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Efficient Direction-Of-Arrival Estimation for a Mixture of Circular and Noncircular Sources with PM Method

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Abstract. In this paper, the one-dimensional direction of arrival estimation problem for a mixture of circular and noncircular sources is considered. We call this algorithm Mixed-PM algorithm, which is based on classical PM algorithm. Firstly, we introduce the concept of circular and noncircular signals. And then we explain the theory of Mixed-PM algorithm in detail, which is able to distinguish circular signals and noncircular signals in mixed signals. We find that the performance of Mixed-PM algorithm is better than classical PM. In addition, we also studied the performance of Mixed-PM algorithm under different snapshots and different received array elements. Finally, we study the influence of the number of different round signals on the performance of the algorithm, find that the smaller the number of round signals, the better the performance of the algorithm.

1. Introduction

DOA estimation is an extremely important aspect of array signal processing[1]. In the past few decades, there have been a variety of classic estimation algorithm, include MUSIC algorithm[2], Capon algorithm[3], Esprit algorithm[4] and PM algorithm[5]. The complexity of the PM algorithm based on the invariant rotation of the subspace is low and is more convenient in practical application.

In order to improve the performance of the proposed algorithm, the estimation algorithm combining the characteristics of the signal itself has become an important research direction. The noncircular characteristic of noncircular signals can increase the amount of the received matrix data, which is equivalent to increasing the array aperture, so the DOA estimation is more accurate. In the literature [6], a noncircular signal estimation algorithm based on Esprit algorithm is proposed, which uses the array output matrix of the signal and its conjugate, the output matrix is reconstructed and the corresponding covariance data matrix is calculated, and the DOA of the estimated signal is calculated by the principle of the algorithm. Aiming at the wide application of noncircular signals in modern communication systems, the real-valued algorithm using the real number has been proposed[7]. In the literature[8], the real-valued algorithm is proposed, The Euler transform used in the algorithm can use the real-valued operation to estimate the noise subspace.

There are some persons have done work on mixed signals are common injected, such as unitary Esprit algorithm[9], however, which algorithm couldn't distinguish circular and noncircular signals. herefore we propose a low complexity Mixed-PM algorithm based on rotation invariance.

2. Data Model

2.1 Mathematical Model of Arbitrary Noncircular Signals. Steinwandt et al defined a mathematical model of arbitrary noncircular signals as shown in Fig.1[10], which can represent arbitrary noncircular signal model. In the complex plane, the kth received signals can be expressed as

$$S_k = e^{j\phi_k}\left(\sqrt{\frac{1+mk}{2}}S_{I_k} + j\sqrt{\frac{1-mk}{2}}S_{Q_k}\right), k = 1,\cdots,K. \tag{1}$$

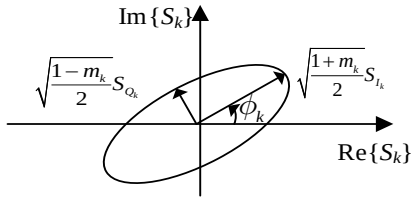

Figure 1. Mathematical model of arbitrary noncircular signals

where ϕ_k is the rotation phase, $m_k(0 \le m_k \le 1)$ represents the magnitude of noncircular coefficients, S_{I_k} and S_{Q_k} denote the in-phase and the quadrature component of the complex signal S_k. For $E\left\{\left|S_{I_k}\right|^2\right\} = 1$ and $E\left\{\left|S_{Q_k}\right|^2\right\} = 1$, m_k meet $E\left\{\left|S_k\right|^2\right\} = 1$, there $E\left\{S_k^2\right\} = e^{j2\phi_k}(m_k + j\sqrt{1-m_k^2}E\left\{S_{I_k}S_{Q_k}\right\})$. In actual linear digital modulation, S_{I_k} and S_{Q_k} are uncorrelated, $E\left\{S_{I_k}S_{Q_k}\right\} = 0$, so $E\left\{S_k^2\right\} = m_k e^{j2\phi_k}$.

