

Slip Effects on Heat and Mass Transfer of a Non-newtonian Fluid in the Micro-channel

Jing Zhu and Jiani Sui

Department of applied Mathematics, University of Science and Technology Beijing, 100083, Beijing, China

Abstract—The effects of velocity-slip and temperature-jump boundary conditions on Non-Newtonian flow and heat transfer in the micro channel have been investigated in this paper. The governing boundary layer equations have been transformed into a system of nonlinear differential equations through the similarity transformation, and the analytical approximations of solutions are derived by homotopy analysis method. The reliability and efficiency of the HAM solutions are verified by the residual errors. Furthermore, the effects of physical factors on the flow and heat are studied and discussed graphically.

Keywords—second grade fluid; slip-flow; velocity-slip; temperature-jump; homotopy analysis method

I. INTRODUCTION

During the recent years, with the rapid development of science and technology in micro- and nano-measuring technologies, it has been found that there are many significant differences between fluid flow at macro-scale and that at micro/nano-scale, such as wall-slip phenomenon. Fluids exhibiting slip are important in technological applications such as in the polishing of artificial heart valves and internal cavities. Therefore, better understanding of the phenomenon of slip is necessary. Mooney^[1] initiated the study of boundary layer flow with partial slip. M. Turkyilmazoglu^[2] has analyzed the heat and mass transfer characteristics of the mixed thermal slip MHD viscous flow. G. Singh and A.J. Chamkha^[3] have studied viscous fluid flow and heat transfer with second-order slip on a vertical permeable shrinking sheet. T.G. Fang and S.S. Yao^[4] have used a newly proposed second order slip flow model to analyze viscous flow over a shrinking sheet, and they have concluded that the second order slip flow model predicts the flow characteristic accurately. The flows of non-Newtonian fluids play an important role in many practical applications. Moreover, modeling viscoelastic flows is important for understanding and predicting the behavior of processes and thus for designing optimal flow configuration and for selecting operating conditions. B. Sahoo^[5] has considered the laminar flow and heat transfer of an electrically conducting second grade fluid with partial slip. C.Ye and X.N. Luo^[6] have come out the high-order numerical methods for heated generalized second grade fluid. I.G. Baoku and many other scholars^[7-12] also have investigated the influence of various kinds of fluid with different slip conditions. They have got a lot of constructive achievements.

In all the above-mentioned studies, little attention has been given to the effects of partial slip on the flow of a second-grade fluid in the micro channel. The main goal of this paper is to investigate the effects of the velocity-slip and

temperature-jump on the flow and heat transfer characteristics of a second-grade fluid in the micro channel. Besides, mass transfer effects are also considered. Graphical results are presented and discussed by homotopy analysis method.

TABLE I. NOMENCLATURE.

Symbol	description	Symbol	Description
u	velocity components along x -axis	Sc	Schmidt number
v	velocity components along y -axis	K	normal stress parameter
a	width of the channel	η	similarity variable
b, k	constants	μ	dynamic viscosity
T	temperature inside the boundary layer	ν	kinematic viscosity of the fluid
B_0	constant magnetic flux density	θ	dimensionless temperature
C	fluid concentration	ϕ	dimensionless concentration
D	coefficient of mass diffusivity	k_1	thermal diffusivity of the fluid
T_w	uniform temperature of the fluid	ρ	fluid density
C_w	uniform concentration of the fluid	c_p	heat capacity of the fluid
M	magnetic parameter	σ	electrical conductivity
Re	Reynolds number	α	second-grade fluid parameter
Pr	Prandtl number	λ	velocity-slip parameter
Ec	Eckert number	β	temperature-jump parameter

II. MATHEMATICAL ANALYSIS

We consider the heat and mass transfer in a boundary layer flow of a second-grade fluid in the micro channel. The coordinate system and scheme of the problem is shown in Fig.1. We consider the velocity-slip and temperature-jump conditions on the wall of the channel, the boundary conditions are:

