

Decentralized Channel Decisions of Green Supply Chain in a Fuzzy Decision Making Environment

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Abstract

This paper considers the greening policies in a decentralized channel between one manufacturer and one retailer in a fuzzy decision making environment. We consider the manufacturing cost and the parameters of demand function as the fuzzy variables. Based on the different market structures, we develop three different fuzzy decentralized decision models. For each case, the expected value, optimistic value and pessimistic value models are formulated, and their optimal solutions are also derived through the fuzzy set theory. Finally, three numerical examples are solved to examine the effectiveness of fuzzy models. The effects of the confidence level of the supply chain member's profits and the fuzziness of parameters on optimal prices, level of green innovation, and fuzzy expected profits of actors are also analyzed.

Keywords: Green supply chain, Game theory, Fuzzy theory, Fuzzy variables

1. Introduction

In a traditional supply chain, chain members usually focus on their total costs and profits, and ignore their operations impacted on environment. This phenomenon is altering rapidly as the problems of environment are affecting the population's living conditions of the world more severely. With the development of the green economy and low-carbon economy, more and more scholars and market administrators have applied the green principles and techniques to develop and solve the green supply chain (GSC) management problems.

Recently, some studies have been done on analyzing the game theoretic models in GSC management. Sheu¹ considered the nuclear power generation problem by using a multi-objective optimization programming approach in GSC management. Using the method of

asymmetrical Nash bargaining game and backward induction, Sheu² analyzed the negotiations problem with government intervention between the GSC members. Ghosh and Shah³ developed a game theoretic model for analyzing the influence of channel structures on greening levels, prices and profits in both cooperation and non-cooperation situations. Ghosh and Shan⁴ also discussed the coordination issues of GMC by using the cost sharing contract. By using evolutionary dynamics, Barari *et al.*⁵ framed integrated and holistic conceptual framework on maximizing the total supply chain's profits that took the practical aspects into account in a GSC. Swami and Shah⁶ established a two-part tariff contract for coordinating GSC, where the models contained price and non-price variables. Similar issues was studied by Swami and Shah⁷, they studied the coordination mechanism of GSC that took the shelf-space allocation of products into

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consideration. Mirzapour Al-e-hashem⁸ proposed a stochastic programming model to investigate the production–distribution planning system after demand fluctuation in a GSC. Tomasin *et al.*⁹ used multiple case studies to investigate the elements that contribute to increase sales of green products in Brazil. Zhang and Liu¹⁰ proposed three types of coordination mechanisms to coordinate the three stage green supply chain.

The works mentioned above discussed the game theoretic models of green supply chain with deterministic market demands and production costs. However, in real world, there exist many in deterministic factors for some new green products. The production costs and market demands of these green products are often lack of historical data. Moreover, with the market uncertainty, the exact data are derived more and more unavailable due to the short of product life cycles. Thus, the fuzzy set theory, instead of probability theory is fit well for dealing with greening policy problem. In recent years, fuzzy sets theory is used by many researches to deal with the modes of supply chain. For instance, Xu and Zhai^{11,12} presented fuzzy newsboy models in which the market demands were fuzzy. Zhou *et al.*¹³ investigated the operations between one-manufacturer and one-retailer, where the customer's demand was fuzziness. Yu and Jin¹⁴ studied the decentralized channel, integrated channel and return contract models with fuzzy retail prices and demands. Wei and Zhao¹⁵ proposed fuzzy models with pricing competition of two retailers in a closed-loop supply chain. Ye and Li¹⁶ investigated the Stackelberg models in which the demand was fuzzy. In addition, Zhao *et al.*¹⁷ studied the pricing competitive strategies in which one manufacturer sold his substitutable products to two competing retailers with fuzzy costs and demands. Zhao *et al.*¹⁸ also studied pricing competition problems with two manufacturers under fuzzy demand environments. Wei and Zhao¹⁹ discussed the problems of pricing decisions in a reverse channel. Zhao and Wang²⁰ studied service and price competition problem with two competitive retailers in a distribution system with fuzzy demand. Yu *et al.*²¹ developed fuzzy newsboy models to obtain the optimal prices between the supplier and the retailer. Recently, Sang²² investigated the coordination mechanism in a multiple supply chain with fuzzy demands and costs. Chang and Yeh²³ analyzed the decentralized and the centralized supply chain system with fuzzy demand under a return policy, and showed that the fuzziness of demand affected the optimal results

of the supply chain members. Khamseh *et al.*²⁴ proposed four different pricing models of complementary products with two competing manufacturers in fuzzy environments. Sang²⁵ proposed one expected value model and two chance-constrained programming models between two competing manufacturers and one common retailer under fuzzy uncertainty.

To the best of our knowledge, there is no work that deals with the optimal decisions of GSC in a fuzzy decision environment. Therefore, in this paper, we will examine how the retailer and the manufacturer should make their own pricing and green level decisions in a fuzzy decision environment. We also explore the impacts of the confidence level and the fuzzy degree of parameters on the equilibrium prices, green level and profits of the supply chain members.

The paper makes three contributions to the extant literature. Firstly, our proposed models extend the study of Ghosh and Shah³ by considering the greening policies under fuzzy uncertainty. The manufacturing costs and market demand are all fuzzy. Secondly, we apply the different ranking measures of fuzzy variables to reflect the attitudes of the participants. They can choose the expected values model to derive their optimal decisions if they are risk neutral. They can choose the optimistic values model to derive their optimal decisions if they are risk preferable. And they can choose the pessimistic values model to derive their optimal decisions if they are risk averse. Thirdly, compared to the method used in conventional environment, our work has some main findings: the profit margin, wholesale price, retail price and level of green innovation are higher, while the manufacturer's profit is lower, and the retailer's profit and supply chain system's profit are higher under fuzzy uncertainty.

The rest of this paper is as follows. Firstly, some useful concepts and propositions about fuzzy set theory are presented in Section 2. Section 3 introduces the notations of the models. Three fuzzy green supply chain models are proposed in Section 4. In Section 5, three numerical examples are given to elucidate the solutions of each model. Section 6 summarizes the work.

2. Preliminaries

The possibility measure Pos was introduced by Nahmias²⁶ and three axioms were given:

Axiom 1. $\text{Pos}(\Theta) = 1$, for the nonempty set Θ .

Axiom 2. $\text{Pos}(\phi) = 0$, for the empty set ϕ .

Axiom 3. $\text{Pos}\left(\bigcup_{i=1}^m A_i\right) = \sup_{1 \leq i \leq m} \text{Pos}(A_i)$, for any events A_i in $P(\Theta)$, where P denotes the power set of Θ .

Definition 1(Liu and Liu²⁷). If $\text{Pos}\{\xi \leq 0\} = 0$, then the fuzzy variable ξ is called positive.

Definition 2 (Liu and Liu²⁷). Let ξ be a fuzzy variable. Then, for any $\alpha \in (0,1]$, $\xi_\alpha^R = \sup\{r \mid \text{Pos}\{\xi \geq r\} \geq \alpha\}$ and $\xi_\alpha^L = \inf\{r \mid \text{Pos}\{\xi \leq r\} \geq \alpha\}$ are said to be the α -optimistic value and the α -pessimistic value of ξ , respectively.

Example 1. Let ξ be a triangular fuzzy variable with $\xi = (a, b, c)$, then ξ_α^R and ξ_α^L are given as

$$\xi_\alpha^R = \alpha b + (1 - \alpha)c \text{ and } \xi_\alpha^L = \alpha b + (1 - \alpha)a.$$

Lemma 1 (Liu and Liu²⁷ and Zhao *et al.*²⁸). Let ξ and η be positive independent fuzzy variables. Then, for any $\alpha \in (0,1]$

- (a) $(\xi + \eta)_\alpha^R = \xi_\alpha^R + \eta_\alpha^R$ and $(\xi + \eta)_\alpha^L = \xi_\alpha^L + \eta_\alpha^L$;
- (b) $(\xi \cdot \eta)_\alpha^R = \xi_\alpha^R \cdot \eta_\alpha^R$ and $(\xi \cdot \eta)_\alpha^L = \xi_\alpha^L \cdot \eta_\alpha^L$;
- (c) $(\xi - \eta)_\alpha^R = \xi_\alpha^R - \eta_\alpha^L$ and $(\xi - \eta)_\alpha^L = \xi_\alpha^L - \eta_\alpha^R$.

Lemma 2. (Liu and Liu²⁷). Let ξ be a fuzzy variable with finite expected value, then

$$E[\xi] = \frac{1}{2} \int_0^1 (\xi_\alpha^L + \xi_\alpha^R) d\alpha.$$

Lemma 3 (Liu and Liu²⁷). Let ξ and η be independent fuzzy variables with finite expected values. Then for any numbers m and n

$$E[m\xi + n\eta] = mE[\xi] + nE[\eta].$$

Definition 2. Let ξ and η be positive independent fuzzy variables, $\xi > \eta$ if and only if for any $\alpha \in (0,1]$, $\xi_\alpha^L > \eta_\alpha^L$ and $\xi_\alpha^R > \eta_\alpha^R$.

Definition 3. Let ξ and η be positive independent fuzzy variables, if $\xi > \eta$, then $E[\xi] > E[\eta]$.

3. Problem Descriptions

Consider a GSC with a manufacturer and a retailer, where the retailer orders greening products from the manufacturer, and then he retails them to end customers. The manufacturer is assumed to produce only one product and the retailer sells only single product.

