

Mathematics in Tiwanaku

The gold number in the gate of the sun

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Abstract— Man has tried to understand nature and represent it in mathematical ways since ancient times. Looking back to the past, instead of finding uncultured civilizations, we found that they understood the beauty of nature, and expressed it by numbers. The Tiwanaku Empire is an example that amused archeologist from all over the world. Its megalithic giant's constructions and its iconography are still at the Bolivian plateau. Here, we focus on one particular relic from this Empire, the Gate of the Sun and its mathematical relation to the gold number.

Keywords-component; *Tiwanaku; Gold number; Golden rectangle; Golden Triangle.*

I. THE TIWANAKU EMPIRE

A. Origins

The empire of Tiwanaku, recognized by UNESCO as Spiritual and Political Centre of the Tiwanaku culture [1], developed around the year 1580 B.C.E. and extended its kingdom until the year 1187 C.E. according to the archaeologist Carlos Ponce Sanginés [2]. Tiwanaku spreads through the territories of Peru, Chile, and Bolivia, but mainly developed in Bolivia [3], a few kilometers of Titicaca Lake, in the city of La Paz, the seat of the Bolivian government.



FIGURE I. THE TIWANAKU EMPIRE COVERING 3 COUNTRIES IN SOUTH AMERICA

Tiwanaku was characterized by a ceremonial infrastructure and religious practices. The systems of thought were disseminated through objects such as ceramics and sculptures, with standardized iconography. For example, in 2006, more

than four hundred pieces of ceramic of great perfection and color were found on the Lake Titicaca, dating from radiocarbon analysis of 1000 C.E.



FIGURE II. CERAMICS ICONOGRAPHY OF THE TIWANAKU CULTURE.

In Tiwanaku, bronze technology was widely used in stone ceremonial architecture in the form of clamps. Laminated gold ornaments, silver, pyrography objects, and basketry in semi-precious stones, bones, and shells.



FIGURE III. ARTHUR POSNANSKY TOGETHER WITH A MONOLITHIC STRUCTURE IN TIWANAKU.

The Spanish chronicler Pedro Cieza de León, in the 16th century, was the first European to describe the archaeological remains of Tiwanaku. But it was in the 19th century that the site was visited.

Alcides D'Orbigny, Ephraim Squier, Charles Weiner, and archaeologists Alphonse Bandelier and Max Uhle, in the early 20th century visited the sacred lands of Tiwanaku. The engineer, explorer, ship's navigator, director of a river navigation company, entrepreneur, author of the book: *Tiwanaku, the Cradle of American Man*, well known and well respected avocational archaeologist Arthur Posnansky, later studied the area as well. During the 20th century the American Wendell

Bennett in the decade of the 30; Carlos Ponce Sanginés from Bolivia between the 60s and 80s; and the North Americans Alan Kolata, David Browman and Marc Berman, studied the ruins. By the growing interest, Bolivians such as Sonia Alcon and Juan Albarracín Jordán in the 90s, and other Americans like Charles Stanish, Nicole Couture, Deborah Blom, Alexei Vranich, John Janusek; The Finn Antti Korpisaari; And the Bolivians Jédu Sagárnaga, Claudia Rivera, and Juan Villanueva, visited the sacred site of Tiwanaku years later.

B. Decadence

The political and religious decay of Tiwanaku may have occurred between 950 and 1100 C.E. Evidence indicates that this empire was bloody destroyed, as reported by the remains of bodies that were desecrated and torn apart.

In the 11th century, the capital of Tiwanaku was abandoned and several minor settlements began to appear in the whole plateau of the Collao, where several groups decided to migrate from the city and group themselves into independent populations.



FIGURE IV. PLATEAU OF THE COLLAO, EXTENDING BETWEEN BOLIVIA AND PERU WHERE THE TIWANAKU EMPIRE WAS DEVELOPED.

It is on the shores of Titikaka Lake of Bolivia, where the remains of this mysterious empire lie. Monuments of colossal size, made of stone with fantastic friezes have captivated Bolivians and foreign archaeologist since the 19th century. This friezes and its mathematical relation with ancient numbers will be addressed in the following section of this article.



FIGURE V. TEMPLE OF KALASASAYA IN TIWANAKU.

II. GEOMETRY IN TIWANAKU

The archaeological remains left by the empire of Tiwanaku can be found in Bolivia. The geometry present on its friezes has attracted the attention of archaeologist from all over the world, by its shape and its antiquity [4]. To describe its geometry, we have to recall some concepts about the mathematical constant known as the gold number.



FIGURE VI. KALASASAYA TEMPLE, SHOWING DIFFERENT SCULPTED FACES, LOCATED IN TIWANAKU, CLOSE TO THE GATE OF THE SUN.

A. Golden Rectangles

For the Greeks, this geometry, whose sides are in a proportion equal to the golden ratio, it was considered of particular beauty and was used in its architecture. The golden rectangle has a very interesting property because new golden rectangles can be obtained by means of an iterative process. Figure 7 describes mathematically this relation

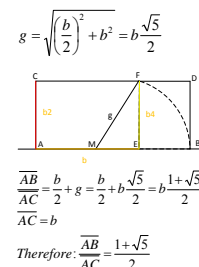


FIGURE VII. CONSTRUCTION OF A GOLDEN RECTANGLE.