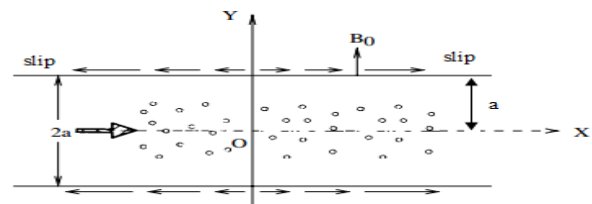


FIGURE I. PHYSICAL MODEL AND COORDINATE SYSTEM OF THE PROBLEM

$$\frac{\partial u}{\partial y}(x,0) = v(x,0) = 0, \frac{\partial T}{\partial y}(x,0) = 0, C(x,0) = C_w, \quad (1)$$

$$u(x,a) = bx + \frac{2-\sigma}{\sigma} \lambda \left[\frac{\partial u}{\partial y} + \frac{k_0}{\mu} \left(2 \frac{\partial u}{\partial x} \frac{\partial u}{\partial y} + v \frac{\partial^2 u}{\partial y^2} + u \frac{\partial^2 u}{\partial x \partial y} \right) \right]_{y=a}, \quad (2)$$

$$v(x,a) = -kab, T(x,a) = T_s = T_w + \beta \frac{\partial T}{\partial y} \Big|_{y=a}, C(x,a) = C_s. \quad (3)$$

The governing boundary layer equations^[13] for the problem are as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (4)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{k_0}{\rho} \left[\frac{\partial}{\partial x} \left(u \frac{\partial^2 u}{\partial y^2} \right) + \frac{\partial u}{\partial y} \frac{\partial^2 v}{\partial y^2} + v \frac{\partial^3 v}{\partial y^3} \right] - \frac{\sigma B_0^2}{\rho} u \quad (5)$$

$$\rho c_p \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k_1 \frac{\partial^2 T}{\partial y^2} + \mu \left(\frac{\partial u}{\partial y} \right)^2 + k_2 \frac{\partial u}{\partial y} \frac{\partial}{\partial y} \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) + \sigma B_0^2 u^2 \quad (6)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} \quad (7)$$

The mathematical analysis of the problem is simplified by introducing the following dimensionless variables:

$$x^* = \frac{x}{a}, y^* = \frac{\eta}{a}, u = -\alpha x^* f'(y^*), v = sf'(y^*), s = ab, \theta(\eta) = \frac{T-T_s}{T_w-T_s}, \phi(\eta) = \frac{C-C_s}{C_w-C_s} \quad (8)$$

Substituting (8) into (1)-(7), we obtain the following ordinary differential with respect to the dimensionless variable η :

$$f''' - Mf' + Re(f'^2 - ff'') - \alpha(2f'f''' - f'^2 - ff^{(4)}) = 0 \quad (9)$$

$$\theta'' + Pr(f'\theta' - f'\theta) + Ec f'^2 + MEc f'^2 + KEc f''(f'f'' - ff''') = 0 \quad (10)$$

$$\phi'' - 2Scf'\phi + Scf\phi' = 0 \quad (11)$$

$$f'(0) = f(0) = 0, f(1) = 1 + \lambda(f'(1)(1 + 3\alpha f'(1)) - \alpha f(1)f'(1)), \quad (12)$$

$$f(1) = K, \theta'(0) = 0, \phi(0) = 1, \theta(1) = 1 + \frac{\beta}{\alpha} \theta'(1), \phi(1) = 0. \quad (13)$$

Here $M = \frac{\sigma B_0^2 a^2}{\mu}, Re = \frac{a^2 b}{\nu}, \alpha = \frac{k_0 b}{\mu}$

$$Pr = \frac{\rho c_p a^2 b}{k_1}, Ec = \frac{\mu}{\rho c_p (T_w - T_s)}, K = \frac{k_2}{\mu}, Sc = \frac{\nu \rho c_p}{\mu}, T_w = T_s + d \left(\frac{x}{l} \right)^2, C_w = C_s + c \left(\frac{x}{l} \right)^2$$

TABLE II. RESULTS OF SKIN FRICTION COEFFICIENT $f''(1)$

λ	M	Re	α	$f''(1)$
1	6	10	0	-0.319016
			0.4	-0.533620
			0.6	-0.625132
			1	-0.802046
1	6	0	0.6	-0.207126
				-0.398486
				-0.885826
				-1.17959
λ	M	Re	α	$f''(1)$
1	0	10	0.6	-0.625132
1.5				-0.374598
2				-0.267381
3				-0.170035
1	0	10	0.6	-0.767444
				-0.717659
				-0.670244
				-0.625132