The complex noncircular coefficients is $\rho_k = \frac{E\left\{S_k^2\right\}}{E\left\{\left|S_k\right|^2\right\}} = \left|\rho_k\right| e^{j\varphi_k} = m_k e^{j2\phi_k}$, where $\left|\rho_k\right| = m_k$, noncircular phase is $\varphi_k = 2\phi_k$. When $m_k = 0$, S_k is a circular signal; When $m_k = 1$, S_k is a strictly noncircular signal. The strictly noncircular signal can be express as $S_k = S_R e^{j\varphi_k}$. where S_R is real valued signal, we use the noncircular signals are strictly noncircular signals.

2.2 Uniform Linear Array (ULA). In this paper, the receiving antenna model is a one-dimensional uniform linear array as shown in figure 2, which are consisted of *M* array antennas. In order to avoid the angle ambiguity during estimating, the spacing *d* between the elements is the half of wavelength λ of the incident signal, can be express as $d = \lambda/2$.

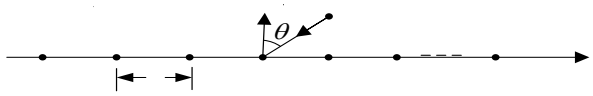

Figure 2. Uniform linear array

Assuming that K narrowband, far-field, uncorrelated the circular and noncircular mixed signals. The incident angle of the signal is $\theta_k(k = 1,2,\cdots,K)$ $\left(0 \le \theta_k \le 360^\circ\right)$. Assuming K_c circular and K_n noncircular signals, they can be defined as $S_{c,k}(t), k = 1,\cdots,K_c$ and $S_{n,k}(t), k = 1,\cdots,K_n$, and the noise in the received signal is additive white Gaussian noise, the mean is zero and the variance is σ^2. The array receive signal $\mathbf{x}(t)$ is expressed as follows

$$\mathbf{x}(t) = \mathbf{A}\tilde{\mathbf{s}}(t) + \mathbf{n}(t). \tag{2}$$

$\mathbf{A}$ is a manifold matrix, divided as $\mathbf{A} = \left[\mathbf{A}_c, \mathbf{A}_n\right]$, $\mathbf{A}_c \in \mathbf{C}^{M\times K_c}$, $\mathbf{A}_n \in \mathbf{C}^{M\times K_n}$, the column vector is $\mathbf{a}(\theta_k)(k = 1,2,\cdots,K)$, where $\mathbf{a}(\theta_k) = \left[a_0(\theta_k),\cdots,a_{M-1}(\theta_k)\right]^T$, $a_i(\theta_k) = e^{-j(2\pi/\lambda)di\sin\theta_k}(i = 0,1,\cdots,M-1)$. $\tilde{\mathbf{s}}(t) = \left[s_{c,1}(t),\cdots,s_{c,K_c}(t),s_{n,1}(t),\cdots,s_{n,K_n}(t)\right]^T$, $\mathbf{n}(t) = \left[n_1(t),\cdots,n_M(t)\right]^T$ is a Gaussian white noise vector.

According to chapter 2.1 $\tilde{\mathbf{s}}(t)$ can be written as follows

$$\tilde{\mathbf{s}}(t) = \Phi\mathbf{s}(t). \tag{3}$$

$$\Phi = diag\left[\underbrace{1,\cdots,1}_{K_c},\varphi_{n,1},\cdots,\varphi_{n,k_n}\right] = \begin{bmatrix}\Phi_c & 0\\ 0 & \Phi_n\end{bmatrix}. \tag{4}$$

where $\Phi_c = \mathbf{I}_{K_c}$, $\Phi_n = diag\left[\phi_{n,1},\cdots,\phi_{n,k_n}\right]$, $diag(\bullet)$ means making vector a diagonal matrix. $\mathbf{s}(t)$ is express as $\mathbf{s}(t) = \left[s_{c,1}(t),\cdots,s_{c,K_c}(t),\bar{s}_{n,1}(t),\cdots,\bar{s}_{n,K_n}(t)\right]^T$. $s_{n,k}(t) = \phi_{n,k}\bar{s}_{n,k}(t)$, $\bar{s}_{n,k}(t)$ is real valued signal. $\phi_{n,k} = e^{j\varphi_k}\left(k = 1,\cdots,K_n\right)$ is noncircular phase. Therefore, $\mathbf{x}(t)$ can be expressed as

