

# A Comprehensive Evaluating System for Water Situation

Wenfa Yuan

North China Electric Power University, Baoding 071000

812781581@qq.com

**Keywords:** Grey model; Water situation; Evaluating System

**Abstract.** We tried to measure and forecast the water situation of a water-deficient region and give plans to change it. In order to achieve this, first, we defined M (Water Meeting Ability) which contains 6 factors (in next paragraph). Second we used two models (grey forecast GM (1, 1) model and linear regression model) to forecast all the factors, chose the one whose error is less individually and get the M. Finally we designed nine programmes based on M to influence these factors individually, then forecast the M and give the result.

## Introduction

The problem of water shortages for a long time is the focus of attention. Fresh water on earth is limited. The space and time distribution is very uneven, which causes the water shortages in some areas. However, with the deterioration of climate, increase of population growth and pollution makes water shortage more serious. Water use has been growing at twice the rate of population over the last century. According to scientists, physical scarcity and economic scarcity are two main factors causing the shortage of water resources. Physical scarcity is where there is inadequate water in a region to meet demand. Economic scarcity is where water exists but poor management and lack of infrastructure limits the availability of clean water.

Modern technology continues to advance, have a batch of new technologies such as desalination system of water and air system, water, etc. To solve the shortage of water resources, we should consider controlling demand, optimizing configuration of water resources in some area and improving the utilization efficiency of water resources. Also we should focus on the potential for new or alternate sources of water.

## The Model of Water Meeting Ability

The model should reflect available water resources and the water demand, so we defined the water meeting ability M. Definition type as follows.

$$M = \frac{\eta W (1 - \gamma \frac{P - P_0}{W}) - C}{C} \quad (1)$$

$W$  means total amount of water resources.  $C$  means total amount of water use.  $\eta$  means the degree of collecting water resource.  $P$  means sewage emissions.  $P_0$  means sewage treatment.  $\gamma$  means  $\gamma(P - P_0)$  available water cannot be used because of the effect of waste water. Among these factors,  $C = C_1 + C_2 + C_3 + C_4$ , total amount of water use can be divided into domestic water, agricultural water, industrial water and ecological water.

In specific areas, we can use the total water resources  $W$ , agricultural water  $C_2$ , industrial water  $C_3$ , ecological water  $C_4$  and sewage emissions  $P$  a few years ago to predict  $W$ ,  $C_2$ ,  $C_3$ ,  $C_4$  and  $P$  a few years later. We can use per capita domestic water consumption and population to predict  $C_1$ . Assuming  $C_1 = \lambda R$  ( $\lambda$  means per capita life water consumption), we can find the change is not obvious, so we take an average according to the data (in appendix).  $R$  means Population in

Shandong province can be predicted by  $R = R_0(1+r)^t$ .  $R_0$  is the population in Shandong province in 2004.  $r$  is the population growth rate. We also take an average according to the data.  $\eta$  has connection with the technology about collection and utilization of water resources in local areas.  $\gamma$  is a constant.

Using gray prediction GM (1, 1) model and linear regression model to predict the values respectively, getting relationship to compare with the source data and calculate the relative error  $\varepsilon$ . Compare  $\varepsilon$  of two models and get the minimum value.

Define  $\left\{ \begin{array}{l} M < 0, \text{ need help} \\ 0 \leq M < 0.5, \text{ poor} \\ 0.5 \leq M < 1, \text{ medium} \\ M \geq 1, \text{ good} \end{array} \right.$  as index. 'M < 0' means water supply that need to be provided.

### The Analysis of Water Scarcity in the Region

**Why Water is Scarce in Shandong Province**. From the data we know that there are two years, when water consumption is greater than the total water resources. Average of annual water consumption is 21.927 billion cubic meters while average of the total water resources is 30.333 billion cubic meters.

