Non-linear Analysis on Optical Flow 9 × 9 High-Contrast Patches

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***Abstract*—We analyze the space of 9×9 optical flow high-contrast patches by computational topological tools. The fact that there are subspaces of 9×9 optical flow high-contrast patches with topology of a circle is experimentally shown. Subspaces of 9×9 optical flow patches with homology of the Klein bottle are not been found by two different ways.**

Keywords-high-contrast patches; persistent homology; optical flow

I. Introduction

A lot of researches about natural image statistics have been made, as difficulty of producing ground truth data for modelling optical flow statistics, there are relatively few about statistics of optical flow. Roth and Black [1] constructed a database of natural scene motions by using of range images and camera motions, and established a rich prior model for optical flow. A new topological property of 3×3 optical flow patches is found by using the nudged elastic band technique [2]. As the optical flow database producing from the Brown range image database, optical flow patches may have similar topological properties as that of range image patches. The authors of papers [2, 3, 4] have found such similar topological features for optical flow $n\times n$ patches with range image patches when $n=3,\ 4,\ 5,\ 6,\ 7$.

In this short note, we enlarge the size of optical flow patches to 9. By using methods of the paper [5], it is shown that 9×9 optical flow patches with density subsets having topology of a circle. As increasing of the size of optical flow patches, the Klein bottle property of the optical flow spaces gradually weaken [3, 4]. Here, we prove that the Klein bottle property of 9×9 optical flow patches may vanish.

II. The optical flow 9×9 patches space

The main space comes from the Roth and Black optical flow database [1]. Figure I is one sample. Our space X_9 consists of 9×9 optical flow patches established by the same way as [5, 6, 7].

A 9×9 patch is put in order

$$\begin{pmatrix} (u_1,v_1) & (u_{10},v_{10}) & \cdots & (u_{64},v_{64}) & (u_{73},v_{73}) \\ (u_2,v_2) & (u_{11},v_{11}) & \cdots & (u_{65},v_{65}) & (u_{74},v_{74}) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ (u_8,v_8) & (u_{17},v_{17}) & \cdots & (u_{71},v_{71}) & (u_{80},v_{80}) \\ (u_9,v_9) & (u_{18},v_{18}) & \cdots & (u_{72},v_{72}) & (u_{81},v_{81}) \end{pmatrix},$$

where u denotes optical flow in the horizontal direction and v express the vertical direction. We take each 9×9 patch as a vector $\boldsymbol{x}=(u_1,\ldots,u_{81},v_1,\ldots,v_{81})\in \mathbb{R}^{162}$.

We randomly take 50,000 patches of X_9, written as XS_9, for convenience of computing.

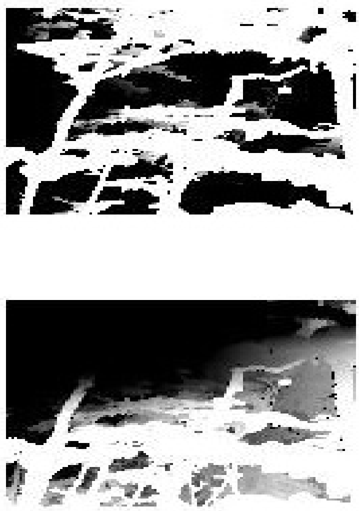

FIGURE I. ONE SAMPLE FROM THE OPTICAL flOW DATABASE. HORIZONTAL MOTION IS ON THE TOP AND VERTICAL MOTION IS ON THE BOTTOM.

III. Results for $XS_9(k,p)$

Persistent homology is a means to detect topological property of a space by a finite sampled points. We use software package Javaplex to compute persistent homology, please refer to [5, 8, 9, 10, 11] for more details.

We take core subset $XS_9(200,30)$, and compute its barcodes, Figure II shows one PLEX results for it. A long Betti$_0$ line and a long Betti$_1$ line are in the plots, this means: $\beta_0 = 1$, $\beta_1 = 1$, having the topology of a circle. We do one hundred trials on $XS_9(200,30)$, in all trial results, the circular behavior $\beta_0 = 1$, $\beta_1 = 1$ is discovered in a large range, and other Betti plot lines are very short, hence the result is steady. When we take core subsets $XS_9(100,30)$, we have the same result.

