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A Numeral Simulation of Atomic Clock Noise

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***Abstract*—A numeral method for simulating three types of noise in atomic clock is presented, and the specific recursive functions to produce the random processes are given. The simulation is simple by computer, and it is useful for designing atomic clock before committing the system to a hardware realization.**

Keywords-noise; atomic clock; simulation

I. Introduction

The noise model of atomic clock is a superposition of five noise types, they are named as white noise phase modulation (W-PM), flicker noise phase modulation (F-PM), white noise frequency modulation (W-FM), flicker noise frequency modulation (F-FM), and random walk noise frequency modulation (RW-FM) [1], and the model is called power-law spectra noise model since the power spectral densities have a power law dependence on frequency [2]. For W-PM, W-FM, and RW-FM, we give a numeral simulation method.

II. Numeral Simulation Method

Allan variance is expressed as

$$\sigma_y^2(\tau) = \frac{1}{2\tau^2} E[(x_n - 2x_{n+N} + x_{n+2N})^2] \tag{1}$$

where x_n is time datum, $\tau = N\tau_m$, τ_m is the interval between points [3].

A. Simulation to W-PM

Let a_n denote the random number of normal distribution, and their mean is zero and the variance is σ_a^2 ; let x_n denote the random number to simulate W-PM. First of all, it is need to calculate the variance of x_n .

For the phase white noise, have $x_n = a_n$, thus substitution it into (1),

$$\sigma_y^2(\tau) = \frac{1}{2\tau^2} E[(a_n - 2a_{n+N} + a_{n+2N})^2] \tag{2}$$

Based on

$$E(a_n, a_m) = \begin{cases} 0, n \neq m \\ \sigma_a^2, n = m \end{cases},$$

we obtain

$$E[(a_n - 2a_{n+N} + a_{n+2N})^2] = 6\sigma_a^2 \tag{3}$$

that is

$$\sigma_a^2 = \frac{1}{3}\tau^2\sigma_y^2(\tau) \tag{4}$$

The noise level is from the Allan variance of conventional atomic clock. Based on known Allan variance to be simulated over a suitable range of time intervals, we select a point (τ_{-2}) which appears typical of the noise type to be simulated, and $\sigma_y^2(\tau_{-2})$ is the corresponding Allan variance, thus, we obtain the variance of the random numbers to simulate the noise

$$\sigma_{-2}^2 = \frac{1}{3}\tau_{-2}^2\sigma_y^2(\tau_{-2}) \tag{5}$$

Then, random numbers to simulate the desired W-PM are

$$x_n = \sigma_{-2} p_n \qquad (\nu = 1, 2, ...) \tag{6}$$

where p_n is random number with standard normal distribution.

Fig.1 shows the simulation data to W-PM.

B. Simulation to W-FM

Let y_n denote the random number to simulate W-FM, thus

$$y_n = a_n .$$

In this case, the variations in frequency affect the rate of change of phase variations. For simple, let the interval of sample be one unit and x_0 be zero, thus

$$y_n = x_n - x_{n-1} = a_n$$

that is

$$x_n = \sum_{i=1}^{n} a_n \tag{7}$$

Substitution it into (1), have

$$\begin{aligned}\sigma_y^2(\tau) &= \frac{1}{2\tau^2} E[(\sum_{i=1}^{n} a_i - 2\sum_{i=1}^{n+N} a_i + \sum_{i=1}^{n+2N} a_i)^2] \\ &= \frac{1}{2\tau^2} E[(\sum_{i=n+N+1}^{n+2N} a_i)^2 + (\sum_{i=1+n}^{n+N} a_i)^2] \\ &= \frac{N}{\tau^2}\sigma_a^2\end{aligned}$$

that is

$$\sigma_a^2 = \frac{\tau^2}{N}\sigma_y^2 \tag{8}$$

On the noise level, we select a point (τ_{-1}) which appears typical of the noise type to be simulated, and $\sigma_y^2(\tau_{-1})$ is the corresponding Allan variance. Thus, we obtain the variance of the random numbers to simulate the noise

$$\sigma_{-1}^2 = \frac{\tau^2}{N}\sigma_y^2(\tau_{-1}) \tag{9}$$

Thus, random numbers to simulate the desired W-FM are

$$x_n = x_{n-1} + \sigma_{-1} p_n \quad (\nu = 1, 2, \dots) \tag{10}$$

Fig.2 shows the simulation phase data to W-FM.