The local skin friction coefficient C_f and Nusselt number Nu are

$$C_f = \left(\frac{1+\alpha}{Re} \right) f''(1), Nu = -\theta'(1), \quad (14)$$

III. APPLICATION OF HAM

Due to the strongly nonlinear and unconventional nature of these problems above, the solving process is extraordinarily complex and we can hardly get the effective solutions. In this paper, we choose the homotopy analysis method (HAM) to get the approximate analytical solution. Homotopy analysis method is put forward by Liao^[14] originally, it has been applied and developed by many experts^[15-19] and it has been proved to be a strong and effective mathematical method to solve weak nonlinear problems. The initial approximations are as follows:

$$f_0 = -\eta^4 + 2\eta^2 - \eta, \theta_0 = \frac{1}{3}\eta^3 - \frac{1}{2}\eta^2 + \frac{7}{6}, \phi_0 = 1 + \frac{1}{2}\eta^2 - \frac{1}{2}\eta. \quad (15)$$

The linear operators are:

$$L_f = f^{iv}, L_\theta = \theta'', L_\phi = \phi'' \tag{16}$$

The m-order deformation equations are constructed as follows:

$$L_f[f_m(\eta) - \chi_m f_{m-1}(\eta)] = qh_f H_f(\eta) R_m(\eta) \tag{17}$$

$$L_\theta[\theta_m(\eta) - \chi_m \theta_{m-1}(\eta)] = qh_\theta H_\theta(\eta) S_m(\eta) \tag{18}$$

$$L_\phi[\phi_m(\eta) - \chi_m \phi_{m-1}(\eta)] = qh_\phi H_\phi(\eta) R_m(\eta) \tag{19}$$

with boundary conditions:

$$f_m(0) = f_m''(0) = f_m(1) = 0, \tag{20}$$

$$\theta_m'(0) = 0, \theta_m(1) = \frac{\beta}{\alpha} \theta_m'(1), \phi_m(0) = \phi_m(1) = 0. \tag{21}$$

$$f_m'(1) = \lambda(f_m^-(1) + 3\alpha 3\alpha(f_0^-(1)f_m^-(1) + f_0^-(1)f_m^-(1) + \sum_{i=1}^{m-1} f_i^-(1)f_{m-i-1}^-(1)) - \alpha(f_0^-(1)f_m^-(1) + f_0^-(1)f_m^-(1) + \sum_{i=1}^{m-1} f_i^-(1)f_{m-i-1}^-(1))), \tag{22}$$

IV. RESULTS AND DISCUSSION

Professor Liao has pointed that the convergence and its speed depend on the auxiliary parameters h_f , h_θ and h_ϕ to a great extent. It is straightforward to choose a proper value of h which ensures that the solution series is convergent. Fig.2 has given the valid ranges: $-1.2 \leq h_\theta \leq -0.5$, $-1.2 \leq h_\phi \leq -0.5$. We can also use the residual errors to help us find the proper h . In this paper, we define the residual error $E_{m,f}$ as the following:

$$E_{m,f} = \int_0^1 (f^* - Mf^* + Re(f'^2 - ff'') - \alpha(2f^* f^* - f'^2 - ff''))^2 d\eta \tag{23}$$

TABLE III. RESULTS OF LOCAL NUSSELT NUMBER Nu WITH $\lambda = 1, M = 6, \beta = 1$.

Re	α	Pr	K	Ec	Nu
5	0.6	0.6	0.3	0.3	0.649855
10					0.444432
15					0.303327
20					0.197048
Re	α	Pr	K	Ec	Nu
10	0.0	0.3	10	0.3	1.622670
	0.4				0.567186
	0.8				0.219214
	1.0				0.096742

TABLE IV. RESULTS OF LOCAL NUSSELT NUMBER Nu WITH $Pr = Ec = 0.3, K = 10, \beta = 1$.