Similar to Ghosh and Shah³, the market demand function faced by the manufacturer and the retailer is considered as a linear form of the retail price p and the level of greening innovation θ , the market demand is

$$\tilde{q} = \tilde{D} - \tilde{\beta}p + \tilde{\gamma}\theta. \tag{1}$$

where the fuzzy parameter \tilde{D} denotes the market potential, the fuzzy parameter $\tilde{\beta}$ denotes the retail price sensitivity of the customer, and the fuzzy parameter $\tilde{\gamma}$ denotes the greening innovation sensitivity of the manufacturer to the demand. The fuzzy parameters \tilde{D} , $\tilde{\beta}$ and $\tilde{\gamma}$ are positive and mutually independent.

Further, let w denote the unit wholesale price of greening product, \tilde{c} the unit fuzzy producing cost of greening product for the manufacturer and m the unit margin profit of product for the retailer. As the retail price p is the sum of margin profit m and wholesale price w , we consider retail price as $p = m + w$. The fuzzy demand function of the greening product is presented as follows

$$\tilde{q} = \tilde{D} - \tilde{\beta}(m + w) + \tilde{\gamma}\theta. \tag{2}$$

It is assumed that the marginal cost of the manufacturer is not affected by the greening innovation. In addition, for achieving greening innovation, it needs fixed investment. The cost of the fixed investment is assumed as a quadratic function of the parameter θ , and is expressed as $\tilde{I}\theta^2$, where \tilde{I} denotes fuzzy fixed investment coefficient. The demand is positive in the real world. Thus we have $\text{Pos}\{\tilde{D} - \tilde{\beta}(m + w) + \tilde{\gamma}\theta < 0\} = 0$, where Pos is a possibility measure. The order quantity of the greening product can be presented as $q = E[\tilde{q}]$.

The fuzzy profit functions of the manufacturer and the retailer can be derived as

$$\Pi_M = (w - \tilde{c})(\tilde{D} - \tilde{\beta}(m + w) + \tilde{\gamma}\theta) - \tilde{I}\theta^2. \tag{3}$$

$$\Pi_R = m(\tilde{D} - \tilde{\beta}(m + w) + \tilde{\gamma}\theta). \tag{4}$$

4. Model Analysis

In this section, we will discuss the operations of the supply chain participants in a GSC, and examine the manufacturer and the retailer how to set their optimal policies with different power structures under fuzzy uncertainty. We study the equilibrium decisions under three non-cooperative games in the decentralized channel: the manufacturer dominates the channel (Manufacturer-Stackelberg game), the retailer leads the

channel (Retailer-Stackelberg game), and the supply chain participants have an equal bargaining power (Vertical-Nash game).

4.1. Manufacturer Stackelberg (MS) game

In the MS game, the retailer has less bargaining power than the manufacturer. That is to say, the manufacturer is the leader in the GSC. Firstly, the manufacturer sets his wholesale price w and level of greening innovation θ condition on the retailer's optimal reaction to his decisions. Then, the retailer sets his own margin profit m . Hence, in this case, we can formulate the fuzzy optimal model as follows

$$\left\{ \begin{array}{l} \max_{w, \theta} E[\Pi_M(w, \theta, m^*)] = E[(w - \tilde{c})(\tilde{D} - \tilde{\beta}(m + w) + \tilde{\gamma}\theta) - \tilde{I}\theta^2] \\ \text{s. t.} \\ \text{Pos}\{w - \tilde{c} < 0\} = 0 \\ \text{where } m^* \text{ derives from problem :} \\ \left\{ \begin{array}{l} \max_m E[\Pi_R(m)] = E[m(\tilde{D} - \tilde{\beta}(m + w) + \tilde{\gamma}\theta)] \\ \text{s. t.} \\ \text{Pos}\{\tilde{D} - \tilde{\beta}(m + w) + \tilde{\gamma}\theta < 0\} = 0 \end{array} \right. \end{array} \right. \quad (5)$$

where $E[\Pi_M(w, \theta, m^*)]$ and $E[\Pi_R(m)]$ represent the fuzzy expected profit of the manufacturer and the retailer.

We first solve the reaction function of the retailer.

Theorem 1. If $\text{Pos}\{\tilde{D} - \tilde{\beta}(m^* + w) + \tilde{\gamma}\theta < 0\} = 0$, then the retailer's optimal reaction function is given as

$$m^*(w, \theta) = \frac{E[\tilde{D}] - E[\tilde{\beta}]w + E[\tilde{\gamma}]\theta}{2E[\tilde{\beta}]} \quad (6)$$

Proof. The fuzzy expected profit $E[\Pi_R(m)]$ is

$$E[\Pi_R(m)] = -E[\tilde{\beta}]m^2 + (E[\tilde{D}] - E[\tilde{\beta}]w + E[\tilde{\gamma}]\theta)m \quad (7)$$

The first order condition

$$\frac{d}{dm} E[\Pi_R(m)] = -2E[\tilde{\beta}]m + E[\tilde{D}] - E[\tilde{\beta}]w + E[\tilde{\gamma}]\theta$$

The second order condition

$$\frac{d^2}{dm^2} E[\Pi_R(m)] = -2E[\tilde{\beta}] < 0$$

Thus, the retailer's fuzzy expected profit $E[\Pi_R(m)]$ is concave in m .

Let the first order condition be zero, the result (6) can be derived.

Theorem 1 is proved. \square

Here, we consider $m^*(w, \theta)$ as the optimal response to the decisions of the manufacturer. Having the follower's information about the decision, the leader sets his optimal decisions. So, the following Proposition can be derived.

Theorem 2. If $(E[\tilde{\gamma}])^2 < 8E[\tilde{\beta}]E[\tilde{I}]$, $\text{Pos}\{w^* - \tilde{c} < 0\} = 0$ and $\text{Pos}\{\tilde{D} - \tilde{\beta}(m^* + w^*) + \tilde{\gamma}\theta^* < 0\} = 0$, then the optimal equilibrium decisions are

$$m^* = \frac{E[\tilde{\gamma}]\left(2E[\tilde{\beta}\tilde{c}]E[\tilde{\gamma}] - E[\tilde{\beta}]\int_0^1(\tilde{\gamma}_\alpha^L\tilde{c}_\alpha^R + \tilde{\gamma}_\alpha^R\tilde{c}_\alpha^L)d\alpha\right)}{2E[\tilde{\beta}]\left(8E[\tilde{\beta}]E[\tilde{I}] - (E[\tilde{\gamma}])^2\right)} + \frac{2E[\tilde{I}]\left(E[\tilde{D}] - E[\tilde{\beta}\tilde{c}]\right)}{8E[\tilde{\beta}]E[\tilde{I}] - (E[\tilde{\gamma}])^2} \quad (8)$$

$$w^* = \frac{E[\tilde{\gamma}]\left(E[\tilde{\beta}\tilde{c}]E[\tilde{\gamma}] - E[\tilde{\beta}]\int_0^1(\tilde{\gamma}_\alpha^L\tilde{c}_\alpha^R + \tilde{\gamma}_\alpha^R\tilde{c}_\alpha^L)d\alpha\right)}{E[\tilde{\beta}]\left(8E[\tilde{\beta}]E[\tilde{I}] - (E[\tilde{\gamma}])^2\right)} + \frac{4E[\tilde{I}]\left(E[\tilde{D}] + E[\tilde{\beta}\tilde{c}]\right)}{8E[\tilde{\beta}]E[\tilde{I}] - (E[\tilde{\gamma}])^2} \quad (9)$$

$$\theta^* = \frac{E[\tilde{\gamma}]\left(E[\tilde{D}] + 3E[\tilde{\beta}\tilde{c}]\right) - 2E[\tilde{\beta}]\int_0^1(\tilde{\gamma}_\alpha^L\tilde{c}_\alpha^R + \tilde{\gamma}_\alpha^R\tilde{c}_\alpha^L)d\alpha}{8E[\tilde{\beta}]E[\tilde{I}] - (E[\tilde{\gamma}])^2} \quad (10)$$

Proof. The fuzzy expected profit $E[\Pi_M]$ is

$$\begin{aligned} E[\Pi_M] &= \frac{1}{2} \int_0^1 \left((w - \tilde{c})(\tilde{D} - \tilde{\beta}(m + w) + \tilde{\gamma}\theta) - \tilde{I}\theta^2 \right)_\alpha^L d\alpha \\ &\quad + \frac{1}{2} \int_0^1 \left((w - \tilde{c})(\tilde{D} - \tilde{\beta}(m + w) + \tilde{\gamma}\theta) - \tilde{I}\theta^2 \right)_\alpha^R d\alpha \\ &= -E[\tilde{\beta}]w^2 - E[\tilde{I}]\theta^2 + E[\tilde{\gamma}]w\theta \\ &\quad + (E[\tilde{D}] + E[\tilde{\beta}\tilde{c}] - E[\tilde{\beta}]m)w \\ &\quad - \frac{1}{2}\theta \int_0^1 (\tilde{\gamma}_\alpha^L\tilde{c}_\alpha^R + \tilde{\gamma}_\alpha^R\tilde{c}_\alpha^L) d\alpha + E[\tilde{\beta}\tilde{c}]m \\ &\quad - \frac{1}{2} \int_0^1 (\tilde{D}_\alpha^L\tilde{c}_\alpha^R + \tilde{D}_\alpha^R\tilde{c}_\alpha^L) d\alpha \end{aligned} \quad (11)$$

Substituting m in Eq.(6) into Eq.(11), the first order conditions can be obtained as

$$\frac{\partial}{\partial w} E[\Pi_M] = -E[\tilde{\beta}]w + \frac{1}{2}E[\tilde{\gamma}]\theta + \frac{1}{2}(E[\tilde{D}] + E[\tilde{\beta}\tilde{c}]).$$

$$\frac{\partial}{\partial \theta} E[\Pi_M] = -2E[\tilde{I}]\theta + \frac{1}{2}E[\tilde{\gamma}]w + \frac{E[\tilde{\beta}\tilde{c}]E[\tilde{\gamma}]}{2E[\tilde{\beta}]} - \frac{1}{2} \int_0^1 (\tilde{\gamma}_\alpha^L \tilde{c}_\alpha^R + \tilde{\gamma}_\alpha^R \tilde{c}_\alpha^L) d\alpha.$$

Therefore, the Hessian matrix of $E[\Pi_M]$ is

$$H = \begin{bmatrix} -E[\tilde{\beta}] & \frac{1}{2}E[\tilde{\gamma}] \\ \frac{1}{2}E[\tilde{\gamma}] & -2E[\tilde{I}] \end{bmatrix}.$$

Since $E[\tilde{\beta}] > 0$, $E[\tilde{I}] > 0$ and $(E[\tilde{\gamma}])^2 < 8E[\tilde{\beta}]E[\tilde{I}]$, thus H is a definite negative matrix and $E[\Pi_M]$ is jointly concave in w and θ .