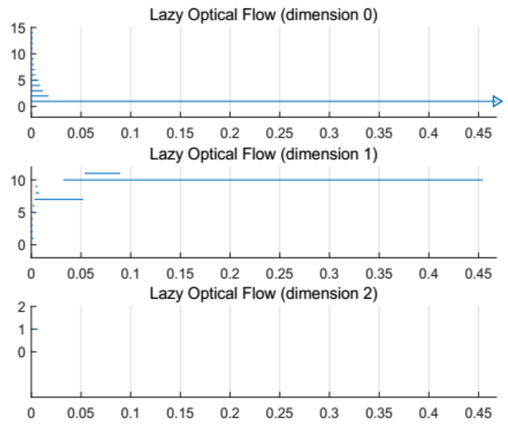

FIGURE II. BARCODES FOR $XS_9(200,30)$

IV. MAIN RESULTS FOR X_9

To detect Klein bottle behavior of 9×9 optical flow patches X_9, we describe main steps of generating a theoretic Klein bottle model. Let $\wp$ be functions with the form $a_2(a_1x+b_1y)^2$ $+b_2(a_1x+b_1y)$, where $(a_i,b_i)\in S^1$ $(i=1,2)$, S^1 being the unit circle. Let: $g: S^1\times S^1 \to \wp$ be given by (a_1,b_1,a_2,b_2) a $a_2(a_1x+b_1y)^2$ $+b_2(a_1x+b_1y)$ ([5]), and $h_9: \wp \to S^{161}$ defined by a composite of evaluating the function at each grid $G_9=\{-8,-7,\ K\ ,8,9\}$ $\times\{-4,-3,\ldots,3,4\}$ subtracting the mean and normalizing.

It is followed from [5] that the image $im(h_9\,|\,\wp)$ is homeomorphic to the Klein bottle.

We uniformly pick 200 points $(\{x_1, K\ , x_{200}\})$ on S^1, all possible tuples (x_i, x_j) compose a point set on the torus $S^1\times S^1$ for gaining a proper theoretic model of the Klein bottle in S^{161}. Then, we map each of the 40000 points into S^{161} by $h_9 \circ g$, the image is represented $K_9(200)$. Figure III is one PLEX result of $K_9(200)$, which shows $\beta_0=1$, $\beta_1=2$ and $\beta_2=1$, that are the Betti numbers of the Klein bottle. Therefor $K_9(200)$ is a proper approach of the Klein bottle in S^{161}.

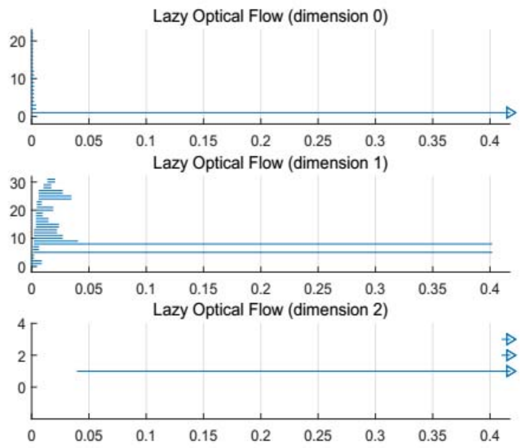

FIGURE III. BARCODES FOR $K_9(200)$

To detect Klein bottle behavior of 9×9 optical flow patches, we use two kinds of subspaces of X_9 obtained by following.

(1) For each point p in $K_9(200)$ we compute the Euclidean distance from p to every point of X_9, and then choose t closest points to the point p. The obtained subspace of X_9 is represented by $Kopt_9(200,t)$.

(2) For each point of X_9, we calculate the Euclidean distance from p to the set $K_9(200)$, then we resort points of X_9 in order to increasing of their Euclidean distances to $K_9(200)$, the subspace $XP_9(200,t)$ of X_9 is taken by the top t percent of the closest distances.

To discover whether a subspace of X_9 has the homology of the Klein bottle, we consider the subspace $Kopt_9(200,11)$. We many tests on $Kopt_9(200,11)$, Figure IV shows one PLEX result for $Kopt_9(200,11)$, which has the Klein bottle behavior. Figure V shows the other PLEX result of $Kopt_9(200,11)$, which shows no the Klein bottle behavior.

We also consider $Kopt_9(w,t)$ for w = 180, 280, and t =1, 3, 5, 7, 9, 11, 13, we ran many experiments on them, which give similar results as for $Kopt_9(200,11)$.