λ	M	Re	α	Nu
1	6	10	0.6	0.354587
1.5				0.526216
2				0.602612
3				0.673742
λ	M	Re	α	Nu
1	0	10	0.6	0.011393
	2			0.025382
	4			0.165122
	6			0.317860

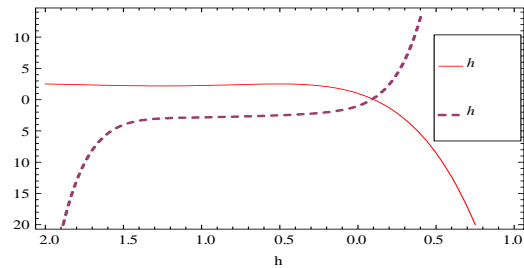


FIGURE II. h -CURVE OF $\theta'(0)$ AND $\phi'(0)$

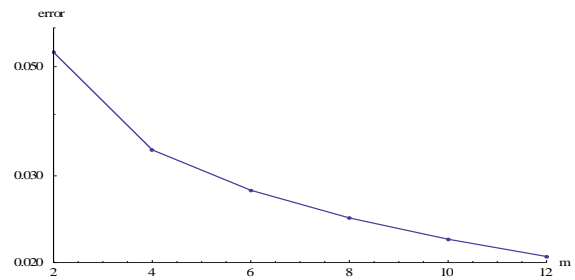


FIGURE III. THE LOGARITHMS OF THE RESIDUAL ERRORS $M = Re = Ec = Pr = K = 1$

Fig.3 shows that as the order of the HAM approximation becomes higher, the residual error becomes smaller.

Table.2 shows the magnitude of the wall velocity gradient $|f''(1)|$ for different values of Re , α , M and λ . it is observed that as the velocity-slip parameter λ and magnetic parameter M increase, the value of $|f''(1)|$ becomes lower. That is to say the slip coefficient tends to decrease wall shear stress. It also can be seen that the combined effect of increasing the values of α and Re is to increase wall shear stress.

Table.3-4 are made to analyze the effects of Re , α , M , λ , Nu , Ec , Pr , K on the temperature gradient on the wall $|\theta'(1)|$. The increase of λ and M lead to the increase of $|\theta'(1)|$.

That is to say the local Nusselt number $|\theta'(1)|$ increases when λ and M increases. It also is interesting to note that

decreasing α and Re tend to have high heat transfer rate. To reach a high heat transfer rate more slip and magnetic field by a liquid with a lower α and Re number is needed.

On observing the data in Table.4, it is observed that λ, α, M and Sc have little effect on Sherwood number $|\phi'(1)|$. The increase of λ and M lead to the increase of $|\phi'(1)|$. The Schmidt number Sc tends to increase the Sherwood number by increasing concentration gradient on the wall $|\phi'(1)|$. The reason for this trend is that the concentration boundary layer becomes thin for large Sc number. On the other hand α tends to decrease it.

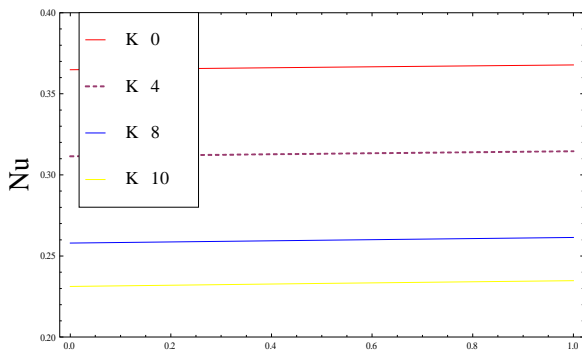


FIGURE IV. $|\theta'(1)|$ WHEN $\lambda = 1, \alpha = 0.6, Re = 10, M = 6, Pr = 0.6, Ec = 0.3$

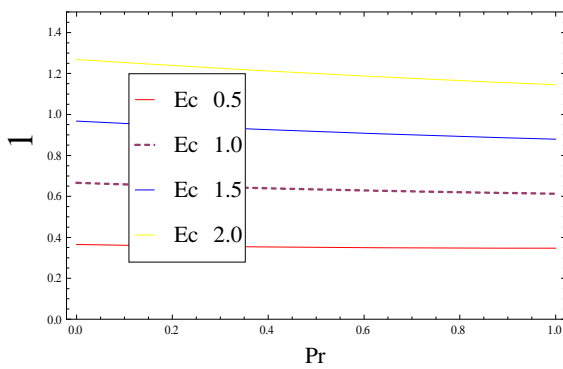


FIGURE V. $|\theta'(1)|$ WHEN $\lambda = 1, \alpha = 0.6, Re = 10, M = 6, K = 10, \beta = 1$

Fig.4 shows the influence of temperature-jump parameter β and Suction/injection K on the temperature gradient on the wall $|\theta'(1)|$. It is indicated that increasing β tends to have high heat transfer rate but K decreases heat transfer rate. As a result, Nusselt number decreases by K parameter.