$$\mathbf{x}(t) = \mathbf{A}\Phi\mathbf{s}(t) + \mathbf{n}(t). \tag{5}$$

3. Mixed-PM Algorithm

Build the opposition matrix as follows

$$\mathbf{J} = \begin{bmatrix} & & & 1\\ & & 1 & \\ & \iddots & & \\ 1 & & & \end{bmatrix}_{M\times M}. \tag{6}$$

The new received signals matrix is express as follows

$$\mathbf{z}(t) = \begin{bmatrix}\mathbf{x}(t)\\ \mathbf{J}\mathbf{x}^*(t)\end{bmatrix} = \begin{bmatrix}\mathbf{A}(\theta)\Phi\mathbf{s}(t)\\ \mathbf{J}\mathbf{A}^*(\theta)\Phi^*\mathbf{s}^*(t)\end{bmatrix} + \begin{bmatrix}\mathbf{n}(t)\\ \mathbf{J}\mathbf{n}^*(t)\end{bmatrix} = \mathbf{B}\breve{\mathbf{s}}(t) + \breve{\mathbf{n}}(t). \tag{7}$$

where $\breve{\mathbf{n}}(t) = \begin{bmatrix}\mathbf{n}(t)\\ \mathbf{J}\mathbf{n}^*(t)\end{bmatrix}$. $\mathbf{B} = \begin{bmatrix}\mathbf{A}_c & \mathbf{O}_{M\times K_c} & \mathbf{A}_n\Phi_n\\ \mathbf{O}_{M\times K_c} & \mathbf{J}\mathbf{A}_c^* & \mathbf{J}\mathbf{A}_n^*\Phi_n^*\end{bmatrix} \in \mathbf{C}^{2M\times(2K_c+K_n)}$ is the new steering matrix. $\breve{\mathbf{s}}(t) = \left[\mathbf{s}_{c,k}(t) \quad \mathbf{s}_{c,k}^*(t) \quad \bar{\mathbf{s}}_{n,k}(t)\right]^T \in \mathbf{C}^{(2K_c+K_n)\times 1}$. The covariance matrix of $\mathbf{z}(t)$ is shown as follows

$$\mathbf{R} = E\left[\mathbf{z}\mathbf{z}^H\right] = \mathbf{B}\mathbf{R}_s\mathbf{B}^H + \sigma^2\mathbf{I}_{2M}. \tag{8}$$

where $\mathbf{R}_s = E\left[\breve{\mathbf{s}}\breve{\mathbf{s}}^H\right]$ is covariance matrix of $\breve{\mathbf{s}}(t)$, the matrix $\mathbf{R}_s$ is a full rank matrix. The steering matrix $\mathbf{A}$ is a full rank matrix so the matrix $\mathbf{B}$ is. $\mathbf{B}$ is divided as $\mathbf{B} = \left[\mathbf{B}_1 \quad \mathbf{B}_2\right]^T$, where $\mathbf{B}_1 \in \mathbf{C}^{(K+K_c)\times(K+K_c)}$, $\mathbf{B}_2 \in \mathbf{C}^{(2M-K-K_c)\times(K+K_c)}$. $\mathbf{B}_1$ matrix is a full rank matrix. So $\mathbf{B}_2 = \mathbf{P}_c^H\mathbf{B}_1$, where $\mathbf{P}_c \in \mathbf{C}^{(2M-K-K_c)\times(K+K_c)}$. Then $\left[\mathbf{I}_{K+K_c} \quad \mathbf{P}_c^H\right]^T\mathbf{B}_1 = \mathbf{B}$. Make $\mathbf{P} = \left[\mathbf{I}_{K+K_c} \quad \mathbf{P}_c^H\right]^T$. Then