Let the first order conditions be zero, we can have Eqs.(9) and (10).

Substituting w^* in Eq. (9) and θ^* in Eq. (10) into Eq.(6), we can get Eq.(8).

Theorem 2 is proved. \square

The chance-constrained programming proposed by Liu and Iwamura^{29, 30}, has a big role in formulating fuzzy models. Given a confidence level α , the decision makers try to optimize the critical value subject to chance constraint. Hence, in this case, we can formulate the maximax chance-constrained programming model as follows

$$\begin{cases} \max_{w, \theta} \bar{\Pi}_M \\ \text{s. t.} \\ \text{Pos}\{(w - \tilde{c})(\tilde{D} - \tilde{\beta}(m^* + w) + \tilde{\gamma}\theta) - \tilde{I}\theta^2 \geq \bar{\Pi}_M\} \geq \alpha \\ \text{Pos}\{w - \tilde{c} < 0\} = 0 \\ \text{where } m^* \text{ derives from problem :} \\ \begin{cases} \max_m \bar{\Pi}_R \\ \text{s. t.} \\ \text{Pos}\{m(\tilde{D} - \tilde{\beta}(m + w) + \tilde{\gamma}\theta) \geq \bar{\Pi}_R\} \geq \alpha \\ \text{Pos}\{\tilde{D} - \tilde{\beta}(m + w) + \tilde{\gamma}\theta < 0\} = 0 \end{cases} \end{cases} \quad (12)$$

where α is a given confidence level for manufacturer's and retailer's profits, $\bar{\Pi}_R$ is the maximum value that $\Pi_R(m)$ obtains with at least possibility α , and $\bar{\Pi}_M$ is the maximum value that $\Pi_M(w, \theta, m^*)$ obtains with at least possibility α . Therefore, model (12) equals to the following model (13), in which supply chain actors attempt to maximize the α -optimistic value of profits

$(\Pi_M(w, \theta, m^*))_\alpha^R$ and $(\Pi_R(m))_\alpha^R$ by choosing the optimal solutions, that is

$$\begin{cases} \max_{w, \theta} (\Pi_M(w, \theta, m^*))_\alpha^R = ((w - \tilde{c})(\tilde{D} - \tilde{\beta}(m^* + w) + \tilde{\gamma}\theta) - \tilde{I}\theta^2)_\alpha^R \\ \text{s. t.} \\ \text{Pos}\{w - \tilde{c} < 0\} = 0 \\ \text{where } m^* \text{ derives from problem :} \\ \begin{cases} \max_m (\Pi_R(m))_\alpha^R = (m(\tilde{D} - \tilde{\beta}(m + w) + \tilde{\gamma}\theta))_\alpha^R \\ \text{s. t.} \\ \text{Pos}\{\tilde{D} - \tilde{\beta}(m + w) + \tilde{\gamma}\theta < 0\} = 0 \end{cases} \end{cases} \quad (13)$$

Theorem 3. Let $(\Pi_R(m))_\alpha^R$ be the retailer's α -optimistic value of profit. The wholesale price w and the level of greening innovation θ set by manufacturer are fixed. If $\text{Pos}\{\tilde{D} - \tilde{\beta}(m^* + w) + \tilde{\gamma}\theta < 0\} = 0$, then optimal reaction function is given as

$$m^*(w, \theta) = \frac{\tilde{D}_\alpha^R - \tilde{\beta}_\alpha^L w + \tilde{\gamma}_\alpha^R \theta}{2\tilde{\beta}_\alpha^L}. \quad (14)$$

Proof. The retailer's profit is

$$(\Pi_R(m))_\alpha^R = -\tilde{\beta}_\alpha^L m^2 + (\tilde{D}_\alpha^R - \tilde{\beta}_\alpha^L w + \tilde{\gamma}_\alpha^R \theta)m \quad (15)$$

The first order condition

$$\frac{d}{dm} (\Pi_R(m))_\alpha^R = -2\tilde{\beta}_\alpha^L m + \tilde{D}_\alpha^R - \tilde{\beta}_\alpha^L w + \tilde{\gamma}_\alpha^R \theta.$$

The second order condition

$$\frac{d^2}{dm^2} (\Pi_R(m))_\alpha^R = -2\tilde{\beta}_\alpha^L < 0.$$

Thus, for any $\alpha \in (0, 1]$, $(\Pi_R(m))_\alpha^R$ is concave in m .

Let $\frac{d}{dm} (\Pi_R(m))_\alpha^R = 0$, we can get the retailer's optimal margin profit as in Eq. (14).

Theorem 3 is proved. \square

Theorem 4. Let $(\Pi_M(w, \theta, m^*))_\alpha^R$ be the manufacturer's α -optimistic value of profit. If $(\tilde{\gamma}_\alpha^R)^2 < 8\tilde{\beta}_\alpha^L \tilde{I}_\alpha^L$, $\text{Pos}\{\tilde{D} - \tilde{\beta}(m^* + w) + \tilde{\gamma}\theta < 0\} = 0$ and $\text{Pos}\{w - \tilde{c} < 0\} = 0$, then the optimal equilibrium decisions are

$$m^* = \frac{2\tilde{I}_\alpha^L (\tilde{D}_\alpha^R - \tilde{\beta}_\alpha^L \tilde{c}_\alpha^L)}{8\tilde{\beta}_\alpha^L \tilde{I}_\alpha^L - (\tilde{\gamma}_\alpha^R)^2}. \quad (16)$$

$$w^* = \frac{4\tilde{I}_\alpha^L (\tilde{D}_\alpha^R + \tilde{\beta}_\alpha^L \tilde{c}_\alpha^L) - (\tilde{\gamma}_\alpha^R)^2 \tilde{c}_\alpha^L}{8\tilde{\beta}_\alpha^L \tilde{I}_\alpha^L - (\tilde{\gamma}_\alpha^R)^2}. \quad (17)$$

$$\theta^* = \frac{\tilde{\gamma}_\alpha^R (\tilde{D}_\alpha^R - \tilde{\beta}_\alpha^L \tilde{c}_\alpha^L)}{8\tilde{\beta}_\alpha^L \tilde{I}_\alpha^L - (\tilde{\gamma}_\alpha^R)^2}. \quad (18)$$

Proof. The manufacturer's profit is

$$\begin{aligned} (\Pi_M(w, \theta, m^*))_\alpha^R &= ((w - \tilde{c})(\tilde{D} - \tilde{\beta}(m^* + w) + \tilde{\gamma}\theta) - \tilde{I}\theta^2)_\alpha^R \\ &= (w - \tilde{c}_\alpha^L)(\tilde{D}_\alpha^R - \tilde{\beta}_\alpha^L(m^* + w) + \tilde{\gamma}_\alpha^R\theta) - \tilde{I}_\alpha^L\theta^2. \end{aligned} \quad (19)$$

Substituting $m^*(w, \theta)$ in Eq. (14) into Eq. (19), the first order conditions can be obtained as

$$\begin{aligned} \frac{\partial}{\partial w} (\Pi_M(w, \theta, m^*))_\alpha^R &= -\tilde{\beta}_\alpha^L w + \frac{1}{2}\tilde{\gamma}_\alpha^R\theta + \frac{1}{2}(\tilde{D}_\alpha^R + \tilde{\beta}_\alpha^L \tilde{c}_\alpha^L). \\ \frac{\partial}{\partial \theta} (\Pi_M(w, \theta, m^*))_\alpha^R &= -2\tilde{I}_\alpha^L\theta + \frac{1}{2}\tilde{\gamma}_\alpha^R w - \frac{1}{2}\tilde{\gamma}_\alpha^R \tilde{c}_\alpha^L. \end{aligned}$$

Therefore, the Hessian matrix of $(\Pi_M(w, \theta, m^*))_\alpha^R$ is

$$H = \begin{bmatrix} -\tilde{\beta}_\alpha^L & \frac{1}{2}\tilde{\gamma}_\alpha^R \\ \frac{1}{2}\tilde{\gamma}_\alpha^R & -2\tilde{I}_\alpha^L \end{bmatrix}.$$

Since $\tilde{\beta}$, \tilde{I} are positive fuzzy variables and $(\tilde{\gamma}_\alpha^R)^2 < 8\tilde{\beta}_\alpha^L \tilde{I}_\alpha^L$, thus, for any $\alpha \in (0, 1]$, H is a definite negative matrix and $(\Pi_M)_\alpha^R$ is jointly concave in w and θ .