(Unit:  $10^8$  cubic meters)

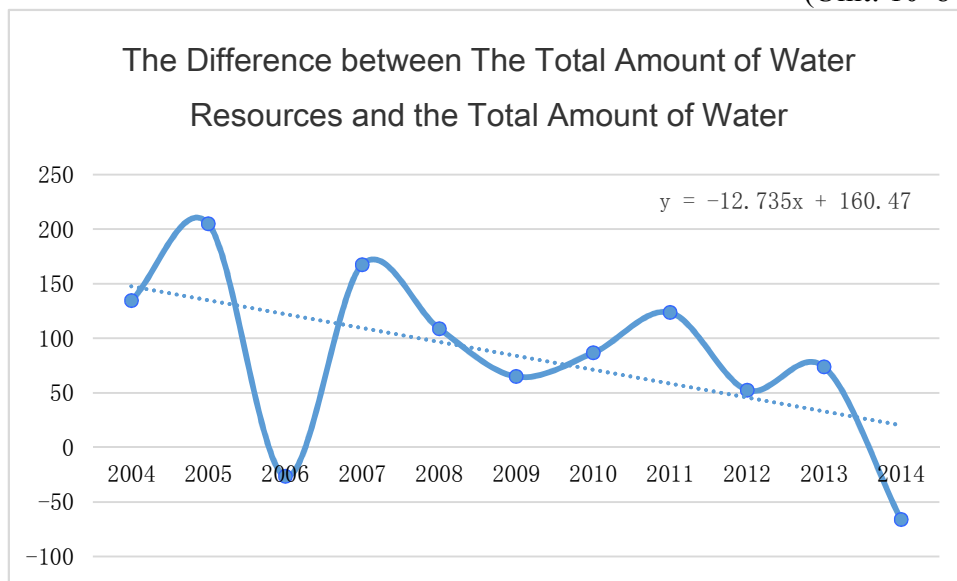


Figure 1. The Difference

From the chart, we can see the trend of water resources in this area is falling, but the trend of total water is rising. The trend of their differences is failing. Annual surface runoff is about more than 80% concentrated in July and August, and water resources in the province is concentrated distribution, so Shandong perennial experiences the difficulty of winter and spring irrigation agriculture, it decreases the degree of collecting water resource. According to resent report, surface water has been polluted frequently.

Due to seawater desalination and other forms of water supply engineering limited by the cost and the technology, they are difficult to large-scale development, water supply of them accounting for only about 2% of the total water supply.

**How Water is Scarce in Shandong Province.** According to historical data we find M from 2004 to 2014:

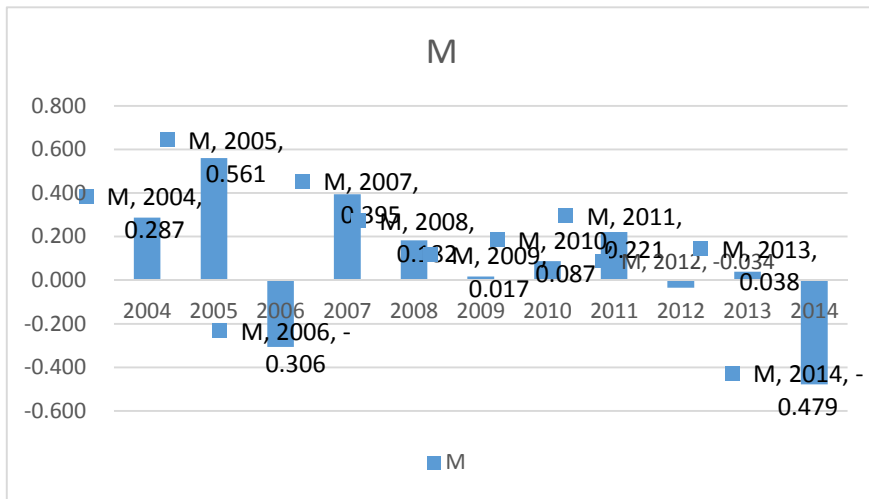


Figure 2. The M

According to sheet we can find that Shandong Province's average of M is 0.088, only one year to be medium region and many years to be poor or need help region.

**The Forecast and Analysis of What the Water Situation will be in Fifteen Years in Shandong Province**

**Grey Forecasting GM (1, 1) Model.** Use GM (1, 1) model and linear regression model to calculate results and the corresponding errors.

**The Establishment of the Model.** Use grey prediction which is based on GM (1, 1) model and grey differential equation. It can forecast the development trend while the amount of data and the relevant information are little.