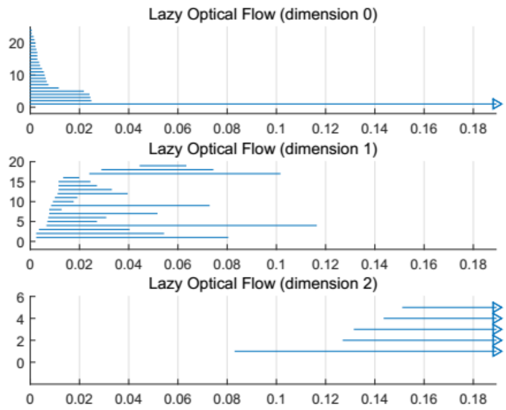

FIGURE IV. BARCODES FOR $Kopt_9(200,11)$

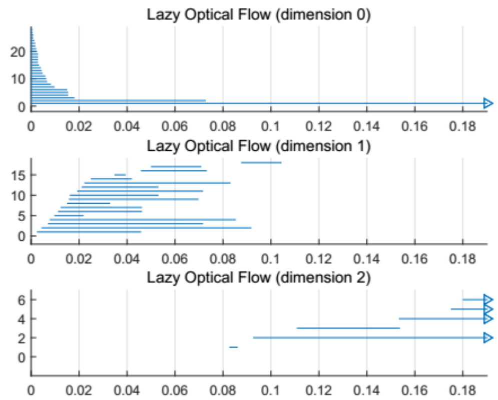

FIGURE V. BARCODES FOR $Kopt_9(200,11)$

When we take subsets $XP_9(200,t)$ for t =10, 15, 20, 25, 30, 35, and 40, we do many tests on them with different parameters, we cannot discover the Klein bottle behavior. If we take the union $XP_9(200,20)\cup Kopt_9(200,1)$, we run 150 tests on it, there are 45 tests with the Klein bottle behavior, but the others show no the Klein bottle behavior. Figures VI~VIII give three PLEX results for $XP_9(200,20)\cup Kopt_9(200,1)$. Figure 6 shows $\beta_0=1$, $\beta_1=2$, $\beta_2=1$ in [0.105, 0.172], Figure 7 has $\beta_0=1$, $\beta_1=2$, $\beta_2=1$ in a very small range [0.110, 0.125], but Figure 8 has no the Klein bottle behavior.

Therefor we can deduce that the Klein bottle behavior of the spaces X_9 gradually eliminates.

V. CONCLUSIONS

In this short note we utilize persistent homology technique to show that there are core subsets of the high-contrast 9×9 patches modeled as a circle. The Klein bottle's behavior of 9×9 optical flow patches gradually vanish. Our result shows that we need not to study larger $n\times n$ optical flow high-contrast patches for $n\geq 10$.

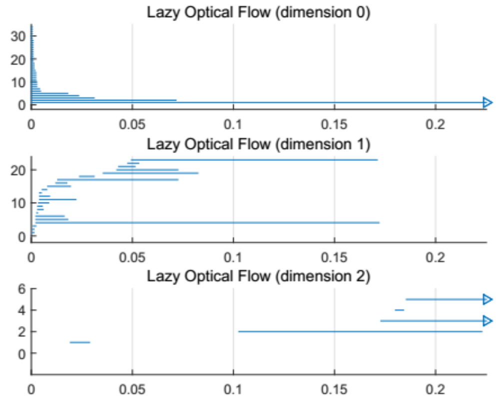

FIGURE VI. BARCODES FOR $XP_9(200,20)\cup Kopt_9(200,1)$

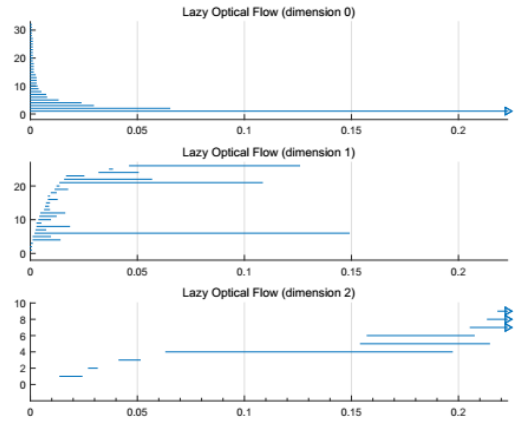

FIGURE VII. BARCODES FOR $XP_9(200,20)\cup Kopt_9(200,1)$

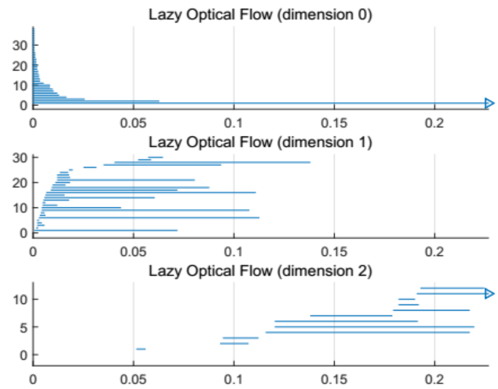

FIGURE VIII. BARCODES FOR $XP_9(200,20)\cup Kopt_9(200,1)$

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