Let the first order conditions be zero, we can have Eqs.(16) and (17).

Substituting w^* in Eq. (16) and θ^* in Eq. (17) into Eq.(14), we can get Eq.(18).

Theorem 4 is proved. \square

The optimal optimistic profits of the manufacturer and the retailer can be easily obtained as follows

$$\begin{aligned} (\Pi_M)_\alpha^R &= \frac{\tilde{I}_\alpha^L (\tilde{D}_\alpha^R - \tilde{\beta}_\alpha^L \tilde{c}_\alpha^L)^2}{8\tilde{\beta}_\alpha^L \tilde{I}_\alpha^L - (\tilde{\gamma}_\alpha^R)^2}. \\ (\Pi_R)_\alpha^R &= \frac{4\tilde{\beta}_\alpha^L (\tilde{I}_\alpha^L)^2 (\tilde{D}_\alpha^R - \tilde{\beta}_\alpha^L \tilde{c}_\alpha^L)^2}{(8\tilde{\beta}_\alpha^L \tilde{I}_\alpha^L - (\tilde{\gamma}_\alpha^R)^2)^2}. \end{aligned}$$

Similarly, in the MS game, we can formulate the minimax chance-constrained programming model of GSC as follows

$$\begin{cases} \max_{w, \theta} \min_{\Pi_M} \bar{\Pi}_M \\ \text{s. t.} \\ \text{Pos}\{(w - \tilde{c})(\tilde{D} - \tilde{\beta}(m^* + w) + \tilde{\gamma}\theta) - \tilde{I}\theta^2 \leq \bar{\Pi}_M\} \geq \alpha \\ \text{Pos}\{w - \tilde{c} < 0\} = 0 \\ \text{where } m^* \text{ derives from problem} \end{cases} \quad (20)$$

$$\begin{cases} \max_m \min_{\Pi_R} \bar{\Pi}_R \\ \text{s. t.} \\ \text{Pos}\{m(\tilde{D} - \tilde{\beta}(m + w) + \tilde{\gamma}\theta) \leq \bar{\Pi}_R\} \geq \alpha \\ \text{Pos}\{\tilde{D} - \tilde{\beta}(m + w) + \tilde{\gamma}\theta < 0\} = 0 \end{cases}$$

where α is a given confidence level for manufacturer's and retailer's profits, $\bar{\Pi}_R$ is the maximum value that $\Pi_R(m)$ obtains with at least possibility α , and $\bar{\Pi}_M$ is the maximum value that $\Pi_M(w, \theta, m^*)$ obtains with at least possibility α . Therefore, model (20) equals to the following model (21), in which supply chain actors attempt to maximize the α -pessimistic value of profits $(\Pi_M(w, \theta, m^*))_\alpha^L$ and $(\Pi_R(m))_\alpha^L$ by choosing optimal solutions, that is

$$\begin{cases} \max_{w, \theta} (\Pi_M(w, \theta, m^*))_\alpha^L = ((w - \tilde{c})(\tilde{D} - \tilde{\beta}(m^* + w) + \tilde{\gamma}\theta) - \tilde{I}\theta^2)_\alpha^L \\ \text{s. t.} \\ \text{Pos}\{w - \tilde{c} < 0\} = 0 \\ \text{where } m^* \text{ derives from problem:} \\ \begin{cases} \max_m (\Pi_R(m))_\alpha^L = (m(\tilde{D} - \tilde{\beta}(m + w) + \tilde{\gamma}\theta))_\alpha^L \\ \text{s. t.} \\ \text{Pos}\{\tilde{D} - \tilde{\beta}(m + w) + \tilde{\gamma}\theta < 0\} = 0 \end{cases} \end{cases} \quad (21)$$

Theorem 5. Let $(\Pi_R(m))_\alpha^L$ and $(\Pi_M(w, \theta, m^*))_\alpha^L$ be the α -pessimistic value of the profits for the retailer and the manufacturer. If $(\tilde{\gamma}_\alpha^L)^2 < 8\tilde{\beta}_\alpha^R \tilde{I}_\alpha^R$, $\text{Pos}\{w^* - \tilde{c} < 0\} = 0$ and $\text{Pos}\{\tilde{D} - \tilde{\beta}(m^* + w^*) + \tilde{\gamma}\theta^* < 0\} = 0$, then the optimal equilibrium decisions in the MS case are

$$m^* = \frac{2\tilde{I}_\alpha^R (\tilde{D}_\alpha^L - \tilde{\beta}_\alpha^R \tilde{c}_\alpha^R)}{8\tilde{\beta}_\alpha^R \tilde{I}_\alpha^R - (\tilde{\gamma}_\alpha^L)^2}. \quad (22)$$

$$w^* = \frac{4\tilde{I}_\alpha^R (\tilde{D}_\alpha^L + \tilde{\beta}_\alpha^R \tilde{c}_\alpha^R) - (\tilde{\gamma}_\alpha^L)^2 \tilde{c}_\alpha^R}{8\tilde{\beta}_\alpha^R \tilde{I}_\alpha^R - (\tilde{\gamma}_\alpha^L)^2}. \quad (23)$$

$$\theta^* = \frac{\tilde{\gamma}_\alpha^L (\tilde{D}_\alpha^L - \tilde{\beta}_\alpha^R \tilde{c}_\alpha^R)}{8\tilde{\beta}_\alpha^R \tilde{I}_\alpha^R - (\tilde{\gamma}_\alpha^L)^2}. \quad (24)$$

Proof. Similar to Theorems 3 and 4. \square

The optimal pessimistic profits of the manufacturer and the retailer are given by

$$\begin{aligned}
 (\Pi_M)_\alpha^L &= \frac{\tilde{I}_\alpha^R (\tilde{D}_\alpha^L - \tilde{\beta}_\alpha^R \tilde{c}_\alpha^R)^2}{8\tilde{\beta}_\alpha^R \tilde{I}_\alpha^R - (\tilde{\gamma}_\alpha^L)^2} \\
 (\Pi_R)_\alpha^L &= \frac{4\tilde{\beta}_\alpha^R (\tilde{I}_\alpha^R)^2 (\tilde{D}_\alpha^L - \tilde{\beta}_\alpha^R \tilde{c}_\alpha^R)^2}{(8\tilde{\beta}_\alpha^R \tilde{I}_\alpha^R - (\tilde{\gamma}_\alpha^L)^2)^2}
 \end{aligned}$$

Remark 1. If $\alpha=1$, it is clear the parameters \tilde{c} , \tilde{D} , $\tilde{\beta}$, $\tilde{\gamma}$ and \tilde{I} reduce to crisp numbers, then the optimal solutions in Theorems 4 and 5 are

$$\begin{aligned}
 m^* &= \frac{2I(D - \beta c)}{8\beta I - \gamma^2} \\
 w^* &= \frac{4I(D + \beta c) - \gamma^2 c}{8\beta I - \gamma^2} \\
 \theta^* &= \frac{\gamma(D - \beta c)}{8\beta I - \gamma^2}
 \end{aligned}$$

4.2. Retailer Stackelberg (RS) game

In the RS game, the manufacturer has more bargaining power than the retailer. That is to say, the retailer is the leader in the GSC. In this case, firstly, the retailer sets his margin profit m condition on the manufacturer's optimal reaction to his decisions. Then, the manufacturer sets his wholesale price w and level of greening innovation θ . Hence, in this case, we can formulate the fuzzy optimal model as follows

$$\begin{cases}
 \max_m E[\Pi_R(m, w^*, \theta^*)] = E[m(\tilde{D} - \tilde{\beta}(m+w) + \tilde{\gamma}\theta)] \\
 \text{s. t.} \\
 \text{where } w^* \text{ and } \theta^* \text{ derive from problem:} \\
 \begin{cases}
 \max_{w, \theta} E[\Pi_M(w, \theta)] = E[(w - \tilde{c})(\tilde{D} - \tilde{\beta}(m+w) + \tilde{\gamma}\theta) - \tilde{I}\theta^2] \\
 \text{s. t.} \\
 \text{Pos}\{w - \tilde{c} < 0\} = 0 \\
 \text{Pos}\{\tilde{D} - \tilde{\beta}(m+w) + \tilde{\gamma}\theta < 0\} = 0
 \end{cases}
 \end{cases}
 \quad (25)$$

We first solve the reaction function of the manufacturer.