Let the original sequence  $x^{(0)} = (x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(n))$  add to sum and get initial value generating sequence:

$$x^{(1)}(k) = \sum_{m=1}^k x^{(0)}(m) \tag{2}$$

$$z^{(1)}(k) = 0.5(x^{(1)}(k) + x^{(1)}(k-1)), k = 2, 3, \dots, n \tag{3}$$

Establish a single variable  $x^{(1)}$  first-order differential equation GM (1, 1) model

$$\frac{dx^{(1)}}{dt} + ax^{(1)} = b \tag{4}$$

'a' and 'b' are coefficients to be indentified. Discretize (2) and use  $x^{(0)}(m)$  in place of the value

while  $\frac{dx^{(1)}}{dt}$  is at 't=m'.

Use  $z^{(1)}(k) = 0.5(x^{(1)}(k) + x^{(1)}(k-1))$  in place of the background value of  $\frac{dx^{(1)}}{dt}$  and get (3).

$$x^{(0)}(k) + az^{(1)}(k) = b, (k = 2, 3, \dots, n) \tag{5}$$

$x^{(0)}(k)$  : Grey derivative.

$a$  : The development coefficient.

$z^{(1)}(k)$  : Vernacular background value.

$b$  : Ash effect degree.

$$\begin{cases} C = \sum_{k=2}^n z^{(1)}(k) \\ D = \sum_{k=2}^n x^{(0)}(k) \\ E = \sum_{k=2}^n z^{(1)}(k)x^{(0)}(k) \\ F = \sum_{k=2}^n (z^{(1)}(k))^2 \end{cases} \quad (6)$$

Use the least square method to get identification coefficient from (3).

$$\begin{cases} a = \frac{CD - (n-1)E}{(n-1)F - C^2} \\ b = \frac{DF - CE}{(n-1)F - C^2} \end{cases} \quad (7)$$

Put 'a' and 'b' into (3) and solve the (3) in the initial conditions. Get the predictive value  $\tilde{x}^{(1)}(k+1)$  of the GM (1, 1) bleaching response function  $x^{(1)}(k+1)$ :

$$\tilde{x}^{(1)}(k+1) = [x^{(0)}(1) - \frac{b}{a}]e^{-ak} + \frac{b}{a} \quad (8)$$

Do a b-b to  $\tilde{x}^{(1)}$  and get the predictive value  $\tilde{x}^{(0)}(k+1)$ :

$$\tilde{x}^{(0)}(k+1) = \tilde{x}^{(1)}(k+1) - \tilde{x}^{(1)}(k) \quad (9)$$

**Solution of the Model.** Use Grey forecasting mode to predict data after 15 years, according to the total water resources W, the total water P and pollution emissions P (in the following table)

### Unary Linear Regression Model

**The Establishment of Model.** Define two variables x and y, the correlation between x and y is :

$$Y = a + bx + \theta, E(\theta) = 0, D(\theta) = \sigma^2 \quad (10)$$

a, b is two unknown parameters, called regression coefficient; 'x' is regression variables;  $\theta$  is a random variable whose mean is zero.

Under the condition of  $x = x_1, x_2, \dots, x_n$ , we conduct experiment on y and get  $(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$ . Setting them as a point, the image drawn on the rectangular coordinate system is called the obtained image scatter plot.

According to the distribution of these points, we get empirical formula:  $\hat{Y} = \hat{a} + \hat{b}x$ ,

$\hat{a}$  is regression constant,  $\hat{b}$  is regression coefficient

$$y_i = a + bx_i + \theta_i, (i = 1, 2, \dots, n) \quad (11)$$

$\theta_i$  means error term. The error sum of squares is:

$$\sum_{i=1}^n \theta_i^2 = \sum_{i=1}^n [y_i - (a + bx_i)]^2 \quad (12)$$

Set it as  $Q(a, b)$ . The problem finding a straight line which is the closest observation point becomes to find  $\hat{a}$  and  $\hat{b}$  who can make  $Q(a, b)$  reach the minimum.