B. Proportions of Phi

The golden ratio or gold number *phi* (Φ) appears regularly in the geometry of nature, as in the growth patterns of organs and biological organisms.

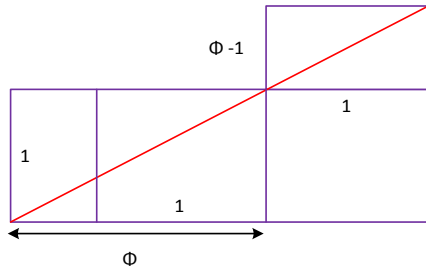


FIGURE VIII. THE GENERATION OF GOLDEN RECTANGLES

The gold number was used during the Renaissance, particularly in the plastic arts and architecture. It was at the beginning of the 16th century that it was called for the first time "Divine Proportion"

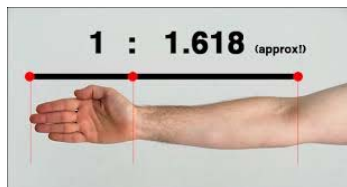


FIGURE IX. THE DIVINE PROPORTION IN THE HUMAN BODY.

C. The Gold Number

It was also known as the golden ratio is was usually represented by the Greek letter *phi* (Φ) in honor of Phidias, the architect who designed the Parthenon. This number is also attributed to the Pythagorean School since they used a five-pointed star as a symbol.

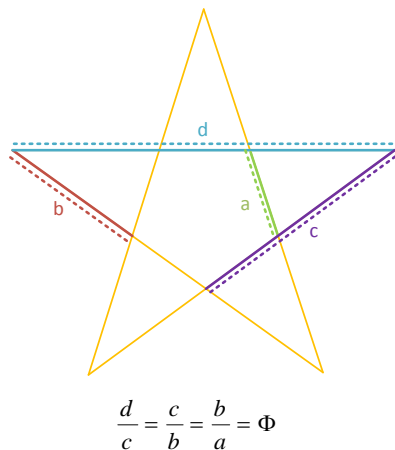


FIGURE X. THE FIVE-POINTED STAR AND THE RELATION WITH THE GOLD NUMBER.

Leonardo da Vinci in his "Vitruvian man" indicates different golden ratios. Golden rectangles built on the basis of the Gold number are proportions that have been frequently used in the art.

D. Golden Triangles

This type of triangle is isosceles with angles 36°, 72° and 72°. The succession of golden triangles can be seen as a dynamic process from outside to inside, that is, generation of smaller triangles from larger triangles.

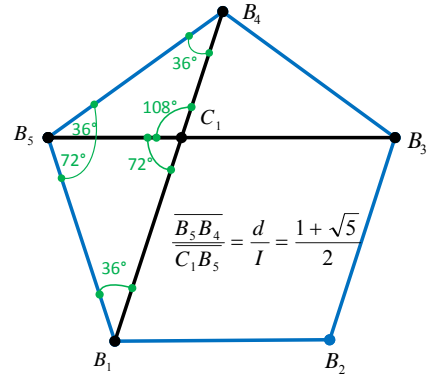


FIGURE XI. THE PENTAGON AND THE INTERNAL GOLDEN TRIANGLES.

After recalling the concepts about the gold number; we will focus on the ruins of Tiwanaku, where the Gate of the Sun is the archaeological piece of interest in this article.

The geometry on its frieze and the dimensions of the structure are related to the mathematical definitions indicated above. In the following lines, we present a close look to show the mathematical relations with *phi*, the gold number.

III. THE GATE OF THE SUN

There are references to the Gate of the Sun since the 16th century. At some point fallen and broken was several times lifted. Nowadays, it is located in the platform in front of other archaeological masterpiece called the temple of Kalasasaya on the shores of Titikaka Lake, even though there is no evidence that formed part of it [5]. All the engravings of its frieze were the object of countless studies that have tried to demonstrate that it represents some calendar: terrestrial, lunar or Venusian. In these studies, there is no reference to the assessment of their geometric composition.

A. Gate or Monument

The composition of the front face of the Gate of the Sun in Tiwanaku presents, anthropometric, and bilateral symmetry. On the back side, there are also harmonic proportions but with less significance. These proportions were not found before because the study of the content of its engraving frieze was believed to be a calendar, idea that has been so widely extended, that any other interpretations were ignored (however, the presence of these proportions does not exclude the possible existence of some type of calendar). The Gate of the Sun may not be part of a temple, but its content could also be considered as the expression of a monument in itself; a work that testifies the importance of knowledge, art, and science [6], [7].

B. Proportions Found at the Gate of The Sun

For the analysis of the whole Gate, it has been assumed that the lower part is 2 cm below the current floor level, and we consider the integral pattern of the design [8].

The first proportion to be found refers to the golden triangle, as seen in figure 12. From the navel of the central frieze, it can be seen that two lines are projected to the base of the gate, the relation between the orthogonal line (ϕ) to the base of the triangle ($1/\phi$) gives the relation ϕ^2 .