The effect of Prandtl number parameter Pr on the Nusselt number Nu is shown by the Fig.5. It comes out that

the Nusselt number Nu becomes lower with the increase of the Prandtl number parameter Pr . This effect becomes little pronounced as Nu increases. It is not surprising as one knows that Nu is independent of Pr for no-slip laminar flow. Fig.5 also gives us the influence of the Eckert number Ec on the Nusselt number Nu . From this graph, we find that the Nusselt number Nu increases as the value of the Eckert number Ec increase.

From Fig.6, it is observed that as the temperature-jump parameter β increases, the value of $\theta(\eta)$ becomes lower. That is to say an increase in temperature-jump increases the thermal boundary layer thickness. Fig.7 shows variation of concentration in response to the change of Schmidt number parameter Sc . With the increase of Sc , the concentration graph increases. This matches the well-known fact that when Schmidt number becomes larger, mass transfer rate becomes faster.

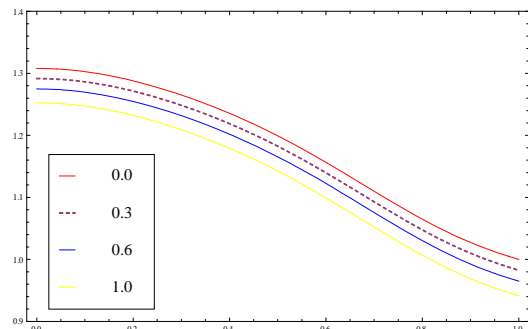


FIGURE VI. $\theta(\eta)$ WITH β WHEN $\lambda = 1, \alpha = 0.6, Re = 10, M = 6, K = 10, Pr = 0.6, Ec = 0.3$

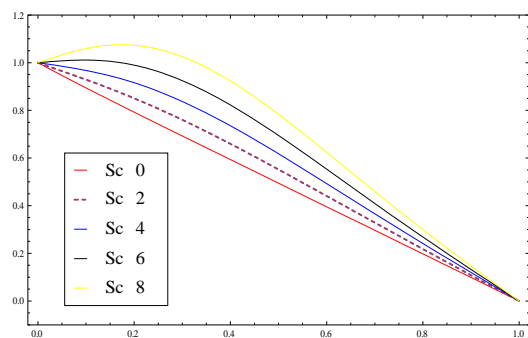


FIGURE VII. $\phi(\eta)$ WITH Sc WHEN $M = 6, Re = 10, \lambda = 1, \alpha = 0.6$

V. CONCLUSIONS

Slip flow and temperature jump boundary conditions are applied to solve the momentum and energy equations in a channel for the MHD second grade fluid. The nonlinear problems of velocity, energy and mass equation are solved by

HAM. The effects of various parameters such as the magnetic parameter M , the Reynolds number Re , the velocity-slip parameter λ , the second-grade fluid parameter α , the Eckert number Ec , the Prandtl number Pr , the Schmidt number Sc and the temperature-jump parameter α on velocity and thermal fields in the micro channel are studied through graphs and tables in detail. Some of the findings drawn from the investigation are listed as follows:

i) The skin friction coefficient C_f increases by increasing Re and α but decreases with an increase of M and λ ;

ii) The Nusselt number Nu is an increasing function of M, λ, α, Ec and an decreasing function of Re, α, Pr, K and Ec .