Theorem 6. Let $E[\Pi_M(w, \theta)]$ be the manufacturer's fuzzy expected profit. If $(E[\tilde{\gamma}])^2 < 4E[\tilde{\beta}]E[\tilde{I}]$, $\text{Pos}\{\tilde{D} - \tilde{\beta}(m+w^*) + \tilde{\gamma}\theta^* < 0\} = 0$ and $\text{Pos}\{w^* - \tilde{c} < 0\} = 0$, then the manufacturer's optimal reaction functions are

$$w^{**}(m) = \frac{4E[\tilde{I}](E[\tilde{D}] + E[\tilde{\beta}\tilde{c}])}{2(4E[\tilde{\beta}]E[\tilde{I}] - (E[\tilde{\gamma}])^2)} + \frac{(E[\tilde{D}] - E[\tilde{\beta}\tilde{c}])}{2E[\tilde{\beta}]}$$

$$\begin{aligned}
 &= \frac{E[\tilde{\gamma}] \int_0^1 (\tilde{\gamma}_\alpha^L \tilde{c}_\alpha^R + \tilde{\gamma}_\alpha^R \tilde{c}_\alpha^L) d\alpha + 4E[\tilde{\beta}]E[\tilde{I}]m}{2(4E[\tilde{\beta}]E[\tilde{I}] - (E[\tilde{\gamma}])^2)} \quad (26) \\
 \theta^{**}(m) &= \frac{E[\tilde{\gamma}](E[\tilde{D}] + E[\tilde{\beta}\tilde{c}])}{4E[\tilde{\beta}]E[\tilde{I}] - (E[\tilde{\gamma}])^2} \\
 &= \frac{E[\tilde{\beta}] \int_0^1 (\tilde{\gamma}_\alpha^L \tilde{c}_\alpha^R + \tilde{\gamma}_\alpha^R \tilde{c}_\alpha^L) d\alpha + E[\tilde{\beta}]E[\tilde{\gamma}]m}{4E[\tilde{\beta}]E[\tilde{I}] - (E[\tilde{\gamma}])^2} \quad (27)
 \end{aligned}$$

Proof. From Eq. (11), the first order conditions can be obtained as

$$\begin{aligned}
 \frac{\partial}{\partial w} E[\Pi_M] &= -2E[\tilde{\beta}]w + E[\tilde{\gamma}]\theta + E[\tilde{D}] + E[\tilde{\beta}\tilde{c}] - E[\tilde{\beta}]m \\
 \frac{\partial}{\partial \theta} E[\Pi_M] &= -2E[\tilde{I}]\theta + E[\tilde{\gamma}]w - \frac{1}{2} \int_0^1 (\tilde{\gamma}_\alpha^L \tilde{c}_\alpha^R + \tilde{\gamma}_\alpha^R \tilde{c}_\alpha^L) d\alpha
 \end{aligned}$$

Therefore the Hessian matrix of $E[\Pi_M]$ is

$$H = \begin{bmatrix} -2E[\tilde{\beta}] & E[\tilde{\gamma}] \\ E[\tilde{\gamma}] & -2E[\tilde{I}] \end{bmatrix}$$

Since $E[\tilde{\beta}] > 0$, $E[\tilde{I}] > 0$ and $(E[\tilde{\gamma}])^2 < 4E[\tilde{\beta}]E[\tilde{I}]$, thus H is a definite negative matrix and $E[\Pi_M]$ is jointly concave in w and θ .

Let the first order conditions be zero, we can have Eqs.(26) and (27).

Theorem 6 is proved. \square

Here, $w(m)$ and $\theta(m)$ can be regarded as the manufacture's response functions to the margin profit m . The retailer sets his optimal decisions. So, the following Proposition can be derived.

Theorem 7. Let $E[\Pi_R(m)]$ be the retailer's fuzzy expected profit. If $(E[\tilde{\gamma}])^2 < 4E[\tilde{\beta}]E[\tilde{I}]$, $\text{Pos}\{w^* - \tilde{c} < 0\} = 0$ and $\text{Pos}\{\tilde{D} - \tilde{\beta}(m^* + w^*) + \tilde{\gamma}\theta^* < 0\} = 0$, then the optimal equilibrium decisions are

$$m^{**} = \frac{E[\tilde{\gamma}](2E[\tilde{\beta}\tilde{c}]E[\tilde{\gamma}] - E[\tilde{\beta}]) \int_0^1 (\tilde{\gamma}_\alpha^L \tilde{c}_\alpha^R + \tilde{\gamma}_\alpha^R \tilde{c}_\alpha^L) d\alpha}{8(E[\tilde{\beta}])^2 E[\tilde{I}]} + \frac{(E[\tilde{D}] - E[\tilde{\beta}\tilde{c}])}{2E[\tilde{\beta}]} \quad (28)$$

$$w^{**} = \frac{E[\tilde{I}](E[\tilde{D}] + 3E[\tilde{\beta}\tilde{c}])}{4E[\tilde{\beta}]E[\tilde{I}] - (E[\tilde{\gamma}])^2} - \frac{E[\tilde{\gamma}](2E[\tilde{\beta}\tilde{c}]E[\tilde{\gamma}] + E[\tilde{\beta}]\int_0^1(\tilde{\gamma}_\alpha^L\tilde{c}_\alpha^R + \tilde{\gamma}_\alpha^R\tilde{c}_\alpha^L)d\alpha)}{4E[\tilde{\beta}](4E[\tilde{\beta}]E[\tilde{I}] - (E[\tilde{\gamma}])^2)} \quad (29)$$

$$\theta^{**} = \frac{E[\tilde{\gamma}](E[\tilde{D}] + 3E[\tilde{\beta}\tilde{c}])}{2(4E[\tilde{\beta}]E[\tilde{I}] - (E[\tilde{\gamma}])^2)} - \frac{(8E[\tilde{\beta}]E[\tilde{I}] - (E[\tilde{\gamma}])^2)\int_0^1(\tilde{\gamma}_\alpha^L\tilde{c}_\alpha^R + \tilde{\gamma}_\alpha^R\tilde{c}_\alpha^L)d\alpha}{8E[\tilde{\gamma}](4E[\tilde{\beta}]E[\tilde{I}] - (E[\tilde{\gamma}])^2)} - \frac{E[\tilde{\beta}\tilde{c}](E[\tilde{\gamma}])^3}{4E[\tilde{\beta}]E[\tilde{I}](4E[\tilde{\beta}]E[\tilde{I}] - (E[\tilde{\gamma}])^2)} \quad (30)$$

Proof. Substituting Eqs. (26) and (27) into Eq. (7), the first order condition is

$$\frac{d}{dm} E[\Pi_R(m)] = -\frac{4(E[\tilde{\beta}])^2 E[\tilde{I}]}{4E[\tilde{\beta}]E[\tilde{\gamma}] - (E[\tilde{I}])^2} m + \frac{2E[\tilde{\beta}]E[\tilde{I}](E[\tilde{D}] - E[\tilde{\beta}\tilde{c}])}{4E[\tilde{\beta}]E[\tilde{\gamma}] - (E[\tilde{I}])^2} + \frac{E[\tilde{\gamma}](2E[\tilde{\beta}\tilde{c}]E[\tilde{\gamma}] - E[\tilde{\beta}]\int_0^1(\tilde{\gamma}_\alpha^L\tilde{c}_\alpha^R + \tilde{\gamma}_\alpha^R\tilde{c}_\alpha^L)d\alpha)}{2(4E[\tilde{\beta}]E[\tilde{\gamma}] - (E[\tilde{I}])^2)}$$

The second order condition is

$$\frac{d^2}{dm^2} E[\Pi_R(m)] = -\frac{4(E[\tilde{\beta}])^2 E[\tilde{I}]}{4E[\tilde{\beta}]E[\tilde{\gamma}] - (E[\tilde{I}])^2}$$

Since $E[\tilde{\beta}] > 0, E[\tilde{\gamma}] > 0, E[\tilde{I}] > 0$ and $(E[\tilde{\gamma}])^2 < 4E[\tilde{\beta}]E[\tilde{I}]$, then the second order condition is negative definite. Thus, $E[\Pi_R(m)]$ is concave in m .

Let the first order conditions be zero, we can have Eq.(28).

Substituting m^{**} in Eq. (28) into Eqs. (26) and (27), we can get Eqs. (29) and (30).

Theorem 7 is proved. \square

In the RS game, the maximax chance-constrained programming model of GSC can be formulated as

$$\begin{cases} \max_m (\Pi_R(m))_\alpha^R = (m(\tilde{D} - \tilde{\beta}(m + w^{**}) + \tilde{\gamma}\theta^{**}))_\alpha^R \\ \text{s. t.} \\ \text{where } w^{**} \text{ and } \theta^{**} \text{ derive from problem:} \\ \begin{cases} \max_{w, \theta} (\Pi_M(w, \theta))_\alpha^R = ((w - \tilde{c})(\tilde{D} - \tilde{\beta}(m + w) + \tilde{\gamma}\theta) - \tilde{I}\theta^2)_\alpha^R \\ \text{s. t.} \\ \text{Pos}\{w^{**} - \tilde{c} < 0\} = 0 \\ \text{Pos}\{\tilde{D} - \tilde{\beta}(m + w^{**}) + \tilde{\gamma}\theta^{**} < 0\} = 0 \end{cases} \end{cases} \quad (31)$$

Theorem 8. Let $(\Pi_M(w, \theta))_\alpha^R$ be the manufacturer's α -optimistic value of profit. If $(\tilde{\gamma}_\alpha^R)^2 < 4\tilde{\beta}_\alpha^L\tilde{I}_\alpha^L$, $\text{Pos}\{w^{**} - \tilde{c} < 0\} = 0$ and $\text{Pos}\{\tilde{D} - \tilde{\beta}(m + w^{**}) + \tilde{\gamma}\theta^{**} < 0\} = 0$, then the optimal reaction functions for manufacturer are

$$w^{**}(m) = \frac{2\tilde{I}_\alpha^L(\tilde{D}_\alpha^R + \tilde{\beta}_\alpha^L\tilde{c}_\alpha^L) - 2\tilde{\beta}_\alpha^L\tilde{I}_\alpha^L m - (\tilde{\gamma}_\alpha^R)^2\tilde{c}_\alpha^L}{4\tilde{\beta}_\alpha^L\tilde{I}_\alpha^L - (\tilde{\gamma}_\alpha^R)^2} \quad (32)$$

$$\theta^{**}(m) = \frac{\tilde{\gamma}_\alpha^R(\tilde{D}_\alpha^R - \tilde{\beta}_\alpha^L\tilde{c}_\alpha^L - \tilde{\beta}_\alpha^L m)}{4\tilde{\beta}_\alpha^L\tilde{I}_\alpha^L - (\tilde{\gamma}_\alpha^R)^2} \quad (33)$$

