The Research of Passenger Flow Through Checkpoint Based on Queuing Theory

Jie Yi
North China Electrical Power University

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Abstract. Long lines emerging recently at American airports incurred sharp criticism for the TSA, how to solve the problem becomes a concern both for the TSA and passengers. To explore the flow of passengers through a security checkpoint, propose some measures to modify the current process and some advice for managers,we develop modeling methods based on queuing theory. Queue length l, queuing probability c and average waiting time w can be obtained by solving equations. We then explore the relations between these variables and the passenger flow λ and give an expression $\lambda = w \cdot c / l$.

1. Introduction

Recently, lines in America seems to be insane,which frustrates many travelers.A recent survey conducted among American on the Internet says that compared with queuing at airports, 35% of participants prefer to do shopping, 22% prefer to clean up the mess after holiday picnics,and 18% would like to sleep in the sofa[1].So it is not difficult to see passengers' anxiety considering hours-long security wait time. This algorithm is also a part of our model regarding the passenger flow through the checkpoint.We set up parameters λ : passenger arrival rate, u :service rate,and utilization rate of the service desk ρ as criteria.

2. Research results based on Queuing Theory:

Poisson process is a special random process which is frequently used to describe customer arrival disciplines in queuing theory. The probability of customers arriving in from t_1 to t_2 can be described by following formula,on the premise that the number of customers is n:

$$Pn(t_1, t_2) = P\{N(t_2) - N(t_1) = n\} \quad (t_1 < t_2; n \geq 0) \tag{1}$$

where

$N(t)$ is the number of customers arriving in from 0 to t

$Pn(t_1, t_2)$ follows the Poisson distribution with the parameter λ_1 when it meets one of the three following conditions:(1)stability(2)independence(3)universality[2].

Through a series of deduction,the probability of n customers arriving in during the t interval $P_n(t)$ is described as:

$$P_n(t) = \frac{(\lambda_1 t)^n}{n!} e^{-\lambda_1 t} \quad (t > 0; n = 0,1,2 \cdots) \tag{2}$$

so we can see $N(t)$ follows the Poisson distribution,its mathematical expectations and variances are as follows:

$$E(N(t)) = \sum_{n=0}^{\infty} n \frac{(\lambda_1 t)^n}{n!} e^{-\lambda_1 t} = \lambda_1 t, \quad Var(N(t)) = \lambda_1 t \tag{3}$$

especially,when $t = 1$, we can get $E(N(1)) = \lambda_1$, which represents average number of arrivals per unit time, also called the arrival rate[3].

If the probability density function of random variable T is expressed as

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$$f(t)=\begin{cases}\mu e^{-\mu t} & (t\geq 0)\\ 0 & (t<0)\end{cases} \qquad (\mu>0) \tag{4}$$

then T follows the negative exponential distribution with parameter μ .

The distribution function is described as

$$F(t)=\begin{cases}1-\mu e^{-\mu t} & (t\geq 0)\\ 0 & (t<0)\end{cases} \quad (\mu>0) \tag{5}$$

next

$$E(T)=\frac{1}{\mu} \tag{6}$$

$$Var(T)=\frac{1}{\mu^2} \tag{7}$$

are easy to obtain.Where μ stands customers who have received service and leave the system per unit time,also called the average service rate of each check-in desk.

Assuming that each check-in desk work independently and average service rate is equal,namely $\mu_1=\mu_2=\cdots=\mu_c=\mu$ .So average service rate of the whole service agent is either $c\mu$ when $n\geq c$ or $n\mu$ when $n\prec c$ .We define average utilization of service agent as

$$\rho=\frac{\lambda_1}{c\mu} \tag{8}$$

to prevent an infinite queue, ρ needs to satisfy $\rho<1$.Fig.1 shows the process.

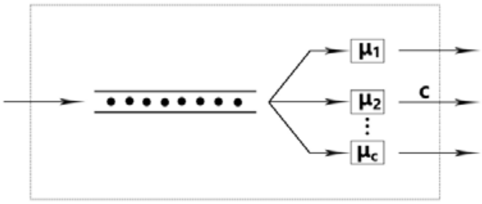

Fig.1 a sketch of service desks

Begin analysis of the queuing system from investigating the relation of system states.The transfer relations between theses states are shown as figure1.

The transition from state 0 to state 1 means that a customer has received service and leaved,the state transition probability is represented by μP_1.The transition from state 2 to state 1 means that one of customers served at the service desk μ_1 and μ_2 ,and the state transition probability is $2\mu P_2$.

Similarly,consider the case where the state transfer from state n to state $n-1$.The state transition probability is $n\mu P_n$ in the case of $n\leq c$ .While if $n>c$, limited number of service desks determine that there are c customers being at service at most,and $n-c$ customers waiting,the state transition probability is therefore changed into $c\mu P_n$.

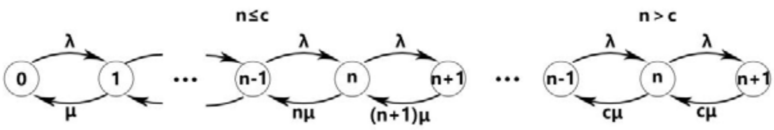

Fig.2 state transition process

From figure2 we can conclude that

$$\begin{cases}\mu P_1=\lambda_1 P_0 & \\ (n+1)\mu P_{n+1}+\lambda_1 P_{n-1}=(\lambda_1+n\mu)P_n & (1\leq n\leq c)\\ c\mu P_{n+1}+\lambda_1 P_{n-1}=(\lambda_1+c\mu)P_n & (n>c)\end{cases} \tag{9}$$

Here $\sum_{i=0}^{\infty} P_i = 1$ and $\rho \le 1$.

Solving the deterministic difference equations using recursion gives state probability as follows

$$P0 = \left[\sum_{k=0}^{c-1} \frac{1}{k!} \left(\frac{\lambda_1}{\mu} \right)^k + \frac{1}{c!} \frac{1}{1-\rho} \left(\frac{\lambda_1}{\mu} \right)^c \right]^{-1} \tag{10}$$

$$Pn = \begin{cases} \frac{1}{n!} \left(\frac{\lambda_1}{\mu} \right)^n P_0 & (n \le c) \\ \frac{1}{c!\, c^{n-c}} \left(\frac{\lambda_1}{\mu} \right)^n P_0 & (n > c) \end{cases} \tag{11}$$

system running indexes are derived as:

average queue length

$$\begin{cases} L_s = L_q + \frac{\lambda_1}{\mu} \\ L_q = \sum_{n=c+1}^{\infty} (n-c) P_n = \frac{(c\rho)^c \rho}{c!(1-\rho)^2} P_0 \end{cases} \tag{12}$$

average waiting time and sojourn time derived from the Little formula are shown as follows respectively

$$W_q = \frac{L_q}{\lambda_1}, \quad W_s = \frac{L_s}{\lambda_1} \tag{13}$$

The results are given in terms of requirements and reality,intermediate data processing are omitted for simplicity,the results are given as follows

average queue length

$$\begin{cases} L_q = \frac{\rho^{k+1}}{(k-\rho) \sum_{n=0}^{k-1} C_n \rho^n} \\ C_n = (k-1)! \frac{n-k}{n!} \end{cases} \tag{14}$$

average waiting time

$$W_q = \frac{L_q}{\lambda_1} \tag{15}$$

queuing probability

$$C(\rho, k) = \frac{k-\rho}{\rho} L_q \tag{16}$$

3. References

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