results of two mechanical models of example 2

4.3 Example 3

In this example, the bending rigidity ratio of the outer sleeve to inner core shaft is small. The outer sleeve and the inner core shaft are in the shape of round steel. Inner core: diameter $\bar{D}_i = 60\text{mm}$, wall thickness $\bar{t}_i = 5\text{mm}$. Sleeve; diameter $\bar{D}_e = 83\text{mm}$, wall thickness $\bar{t}_e = 5\text{mm}$; casing length $\bar{L}_0 = 2\text{m}$, length of extending section of inner core $\bar{L}_1 = 0.2\text{m}$, the gap between inner core and outer casing $\bar{\delta}_g = 22\text{mm}$, the elastic modulus of inner core and outer casing are both $\bar{E}_i = \bar{E}_e = 2.06\times10^5\text{N/mm}^2$; the slenderness ratio of inner core $\lambda_i = 102.43$, the slenderness ration of sleeve $\lambda_e = 51.84$. The corresponding dimensionless quantity is $\delta_g = 0.011$, $r_i = 0.338$, $\beta = 0.129$. The axial displacement of the axial pressure bending of axial compression sleeve and the bending moment of inner core are shown in Figure 5.



(a) The axial force of the inner core—total axial displacement (b) moment of the inner core

Fig. 5 Comparison of the theoretical results of two mechanical models of example 3

From the above three examples, a conclusion is drawn that the total axial displacement of kernel is more practical un-der axial compression when the mechanical model take extension section of the kernel into consideration. The conclusion, shown in Figure 3 to5 (a), is drawn by comparing the three

different degrees of stiffness ration, namely, the small, the medium, and the large one respectively with the mechanical model that takes the extension section of inner core into consideration and the mechanical model in bibliography [14] that sets the extending section aside. What's more, the bending moment of the extension section can be got by the former mechanical model and the joint between the inner core and the outer sleeve can be easily buckled for the bending moment when under axial compression through Figure 3 to 5 (b). However, the bending moment cannot be calculated and the foundation for the design of the limit connection cannot be laid by the latter mechanical model in bibliography [14].

5. Conclusion

In this paper, the calculation formulas are deduced for the axial displacement of inner core and sleeved column, deflection, bending moment and other physical quantities when sleeved column under axial compression and the extension section is taken into consideration in deformation process when it is at point contact. By comparing these formulas with the conclusion in bibliography [14], the following conclusions are drawn:

1)The mechanical model that has taken the extension section of inner core into consideration can reflect the buckling process when sleeved column is at point contact; 2) The force effect of the extension section of inner core that be calculated by corresponding calculation model lays a foundation for the design of the joint between inner core and outer sleeved column; 3) The mechanical model that has taken the extension section of inner core into consideration corresponds with actual situation and it can make sleeved column be better used in engineering practices.

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