Proof. From Eq. (19), the first order conditions can be obtained as

$$\frac{\partial}{\partial w} (\Pi_M(w, \theta, m^*))_\alpha^R = -2\tilde{\beta}_\alpha^L w + \tilde{\gamma}_\alpha^R \theta - \tilde{\beta}_\alpha^L m + \tilde{D}_\alpha^R + \tilde{\beta}_\alpha^L \tilde{c}_\alpha^L$$

$$\frac{\partial}{\partial \theta} (\Pi_M(w, \theta, m^*))_\alpha^R = -2\tilde{I}_\alpha^L \theta + \tilde{\gamma}_\alpha^R w - \tilde{\gamma}_\alpha^R \tilde{c}_\alpha^L$$

Therefore, the Hessian matrix of $(\Pi_M(w, \theta, m^*))_\alpha^R$ is

$$H = \begin{bmatrix} -2\tilde{\beta}_\alpha^L & \tilde{\gamma}_\alpha^R \\ \tilde{\gamma}_\alpha^R & -2\tilde{I}_\alpha^L \end{bmatrix}$$

Since $\tilde{\beta}$, \tilde{I} are positive fuzzy variables and $(\tilde{\gamma}_\alpha^R)^2 < 4\tilde{\beta}_\alpha^L\tilde{I}_\alpha^L$, thus, for any $\alpha \in (0, 1]$, H is a definite negative matrix and $(\Pi_M(w, \theta))_\alpha^R$ is jointly concave in w and θ .

Let the first order conditions be zero, the results (32) and (33) can be derived.

Theorem 8 is proved. \square

Theorem 9. Let $(\Pi_R(m))_\alpha^R$ be the retailer's α -optimistic value of profit. If $(\tilde{\gamma}_\alpha^R)^2 < 4\tilde{\beta}_\alpha^L\tilde{I}_\alpha^L$, $\text{Pos}\{w^{**} - \tilde{c} < 0\} = 0$ and $\text{Pos}\{\tilde{D} - \tilde{\beta}(m^{**} + w^{**}) + \tilde{\gamma}\theta^{**} < 0\} = 0$, then the optimal equilibrium decisions are

$$m^{**} = \frac{\tilde{D}_\alpha^R - \tilde{\beta}_\alpha^L\tilde{c}_\alpha^L}{2\tilde{\beta}_\alpha^L} \quad (34)$$

$$w^{**} = \frac{\tilde{I}_\alpha^L (\tilde{D}_\alpha^R + 3\tilde{\beta}_\alpha^L \tilde{c}_\alpha^L) - (\tilde{\gamma}_\alpha^R)^2 \tilde{c}_\alpha^L}{4\tilde{\beta}_\alpha^L \tilde{I}_\alpha^L - (\tilde{\gamma}_\alpha^R)^2}. \quad (35)$$

$$\theta^{**} = \frac{\tilde{\gamma}_\alpha^R (\tilde{D}_\alpha^R - \tilde{\beta}_\alpha^L \tilde{c}_\alpha^L)}{2(4\tilde{\beta}_\alpha^L \tilde{I}_\alpha^L - (\tilde{\gamma}_\alpha^R)^2)}. \quad (36)$$

Proof. Substituting $w^{**}(m)$ and $\theta^{**}(m)$ in Eqs. (32) and (33) into Eq. (15), the first order condition is

$$\frac{d}{dm}(\Pi_R(m))_\alpha^R = -\frac{4(\tilde{\beta}_\alpha^L)^2 \tilde{I}_\alpha^L}{4\tilde{\beta}_\alpha^L \tilde{I}_\alpha^L - \tilde{\gamma}_\alpha^R} m + \frac{2\tilde{\beta}_\alpha^L \tilde{I}_\alpha^L (\tilde{D}_\alpha^R - \tilde{\beta}_\alpha^L \tilde{c}_\alpha^L)}{4\tilde{\beta}_\alpha^L \tilde{I}_\alpha^L - \tilde{\gamma}_\alpha^R}.$$

The second order condition is

$$\frac{d^2}{dm^2}(\Pi_R(m))_\alpha^R = -\frac{4(\tilde{\beta}_\alpha^L)^2 \tilde{I}_\alpha^L}{4\tilde{\beta}_\alpha^L \tilde{I}_\alpha^L - \tilde{\gamma}_\alpha^R}.$$

Since $\tilde{\beta}$, \tilde{I} and $\tilde{\gamma}$ are positive fuzzy variables, and $(\tilde{\gamma}_\alpha^R)^2 < 4\tilde{\beta}_\alpha^L \tilde{I}_\alpha^L$, then the second order condition is negative definite. Thus, for any $\alpha \in (0, 1]$, $(\Pi_R(m))_\alpha^R$ is concave in m .

Let $\frac{d}{dm}(\Pi_R(m))_\alpha^R = 0$, we can get the retailer's optimal margin profit as in Eq. (34).

Substituting m^{**} in Eq. (34) into Eqs. (32) and (33), we can get Eqs. (35) and (36).

Theorem 9 is proved. \square

The optimal optimistic profits of the manufacturer and the retailer can be easily obtained as follows

$$(\Pi_M)_\alpha^R = \frac{\tilde{I}_\alpha^L (\tilde{D}_\alpha^R - \tilde{\beta}_\alpha^L \tilde{c}_\alpha^L)^2}{4(4\tilde{\beta}_\alpha^L \tilde{I}_\alpha^L - (\tilde{\gamma}_\alpha^R)^2)},$$

$$(\Pi_R)_\alpha^R = \frac{\tilde{I}_\alpha^L (\tilde{D}_\alpha^R - \tilde{\beta}_\alpha^L \tilde{c}_\alpha^L)^2}{2(4\tilde{\beta}_\alpha^L \tilde{I}_\alpha^L - (\tilde{\gamma}_\alpha^R)^2)}.$$

In the RS game, the minimax chance-constrained programming model of GSC can be formulate as follows

$$\left\{ \begin{array}{l} \max_m (\Pi_R(m))_\alpha^L = (m(\tilde{D} - \tilde{\beta}(m + w^{**}) + \tilde{\gamma}\theta^{**}))_\alpha^L \\ \text{s. t.} \\ \text{where } w^{**} \text{ and } \theta^{**} \text{ derive from problem:} \\ \left\{ \begin{array}{l} \max_{w, \theta} (\Pi_M(w, \theta))_\alpha^L = ((w - \tilde{c})(\tilde{D} - \tilde{\beta}(m + w) + \tilde{\gamma}\theta) - \tilde{I}\theta^2)_\alpha^L \\ \text{s. t.} \\ \text{Pos}\{w^{**} - \tilde{c} < 0\} = 0 \\ \text{Pos}\{\tilde{D} - \tilde{\beta}(m + w^{**}) + \tilde{\gamma}\theta^{**} < 0\} = 0 \end{array} \right. \end{array} \right. \quad (37)$$

Theorem 10. Let $(\Pi_R(m))_\alpha^L$ and $(\Pi_M(w, \theta))_\alpha^L$ be the α -pessimistic value of profit for the retailer and the manufacturer. If $(\tilde{\gamma}_\alpha^L)^2 < 4\tilde{\beta}_\alpha^R \tilde{I}_\alpha^R$, $\text{Pos}\{w^{**} - \tilde{c} < 0\} = 0$ and $\text{Pos}\{\tilde{D} - \tilde{\beta}(m^{**} + w^{**}) + \tilde{\gamma}\theta^{**} < 0\} = 0$, then the optimal equilibrium decisions are

$$m^{**} = \frac{\tilde{D}_\alpha^L - \tilde{\beta}_\alpha^R \tilde{c}_\alpha^R}{2\tilde{\beta}_\alpha^R}. \quad (38)$$

$$w^{**} = \frac{\tilde{I}_\alpha^R (\tilde{D}_\alpha^L + 3\tilde{\beta}_\alpha^R \tilde{c}_\alpha^R) - (\tilde{\gamma}_\alpha^L)^2 \tilde{c}_\alpha^R}{4\tilde{\beta}_\alpha^R \tilde{I}_\alpha^R - (\tilde{\gamma}_\alpha^L)^2}. \quad (39)$$

$$\theta^{**} = \frac{\tilde{\gamma}_\alpha^L (\tilde{D}_\alpha^L - \tilde{\beta}_\alpha^R \tilde{c}_\alpha^R)}{2(4\tilde{\beta}_\alpha^R \tilde{I}_\alpha^R - (\tilde{\gamma}_\alpha^L)^2)}. \quad (40)$$

Proof. Similar to Theorems 8 and 9. \square

The optimal pessimistic profits for the supply chain members are given by

$$(\Pi_M)_\alpha^L = \frac{\tilde{I}_\alpha^R (\tilde{D}_\alpha^L - \tilde{\beta}_\alpha^R \tilde{c}_\alpha^R)^2}{4(4\tilde{\beta}_\alpha^R \tilde{I}_\alpha^R - (\tilde{\gamma}_\alpha^L)^2)},$$

$$(\Pi_R)_\alpha^L = \frac{\tilde{I}_\alpha^R (\tilde{D}_\alpha^L - \tilde{\beta}_\alpha^R \tilde{c}_\alpha^R)^2}{2(4\tilde{\beta}_\alpha^R \tilde{I}_\alpha^R - (\tilde{\gamma}_\alpha^L)^2)}.$$