$$\mathbf{P}\mathbf{B}_1 = \mathbf{B}. \tag{9}$$

The matrix $\mathbf{B}$ is divided as follows

$$\mathbf{B} = \left[\mathbf{B}_{a1} \quad \mathbf{B}_{a2} \quad \mathbf{B}_{b1} \quad \mathbf{B}_{b2}\right]^T. \tag{10}$$

$$\mathbf{B} = \left[\mathbf{B}_{a3} \quad \mathbf{B}_{a4} \quad \mathbf{B}_{b3} \quad \mathbf{B}_{b4}\right]^T. \tag{11}$$

where $\mathbf{B}_{a1} \in \mathbf{C}^{(M-1)\times(2K_c+K_n)}$, $\mathbf{B}_{a2} \in \mathbf{C}^{1\times(2K_c+K_n)}$, $\mathbf{B}_{b1} \in \mathbf{C}^{(M-1)\times(2K_c+K_n)}$, $\mathbf{B}_{b2} \in \mathbf{C}^{1\times(2K_c+K_n)}$. $\mathbf{B}_{a3} \in \mathbf{C}^{1\times(2K_c+K_n)}$, $\mathbf{B}_{a4} \in \mathbf{C}^{(M-1)\times(2K_c+K_n)}$, $\mathbf{B}_{b3} \in \mathbf{C}^{1\times(2K_c+K_n)}$, $\mathbf{B}_{b4} \in \mathbf{C}^{(M-1)\times(2K_c+K_n)}$. Make $\mathbf{B}_3 = \begin{bmatrix}\mathbf{B}_{a1} & \mathbf{B}_{b1}\end{bmatrix}^T$, $\mathbf{B}_4 = \begin{bmatrix}\mathbf{B}_{a4} & \mathbf{B}_{b4}\end{bmatrix}^T$.

The matrix $\mathbf{P}$ is divided as follows

$$\mathbf{P} = \begin{bmatrix}\mathbf{P}_{a1} & \mathbf{P}_{a2} & \mathbf{P}_{b1} & \mathbf{P}_{b2}\end{bmatrix}^T. \tag{12}$$

$$\mathbf{P} = \begin{bmatrix}\mathbf{P}_{a3} & \mathbf{P}_{a4} & \mathbf{P}_{b3} & \mathbf{P}_{b4}\end{bmatrix}^T. \tag{13}$$

where $\mathbf{P}_{a1} \in \mathbf{C}^{(M-1)\times(2K_c+K_n)}$, $\mathbf{P}_{a2} \in \mathbf{C}^{1\times(2K_c+K_n)}$, $\mathbf{P}_{b1} \in \mathbf{C}^{(M-1)\times(2K_c+K_n)}$, $\mathbf{P}_{b2} \in \mathbf{C}^{1\times(2K_c+K_n)}$. $\mathbf{P}_{a3} \in \mathbf{C}^{1\times(2K_c+K_n)}$, $\mathbf{P}_{a4} \in \mathbf{C}^{(M-1)\times(2K_c+K_n)}$, $\mathbf{P}_{b3} \in \mathbf{C}^{1\times(2K_c+K_n)}$, $\mathbf{P}_{b4} \in \mathbf{C}^{(M-1)\times(2K_c+K_n)}$ Make $\mathbf{P}_1 = \begin{bmatrix}\mathbf{P}_{a1} & \mathbf{P}_{b1}\end{bmatrix}^T$, $\mathbf{P}_2 = \begin{bmatrix}\mathbf{P}_{a4} & \mathbf{P}_{b4}\end{bmatrix}^T$.