REFERENCES

- [1] M. Mooneym, (2013). Heat and mass transfer of MHD second order slip flow. *Computer & Fluids*, 71, p. 426–434.
- [2] M. Turkyilmazoglu, (2011). Analytic heat and mass transfer of the mixed hydrodynamic thermal slip MHD viscous flow over a stretching sheet. *International Journal of Mechanical Sciences*, 53, p. 886–896.
- [3] G. Singh, A.J. Chamkha, (2013). Dual solutions for second-order slip flow and heat transfer on a vertical permeable shrinking sheet. *Ain Shams Engineering Journal*.
- [4] T.G. Fang, S.S Yao, (2010). Viscous flow over a shrinking sheet with a second order slip flow model. *Commun Nonlinear Sci Numer Simulat*, 15, p.1831–1842.
- [5] B. Sahoo, (2010). Effects of slip, viscous dissipation and Joule heating on the MHD flow and heat transfer of a second-grade fluid past a radially stretching sheet. *Applied Mathematics and Mechanics*, vol.31, p. 159–173. DOI 10.1007/s10483-010-0204-7.
- [6] C. Ye and X. N. Luo, (2012). High-order numerical methods of fractional-order Stokes' problem for heated generalized second grade fluid. *Applied Mathematics and Mechanics*, vol.33, p. 65–80. DOI10.1007/s10483-012-1534-8.
- [7] I.G. Baoku, B.I. Olajuwon, A.O. Mustapha, (2013). Heat and mass transfer on a MHD third grade fluid with partial slip flow past an infinite vertical insulated porous plate in a porous medium. *International Journal of Heat and Fluid Flow*, 40, p. 81–88.
- [8] T. Hayat, M. Awais, (2012). Three-dimensional rotating flow between two porous walls with slip and heat transfer. *International Communications in Heat and Mass Transfer*, 39, p.551–555.
- [9] A.V. Rosca, L. Pop, (2013). Flow and heat transfer over a vertical permeable stretching/shrinking sheet with a second order slip. *International Journal of Heat and Mass Transfer*, 60, p.355-364.
- [10] M. Mahantesh and M. Vajravelu, (2012). Second order slip flow and heat transfer over a stretching sheet with non-linear Navier boundary condition. *International Journal of Thermal Sciences*, 58, p.143–150.
- [11] 11.B. Sahoo, S. Poncet, (2011). Flow and heat transfer of a third-grade fluid past an exponentially stretching sheet with partial slip boundary condition. *International Journal of Heat and Mass Transfer*, 54, p.5010–5019.
- [12] S. SRINIVAS, P. B. A. REDDY and B. S. R. V. PRASAD, (2014). Effects of chemical reaction and thermal radiation on MHD flow over an inclined permeable stretching surface with nonuniform heat source/sink: an application to the dynamics of blood flow. *Journal of Mechanics in Medicine and Biology*, vol.14, p.1450067- 1450091. DOI: 10.1142/S0219519414500675.
- [13] B. I. Olajuwon, (2011). Convection heat and mass transfer in a hydromagnetic flow of a second-grade fluid in the presence of thermal radiation and thermal diffusion. *International communications in Heat and Mass Transfer* 38, p.377–382.
- [14] S.J. Liao, (1992). The Proposed Homophony Analysis Technique for the Solution of Nonlinear Problems. Ph.D. Thesis, Shanghai Jiao Tong University.
- [15] K. Yakushima, M. Yamashita, K. Stub, (2007). An analytic solution of projectile motion with the quadratic resistance law using the homotopy analysis method. *J. Phys.A: Mathematical and Theoretical*, 40, p.8403-8416.
- [16] V. Marinca, N. Herisanu, (2008). Application of optional homotopy asymptotic method for solving nonlinear equations arising in heat transfer. *International communications in Heat and Mass Transfer*, 35, p.710-715.
- [17] M.M. Zhao, (2009). The Further Discussion for Homotopy Analysis Method and Their Modification. Ph.D. Thesis, Lan Zhou University.
- [18] Z. Niu, (2010). A one-step optional homotopy analysis method for nonlinear differential equations. *Commun Nonlinear Sci Numer Simulat*, 15, p.2026-2036.
- [19] S. Srinivas, (2014). A. Vijayalakshmi, T. R. Ramamohan, A. S. Reddy, Hydromagnetic flow of a nanofluid in a porous channel with expanding or contracting walls. *Journal of Porous Media*, 17 (11), p.953–967.