Remark 2. If $\alpha=1$, then the parameters \tilde{c} , \tilde{D} , $\tilde{\beta}$, $\tilde{\gamma}$ and \tilde{I} reduce to crisp numbers, then the optimal solutions in Theorems 9 and 10 are

$$m^{**} = \frac{D - \beta c}{2\beta},$$

$$w^{**} = \frac{I(D + 3\beta c) - \gamma^2 c}{4\beta I - \gamma^2},$$

$$\theta^{**} = \frac{\gamma(D - \beta c)}{2(4\beta I - \gamma^2)}.$$

4.3. Vertical Nash (VN) game

In the VN game, the supply chain participants have an equal bargaining power. Thus, in this condition, the manufacturer makes his wholesale price w and level of greening innovation θ , and the retailer makes his margin profit m simultaneously and independently. The assumption VN game means a Nash equilibrium among the manufacturer and the retailer, hence, in this case, we can formulate the fuzzy optimal model as follows

$$\begin{cases} \max_{w, \theta} E[\Pi_M(w, \theta)] = E[(w - \tilde{c})(\tilde{D} - \tilde{\beta}(m+w) + \tilde{\gamma}\theta) - \tilde{I}\theta^2] \\ \max_m E[\Pi_R(m)] = E[m(\tilde{D} - \tilde{\beta}(m+w) + \tilde{\gamma}\theta)] \\ \text{s. t.} \\ \text{Pos}\{w - \tilde{c} < 0\} = 0 \\ \text{Pos}\{\tilde{D} - \tilde{\beta}(m+w) + \tilde{\gamma}\theta < 0\} = 0 \end{cases} \quad (41)$$

Theorem 11. Let $E[\Pi_M(w, \theta)]$ and $E[\Pi_R(m)]$ be the manufacturer's and retailer's fuzzy expected profits. If $(E[\tilde{\gamma}])^2 < 4E[\tilde{\beta}]E[\tilde{I}]$, $\text{Pos}\{\tilde{D} - \tilde{\beta}(m^{***} + w^{***}) + \tilde{\gamma}\theta^{***} < 0\} = 0$ and $\text{Pos}\{w^{***} - \tilde{c} < 0\} = 0$, then the optimal equilibrium decisions are

$$m^{***} = \frac{E[\tilde{\gamma}]\left(2E[\tilde{\beta}\tilde{c}]E[\tilde{\gamma}] - E[\tilde{\beta}]\int_0^1(\tilde{\gamma}_\alpha^L\tilde{c}_\alpha^R + \tilde{\gamma}_\alpha^R\tilde{c}_\alpha^L)d\alpha\right)}{2E[\tilde{\beta}]\left(6E[\tilde{\beta}]E[\tilde{I}] - (E[\tilde{\gamma}])^2\right)} + \frac{2E[\tilde{I}]\left(E[\tilde{D}] - E[\tilde{\beta}\tilde{c}]\right)}{6E[\tilde{\beta}]E[\tilde{I}] - (E[\tilde{\gamma}])^2} \quad (42)$$

$$w^{***} = \frac{4E[\tilde{I}]\left(E[\tilde{D}] + 2E[\tilde{\beta}\tilde{c}]\right) - E[\tilde{\gamma}]\int_0^1(\tilde{\gamma}_\alpha^L\tilde{c}_\alpha^R + \tilde{\gamma}_\alpha^R\tilde{c}_\alpha^L)d\alpha}{2\left(6E[\tilde{\beta}]E[\tilde{I}] - (E[\tilde{\gamma}])^2\right)} \quad (43)$$

$$\theta^{***} = \frac{2E[\tilde{\gamma}]\left(E[\tilde{D}] + 2E[\tilde{\beta}\tilde{c}]\right) - 3E[\tilde{\beta}]\int_0^1(\tilde{\gamma}_\alpha^L\tilde{c}_\alpha^R + \tilde{\gamma}_\alpha^R\tilde{c}_\alpha^L)d\alpha}{2\left(6E[\tilde{\beta}]E[\tilde{I}] - (E[\tilde{\gamma}])^2\right)} \quad (44)$$

Proof. From Eq. (7), the first order condition is

$$\frac{d}{dm}E[\Pi_R(m)] = -2E[\tilde{\beta}]m + E[\tilde{D}] - E[\tilde{\beta}]w + E[\tilde{\gamma}]\theta.$$

The second order condition is

$$\frac{d^2}{dm^2}E[\Pi_R(m)] = -2E[\tilde{\beta}] < 0.$$

Thus, the retailer's fuzzy expected profit $E[\Pi_R(m)]$ is strictly concave in m .

From Eq. (11), the first order conditions are

$$\frac{\partial}{\partial w}E[\Pi_M] = -2E[\tilde{\beta}]w + E[\tilde{\gamma}]\theta + E[\tilde{D}] + E[\tilde{\beta}\tilde{c}] - E[\tilde{\beta}]m.$$

$$\frac{\partial}{\partial \theta}E[\Pi_M] = -2E[\tilde{I}]\theta + E[\tilde{\gamma}]w - \frac{1}{2}\int_0^1(\tilde{\gamma}_\alpha^L\tilde{c}_\alpha^R + \tilde{\gamma}_\alpha^R\tilde{c}_\alpha^L)d\alpha.$$

Therefore, the Hessian matrix of $E[\Pi_M]$ is

$$H = \begin{bmatrix} -2E[\tilde{\beta}] & E[\tilde{\gamma}] \\ E[\tilde{\gamma}] & -2E[\tilde{I}] \end{bmatrix}.$$

Since $E[\tilde{\beta}] > 0$, $E[\tilde{I}] > 0$ and $(E[\tilde{\gamma}])^2 < 4E[\tilde{\beta}]E[\tilde{I}]$, thus H is a definite negative matrix, and $E[\Pi_M]$ is jointly concave in w and θ .

We can have Eqs. (42), (43) and (44) by setting the first order conditions above be zero.

Theorem 11 is proved. \square

In the VN game, we can also formulate the minimax chance-constrained programming model of GSC as

$$\begin{cases} \max_{w, \theta} (\Pi_M(w, \theta))_\alpha^R = \left((w - \tilde{c})(\tilde{D} - \tilde{\beta}(m+w) + \tilde{\gamma}\theta) - \tilde{I}\theta^2\right)_\alpha^R \\ \max_m (\Pi_R(m))_\alpha^R = \left(m(\tilde{D} - \tilde{\beta}(m+w) + \tilde{\gamma}\theta)\right)_\alpha^R \\ \text{s. t.} \\ \text{Pos}\{w - \tilde{c} < 0\} = 0 \\ \text{Pos}\{\tilde{D} - \tilde{\beta}(m+w) + \tilde{\gamma}\theta < 0\} = 0 \end{cases} \quad (45)$$

Theorem 12. Let $(\Pi_M(w, \theta))_\alpha^R$ and $(\Pi_R(m))_\alpha^R$ be the manufacturer's and the retailer's α -optimistic value of profits. If $(\tilde{\gamma}_\alpha^R)^2 < 4\tilde{\beta}_\alpha^L\tilde{I}_\alpha^L$, $\text{Pos}\{w^{***} - \tilde{c} < 0\} = 0$ and $\text{Pos}\{\tilde{D} - \tilde{\beta}(m^{***} + w^{***}) + \tilde{\gamma}\theta^{***} < 0\} = 0$, then the optimal equilibrium decisions are

$$m^{***} = \frac{2\tilde{I}_\alpha^L(\tilde{D}_\alpha^R - \tilde{\beta}_\alpha^L\tilde{c}_\alpha^L)}{6\tilde{\beta}_\alpha^L\tilde{I}_\alpha^L - (\tilde{\gamma}_\alpha^R)^2} \quad (46)$$

$$w^{***} = \frac{2\tilde{I}_\alpha^L(\tilde{D}_\alpha^R + 2\tilde{\beta}_\alpha^L\tilde{c}_\alpha^L) - (\tilde{\gamma}_\alpha^R)^2\tilde{c}_\alpha^L}{6\tilde{\beta}_\alpha^L\tilde{I}_\alpha^L - (\tilde{\gamma}_\alpha^R)^2} \quad (47)$$

$$\theta^{***} = \frac{\tilde{\gamma}_\alpha^R(\tilde{D}_\alpha^R - \tilde{\beta}_\alpha^L\tilde{c}_\alpha^L)}{6\tilde{\beta}_\alpha^L\tilde{I}_\alpha^L - (\tilde{\gamma}_\alpha^R)^2} \quad (48)$$

Proof. From Eq. (15), the first order condition is

$$\frac{d}{dm}(\Pi_R(m))_\alpha^R = -2\tilde{\beta}_\alpha^Lm + \tilde{D}_\alpha^R - \tilde{\beta}_\alpha^Lw + \tilde{\gamma}_\alpha^R\theta.$$

The second order condition is

$$\frac{d^2}{dm^2}(\Pi_R(m))_\alpha^R = -2\tilde{\beta}_\alpha^L < 0.$$

Thus, for any $\alpha \in (0, 1]$, $(\Pi_R(m))_\alpha^R$ is concave in m .