Use the least square method to solve:

$$\begin{cases} \frac{\delta Q}{\delta a} = -2 \sum_{i=1}^n [y_i - (a + bx_i)] = 0 \\ \frac{\delta Q}{\delta b} = -2 \sum_{i=1}^n [y_i - (a + bx_i)]x_i = 0 \end{cases} \quad (13)$$

$$\text{Get } \hat{a} = \bar{y} - \hat{b}\bar{x}, \quad \hat{b} = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^n (x_i - \bar{x})^2}.$$

**The Solution of the Model.** We predicted the data in the future 15 years by using unary linear regression model.

Get regression constant and regression coefficient:

Table 1 a and b

	W	$C_2$	$C_3$	$C_4$	P	$P_0$
a	378.3515	162.5995	22.9587	1.2798	22.9762	18.0858
b	-12.5042	-1.2199	0.5431	0.5111	2.6738	1.6260

### Choose Model and Calculate the Related Expression

According to  $\varepsilon^{(avg)}$ , total water resources W, ecological water  $C_4$ , waste water emissions P and processing  $P_0$ .

Let  $t=0$  in 2004.

we get :

$$W(t) = 378.35 - 12.50t \quad (14)$$

$$C_1(t) = 32.31 \times 1.005^t \quad (15)$$

$$C_2(t) = -15042(e^{-0.011t} - e^{-0.011(t-1)}) \quad (16)$$

$$C_3(t) = 669.7(e^{0.033t} - e^{0.033(t-1)}) \quad (17)$$

$$C_4(t) = 1.28 + 0.51t \quad (18)$$

$$P(t) = 22.98 + 2.67t \quad (19)$$

$$P_0(t) = 18.09 + 1.63t \quad (20)$$

### Calculate Water Meeting Ability

Assume  $\eta = 80\%$ ,  $\gamma = 0.5$ .

The per capita domestic water consumption is 35.2 cubic meters and the natural population growth rate is 5‰ according the data in Shandong.

After calculating, we get:

**Table 2 result**

year	Total water resources $W$	Life water consumption $C_1$	Agricultural water $C_2$	Industrial water $C_3$	Ecological water $C_4$	Emissions $P$	Pressing $P_0$	Water meeting ability $M$
2015	228.3	34.14	146.30	31.01	7.41	55.06	37.6	-0.197
2016	215.8	34.31	144.72	32.05	7.92	57.74	39.22	-0.246
2017	203.29	34.48	143.16	33.12	8.44	60.41	40.85	-0.294
2018	190.79	34.65	141.62	34.22	8.95	63.08	42.48	-0.342
2019	178.28	34.82	140.09	35.37	9.46	65.76	44.1	-0.390
2020	165.78	35.00	138.58	36.55	9.97	68.43	45.73	-0.439
2021	153.28	35.17	137.08	37.77	10.48	71.1	47.35	-0.487
2022	140.77	35.35	135.60	39.03	10.99	73.78	48.98	-0.535
2023	128.27	35.53	134.14	40.33	11.5	76.45	50.61	-0.583
2024	115.76	35.70	132.69	41.68	12.01	79.13	52.23	-0.631
2025	103.26	35.88	131.26	43.07	12.52	81.8	53.86	-0.679
2026	90.76	36.06	129.84	44.51	13.03	84.47	55.48	-0.727
2027	78.25	36.24	128.44	45.99	13.55	87.15	57.11	-0.774
2028	65.75	36.42	127.05	47.53	14.06	89.82	58.74	-0.822
2029	53.24	36.60	125.68	49.11	14.57	92.5	60.36	-0.868

### Analysis of the Results

Judging from the final results ,  $M$  (water meeting ability) will be in a state of growth, which is obvious in the next 35 years. After 35 years, the growth becomes slow. In the next 5 years, water meeting ability will be greater than zero. Finally Shandong Province will become less susceptible to water scarcity.

### Reference

- [1] Fengxin Kang, Junxiang Xu, Zhongxiang Zhang, Potential Evaluation of Groundwater Resources in Shandong Province. *The Land and Resources in Shandong*, vol. 26, no. 8 August 2010.
- [2] Dejuan Jiang, Huixiao Wang, Huijuan Li, The Classification and Calculation Method of Ecological Environment Water Demand Review. *Progress of Geographical Science*. (4), 22 July 2003.
- [3] Jun Lin, Bolin Han. *Mathematical Modeling Tutorial*. Beijing Science Press.
- [4] Qiyuan Jiang. Jinxing Xie, Jun Ye. *Mathematical Model (The Third Edition)*. Higher Education Press.
- [5] LiMei, Lipei Cheng, YuXiao Jing, Investigation on the Collection Pattern and Treatment Technology of Urban Rainfall. *Journal of Shandong JianZhu University*, 2007, 12