According to the formula (10) we can get the equation as follows

$$\mathbf{P}_1\mathbf{B}_1 = \mathbf{B}_3. \tag{14}$$

$$\mathbf{P}_2\mathbf{B}_1 = \mathbf{B}_4. \tag{15}$$

The formula as follows can be proved $\mathbf{B}_4 = \mathbf{B}_3\Lambda(\theta)$, where $\Lambda(\theta) \in \mathrm{C}^{(2K_c+K_n)\times(2K_c+K_n)}$ as follows

$$\Lambda(\theta) = diag\left(e^{-j2\pi d\sin\theta_{c,1}/\lambda},\cdots,e^{-j2\pi d\sin\theta_{c,K_c}/\lambda},e^{j2\pi d\sin\theta_{c,1}/\lambda},\cdots,e^{j2\pi d\sin\theta_{c,K_c}/\lambda},e^{-j2\pi d\sin\theta_{n,1}/\lambda}\cdots,e^{-j2\pi d\sin\theta_{n,K_n}/\lambda}\right). \tag{16}$$

So if we can get the eigenvalue of $\Lambda(\theta)$, then obtains the incident angle of the received signals. We can get the formula as follows

$$\mathbf{P}_2\mathbf{B}_1 = \mathbf{P}_1\mathbf{B}_1\Lambda\left(\theta\right). \tag{17}$$

$$\Rightarrow \mathbf{P}_1^{+}\mathbf{P}_2 = \mathbf{B}_1\Lambda\left(\theta\right)\mathbf{B}_1^{-1}. \tag{18}$$

Therefore, we can find the eigenvalue decomposition of $\mathbf{P}_1^{+}\mathbf{P}_2$ to get the eigenvalue decomposition of $\Lambda(\theta)$. Make $\lambda_{mk}, mk = 1,2,\cdots,K+K_c$ be the eigenvalue decomposition of $\Lambda(\theta)$, then the estimated value $\hat{\theta}_k$ of the incident angle θ_k is $\hat{\theta}_k = \arcsin(angle(\lambda_{mk}))$.

The matrix $\mathbf{R}$ is divided as $\mathbf{R} = \left[\mathbf{G},\mathbf{H}\right]$, $\mathbf{G} \in \mathbf{C}^{2M\times(K+K_c)}$, $\mathbf{H} \in \mathbf{C}^{2M\times(2M-K-K_c)}$. The propagation operator matrix $\mathbf{P}_c$, it can be proved that the following formula

$$\mathbf{P}_c = \left[\mathbf{G}^{+}\mathbf{H}\right]. \tag{19}$$

We can know that the number of the estimated angle of incidence is more than actual incidence K_c signals. These K_c signals are the estimated angle of the estimated angle of the incident angle after conjugation of the circular signal. So we can determine the incidence angles of the circular signal according to the two estimated angles are the same. The other angles are noncircular incidence angles. Which way can achieve the purpose of distinguish circular and noncircular signals.

4. Simulation Result

During the simulation, M represents the number of received arrays. The receiving antenna is a $1\times M$ uniform linear array. L is the number of snapshots, K is the number of sources, K_c is the

number of circular signals, K_n is the number of noncircular, *SNR* is noise-signal ratio. We adopt the root mean square error (RMSE) as the algorithm performance evaluation criteria, defined as follows

$$RMSE = \frac{1}{K}\sum_{k=1}^{K}\sqrt{\frac{1}{D}\sum_{i=1}^{D}(\hat{\theta}_k - \theta_k)^2}. \tag{20}$$

where D is Monte Carlo simulation times. $\hat{\theta}_k$ is the estimated value of the incident angle for the kth source. We use the QPSK signals as the circular signal, and ASK signals as the noncircular signal. If no special instructions are available, the simulation parameters are set $K = 4$, $\left(10^\circ, 30^\circ, 50^\circ, 70^\circ\right)$, $K_c = 2$, $\left(10^\circ, 30^\circ\right)$, $SNR = \left[0,5,10,15,20,25,30\right]$, $M = 8$, $L = 400$, $D = 1000$.

1. $M = 4$ $SNR = 20$ the simulation results are shown in fig 3. We can find that the circular incident angles $\left(10^\circ, 30^\circ\right)$. In addition, we know that Mixed-PM algorithm can effectively estimate the received signals with the number of elements equal to the number of sources, which is an advantage.

2. We can get that the performance of noncircular signals are better than the circular's from fig 4.

3. $L = \left[50,100,200,400\right]$, we can find that with the increase in the number of snapshots, the performance of the algorithm is getting better from fig 5.