Extending the lines from the navel to the vertices of the Gate of the Sun, we find that two golden triangles are formed. At the same time, projecting the lines from the navel to the surface below the horizontal line of the rectangular windows, two triangles are formed again. All of them satisfying the relation of golden triangles, as shown in figure 13.

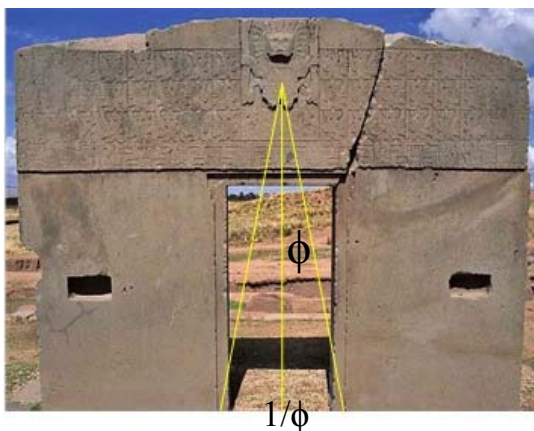


FIGURE XII. THE MATHEMATICAL PROPORTION BETWEEN THE HEIGHT AND THE BASE OF THE GATE OF THE SUN

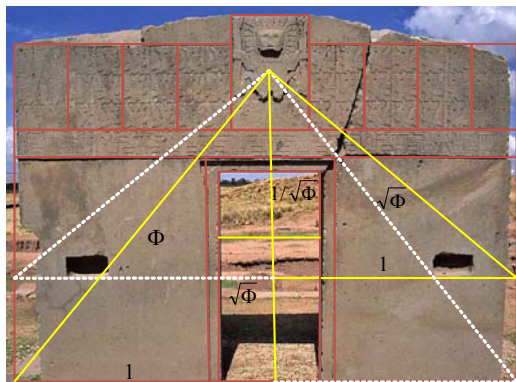


FIGURE XIII. THE PROPORTION OF GOLDEN TRIANGLES ON THE FRONT FACE OF THE GATE OF THE SUN.

Dividing the surface of the Gate of the Sun in rectangles and according to the iconography of the front face, the proportions found correspondent to several golden rectangles as shown in figure 14.

Using the geometry of the Vitruvian Man of Leonardo Da Vinci [9], it is possible to find that the human proportions related to the golden number are intersected at several points at different axes of the lines drawn on the gate. For a better

comprehension, the drawing by Leonardo da Vinci illustrates this manifestation in figure 15.

Finally, the navel of the central figure of the frieze, which constitutes the main point of reference in the composition of the work, is at a height of $6/\phi$ the ground (measured with the same scale of the square and the circle and, therefore, the human proportions), although it could only be a spontaneous effect of the proportions of the whole.

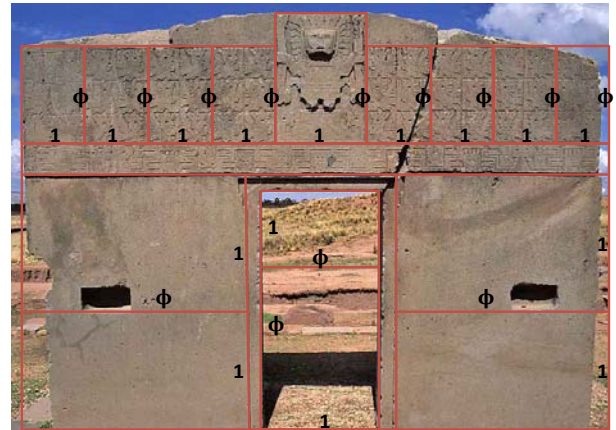


FIGURE XIV. GOLDEN RECTANGLE PROPORTIONS ON THE FRONT FACE OF THE GATE OF THE SUN

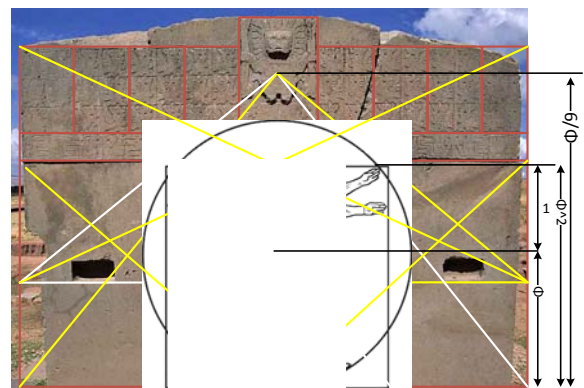


FIGURE XV. THE GEOMETRIC RELATION BETWEEN THE VITRUVIAN MAN AND THE GATE OF THE SUN.

IV. CONCLUSION

Every time man looks to the past finds evidence of astonishing architectural masterpieces, which used mathematical relationships for their construction. One of them is the Gate of the sun of Tiwanaku, Bolivia, where the gold number is manifested as part of the sacred geometry of this monument that has been studied since the 20th century and represents an empire of 2000 years ago. This relic testifies the importance of knowledge, art and science in human life.

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