C. Simulation to RW-FM

Let z_n denote the random number to simulate RW-FM, thus $z_n = a_n$. Since the variations affect the rate of change of frequency variation, we obtain

$$z_n = y_n - y_{n-1} = a_n$$

that is

$$y_n = \sum_{i=1}^{n} a_n$$

Because of $y_n = x_n - x_{n-1}$, have

$$x_n = \sum_{j=1}^{n}\sum_{i=1}^{j} a_n = \sum_{i=1}^{n}(n-i+1)a_i$$

Substitution it into (1), yields:

$$\begin{aligned}\sigma_y^2(\tau) &= \frac{1}{2\tau^2} E[(\sum_{i=1}^{n}(n-i+1)a_i - 2\sum_{i=1}^{n+N}(n+N-i+1)a_i \\ &+ \sum_{i=1}^{n+2N}(n+2N-i+1)a_i)^2] \\ &= \frac{1}{2\tau^2} E[(-\sum_{i=1+n}^{n+N}(n-i+1)a_i + \sum_{i=n+N-1}^{n+2N}(n+2N-i+1)a_i)^2] \\ &= \frac{\sigma_a^2}{2\tau^2}[N^2 + 2(N-1)^2 + 2(N-2)^2 + \cdots + 2\times 1] \\ &= \frac{\sigma_a^2}{6\tau^2}(2N^2+1)N^2\end{aligned}$$

On the noise level, we select a point (τ_1) which appears typical of the noise type to be simulated, and $\sigma_y^2(\tau_1)$ is the corresponding Allan variance. Thus, we obtain the variance of the random numbers to simulate the noise

$$\sigma_1^2 = \frac{\sigma_y^2(\tau_1)}{(2N^2+1)N^2}6\tau^2 \tag{11}$$

Thus, random numbers to simulate the desired RW-FM are

$$z_n = \sigma_1 p_n$$

That is

$$y_n = y_{n-1} + \sigma_1 p_n$$

or

$$x_n = 2x_{n-1} - x_{n-2} + \sigma_1 p_n \quad (\nu = 1, 2, \dots) \tag{12}$$

Fig.3 shows the simulation phase data to RW-FM.

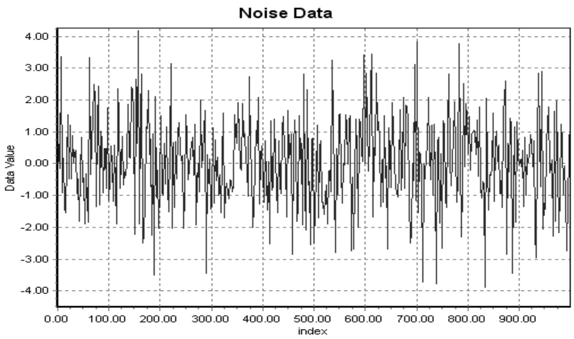

FIGURE I. SIMULATION DATA OF WHITE NOISE PHASE MODULATION

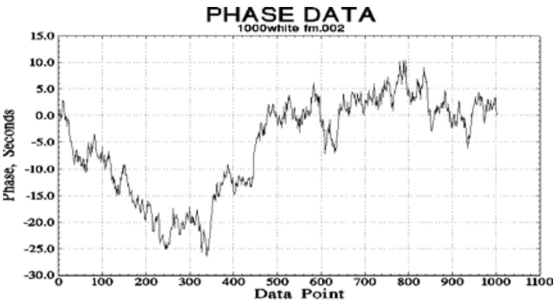

FIGURE II. SIMULATION PHASE DATA OF WHITE NOISE FREQUENCY MODULATION

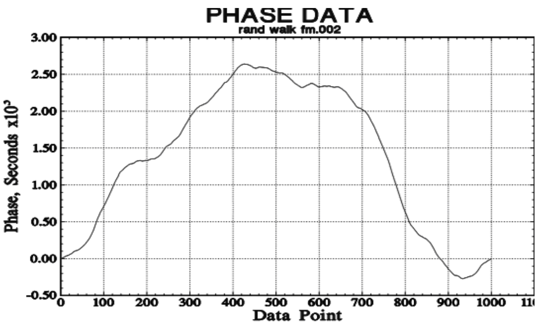

FIGURE III. SIMULATION PHASE DATA OF RANDOM WALK NOISE FREQUENCY MODULATION

Via the simulation method above, one can simulate the three types of noise in atomic clock. An Allan variance can indicate the levels and types of present noise. For a given noise type, we select a value to be the most reliable indication of the Allan variance, as a result, the recursive functions produce these random processes

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