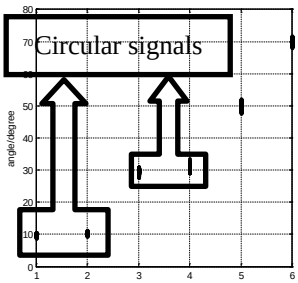

Figure 3.The scatter plot

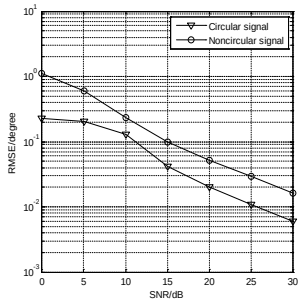

Figure 4.The estimation of mixed signals

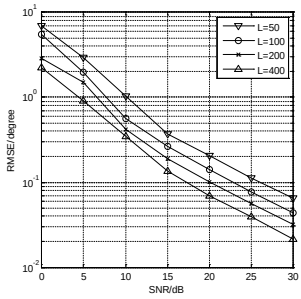

Figure 5.Different L

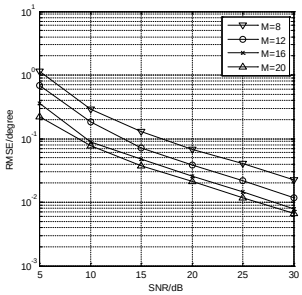

Figure 6.Different M

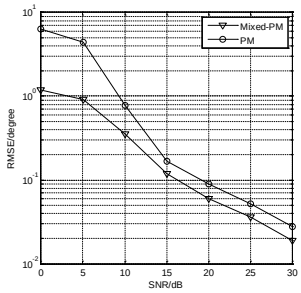

Figure 7.The Mixed-PM and PM

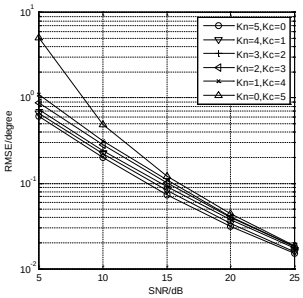

Figure 8.Different K_c and K_n

4. $M = \left[8,10,12,14\right]$, we know form fig 6 that with the increase in the number of array elements, the performance of the algorithm is getting better.

5. We know form fig 7 that the performance of Mixed-PM is better than classical PM.

6. $K_c = \left[0,1,2,3,4\right]$, angles $\left(10^\circ, 30^\circ, 50^\circ, 70^\circ\right)$ in turn selected, as shown in fig 9, we can find the more the number of noncircular signals in mixed signals, the better the performance of Mixed-PM.

5. Conclusions

The complexity of calculating the $\hat{\mathbf{R}}$ is $O(4M^2L)$, and the complexity of calculating $\hat{\mathbf{P}}_c$ is $O(2M(K+K_c)^2 + (K+K_c)^3 + 4M^2(K+K_c))$, then the complexity of calculating the $\hat{\mathbf{P}}_1^+$ $\hat{\mathbf{P}}_2$ is $O(4(M-1)^2(K+K_c) + (K+K_c)^3 + 4(K+K_c)^2(M-1))$, and the complexity of calculating eigenvalues of

$\hat{\mathbf{P}}_1^+ \hat{\mathbf{P}}_2$ is $O((K+K_c)^3)$. The total complexity is $O(4M^2L+8M^2(K+K_c)+3(K+K_c)^3+6M(K+K_c)^2)$.

In this paper, we mainly study the one-dimensional DOA estimation algorithm based on circular and noncircular mixed signals--Mixed-PM algorithm, which is able to distinguish circular signals and noncircular signals in mixed signals. Mixed-PM algorithm can effectively estimate the received signals with the number of elements equal to the number of sources, the performance of which is better than classical PM. The performance of noncircular signals is better than the circular's, and with the increase in the number of snapshots or array elements, the performance of the algorithm is getting better. In addition, the more the number of noncircular signals in the received incident signals, the better the performance of the Mixed-PM algorithm.

6. Acknowledgement

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