From Eq. (19), the first order conditions are

$$\frac{\partial}{\partial w} (\Pi_M(w, \theta, m^*))_\alpha^R = -2\tilde{\beta}_\alpha^L w + \tilde{\gamma}_\alpha^R \theta - \tilde{\beta}_\alpha^L m + \tilde{D}_\alpha^R + \tilde{\beta}_\alpha^L \tilde{c}_\alpha^L.$$

$$\frac{\partial}{\partial \theta} (\Pi_M(w, \theta, m^*))_\alpha^R = -2\tilde{I}_\alpha^L \theta + \tilde{\gamma}_\alpha^R w - \tilde{\gamma}_\alpha^R \tilde{c}_\alpha^L.$$

Therefore, the Hessian matrix of $(\Pi_M(w, \theta, m^*))_\alpha^R$ is

$$H = \begin{bmatrix} -2\tilde{\beta}_\alpha^L & \tilde{\gamma}_\alpha^R \\ \tilde{\gamma}_\alpha^R & -2\tilde{I}_\alpha^L \end{bmatrix}.$$

Since $\tilde{\beta}$, \tilde{I} are positive fuzzy variables and $(\tilde{\gamma}_\alpha^R)^2 < 4\tilde{\beta}_\alpha^L \tilde{I}_\alpha^L$, then the Hessian matrix H is negative definite. Thus, for any $\alpha \in (0, 1]$, $(\Pi_M(w, \theta))_\alpha^R$ is jointly concave in w and θ .

We can have Eqs. (46), (47) and (48) by setting the first order conditions above be zero.

Theorem 12 is proved. \square

The optimal optimistic profits of the manufacturer and the retailer can be easily obtained as follows

$$(\Pi_M)_\alpha^R = \frac{\tilde{I}_\alpha^L \left(4\tilde{\beta}_\alpha^L \tilde{I}_\alpha^L - (\tilde{\gamma}_\alpha^R)^2 \right) (\tilde{D}_\alpha^R - \tilde{\beta}_\alpha^L \tilde{c}_\alpha^L)^2}{\left(6\tilde{\beta}_\alpha^L \tilde{I}_\alpha^L - (\tilde{\gamma}_\alpha^R)^2 \right)^2}.$$

$$(\Pi_R)_\alpha^R = \frac{4\tilde{\beta}_\alpha^L (\tilde{I}_\alpha^L)^2 (\tilde{D}_\alpha^R - \tilde{\beta}_\alpha^L \tilde{c}_\alpha^L)^2}{\left(6\tilde{\beta}_\alpha^L \tilde{I}_\alpha^L - (\tilde{\gamma}_\alpha^R)^2 \right)^2}.$$

In the VN game, the minimax chance-constrained programming model of GSC can be formulated as follows

$$\begin{cases} \max_{w, \theta} (\Pi_M(w, \theta))_\alpha^L = ((w - \tilde{c})(\tilde{D} - \tilde{\beta}(m + w) + \tilde{\gamma}\theta) - \tilde{I}\theta^2)_\alpha^L \\ \max_m (\Pi_R(m))_\alpha^L = (m(\tilde{D} - \tilde{\beta}(m + w) + \tilde{\gamma}\theta))_\alpha^L \\ \text{s. t.} \\ \text{Pos}\{w - \tilde{c} < 0\} = 0 \\ \text{Pos}\{\tilde{D} - \tilde{\beta}(m + w) + \tilde{\gamma}\theta < 0\} = 0 \end{cases} \quad (49)$$

Theorem 13. Let $(\Pi_M(w, \theta))_\alpha^L$ and $(\Pi_R(m))_\alpha^L$ be the manufacturer's and retailer's α -pessimistic value of profits. If $(\tilde{\gamma}_\alpha^L)^2 < 4\tilde{\beta}_\alpha^R \tilde{I}_\alpha^R$, $\text{Pos}\{w^{***} - \tilde{c} < 0\} = 0$ and $\text{Pos}\{\tilde{D} - \tilde{\beta}(m^{***} + w^{***}) + \tilde{\gamma}\theta^{***} < 0\} = 0$, then the optimal equilibrium decisions are

$$m^{***} = \frac{2\tilde{I}_\alpha^R (\tilde{D}_\alpha^L - \tilde{\beta}_\alpha^R \tilde{c}_\alpha^R)}{6\tilde{\beta}_\alpha^R \tilde{I}_\alpha^R - (\tilde{\gamma}_\alpha^L)^2}. \quad (50)$$

$$w^{***} = \frac{2\tilde{I}_\alpha^R (\tilde{D}_\alpha^L + 2\tilde{\beta}_\alpha^R \tilde{c}_\alpha^R) - (\tilde{\gamma}_\alpha^L)^2 \tilde{c}_\alpha^R}{6\tilde{\beta}_\alpha^R \tilde{I}_\alpha^R - (\tilde{\gamma}_\alpha^L)^2}. \quad (51)$$

$$\theta^{***} = \frac{\tilde{\gamma}_\alpha^L (\tilde{D}_\alpha^L - \tilde{\beta}_\alpha^R \tilde{c}_\alpha^R)}{6\tilde{\beta}_\alpha^R \tilde{I}_\alpha^R - (\tilde{\gamma}_\alpha^L)^2}. \quad (52)$$

Proof. Similar to Theorem 12. \square

The optimal pessimistic profits of the manufacturer and the retailer can be obtained as follows

$$(\Pi_M)_\alpha^L = \frac{\tilde{I}_\alpha^R \left(4\tilde{\beta}_\alpha^R \tilde{I}_\alpha^R - (\tilde{\gamma}_\alpha^L)^2 \right) (\tilde{D}_\alpha^L - \tilde{\beta}_\alpha^R \tilde{c}_\alpha^R)^2}{\left(6\tilde{\beta}_\alpha^R \tilde{I}_\alpha^R - (\tilde{\gamma}_\alpha^L)^2 \right)^2}.$$

$$(\Pi_R)_\alpha^L = \frac{4\tilde{\beta}_\alpha^R (\tilde{I}_\alpha^R)^2 (\tilde{D}_\alpha^L - \tilde{\beta}_\alpha^R \tilde{c}_\alpha^R)^2}{\left(6\tilde{\beta}_\alpha^R \tilde{I}_\alpha^R - (\tilde{\gamma}_\alpha^L)^2 \right)^2}.$$

Remark 3. If $\alpha=1$, then the parameters \tilde{c} , \tilde{D} , $\tilde{\beta}$, $\tilde{\gamma}$ and \tilde{I} reduce to crisp numbers, then the optimal solutions in Theorems 12 and 13 are

$$m^{***} = \frac{2I(D - \beta c)}{6\beta I - \gamma^2}.$$

$$w^{***} = \frac{2I(D + 2\beta c) - \gamma^2 c}{6\beta I - \gamma^2}.$$

$$\theta^{***} = \frac{\gamma(D - \beta c)}{6\beta I - \gamma^2}.$$

5. Numerical Examples

Because the optimal equilibrium decisions obtained in above Section are in very complicated forms, we have to conduct numerical examples to illustrate the computational process of fuzzy models proposed in this paper. We will also perform the impacts of the fuzziness of parameters $\tilde{\beta}$, $\tilde{\gamma}$ and the confidence level α on these models.

Consider the case where the retailer orders new greening household appliances from the manufacturer, and then he retails them to the end customers. The data used in the numerical examples are estimated from the Chinese household appliances manufacturing industry. These data have been properly handled before being

adopted. We think these data can represent the real condition due to lack of historical data when supply chain participants make their optimal decisions.

The linguistic descriptions and its corresponding triangular fuzzy variables determined by the experiences of experts are shown in Table 1.

Table 1. The triangular fuzzy variables

Parameter	Linguistic description	Fuzzy variable
\tilde{D}	Large (about 1000)	(900, 1000, 1100)
	Small (about 500)	(400, 500, 600)
$\tilde{\beta}$	Very sensitive (about 50)	(45, 50, 55)
	Sensitive (about 40)	(35, 40, 45)
$\tilde{\gamma}$	Very sensitive (about 40)	(35, 40, 45)
	Sensitive (about 30)	(25, 30, 35)
\tilde{I}	Very sensitive (about 30)	(25, 30, 35)
	Sensitive (about 20)	(15, 20, 25)
\tilde{c}	High (about 13)	(11, 13, 15)
	Medium (about 10)	(9, 10, 11)
	Low (about 7)	(6, 7, 8)

5.1. Discussion 1

We first assume that the parameter \tilde{D} is large, (\tilde{D} is about 1000), price elastic coefficient $\tilde{\beta}$, greening level elastic coefficient $\tilde{\gamma}$ and investment elastic coefficient \tilde{I} are all in sensitive level, ($\tilde{\beta}$ is about 40, $\tilde{\gamma}$ is about 30 and \tilde{I} is about 20), the producing cost \tilde{c} is medium, (\tilde{c} is about 10). The expected values, α -optimistic values and α -pessimistic values related to these triangular fuzzy numbers are listed in Table 2.

Table 2. Triangular fuzzy variables, and their expected, α -optimistic and α -pessimistic values

Triangular fuzzy variable	Expected value	α -optimistic value	α -pessimistic value
$\tilde{D} = (900, 1000, 1100)$	100	$1100 - 100\alpha$	$900 + 100\alpha$
$\tilde{\beta} = (35, 40, 45)$	40	$45 - 5\alpha$	$35 + 5\alpha$
$\tilde{\gamma} = (25, 30, 35)$	30	$35 - 5\alpha$	$25 + 5\alpha$
$\tilde{I} = (15, 20, 25)$	20	$25 - 5\alpha$	$15 + 5\alpha$
$\tilde{c} = (9, 10, 11)$	10	$11 - \alpha$	$9 + \alpha$

From Table 2, we have

$$E[\tilde{\beta}\tilde{c}] = \frac{1}{2} \int_0^1 (\tilde{\beta}_\alpha^L \tilde{c}_\alpha^L + \tilde{\beta}_\alpha^R \tilde{c}_\alpha^R) d\alpha = \frac{1205}{3},$$

$$\int_0^1 (\tilde{\gamma}_\alpha^L \tilde{c}_\alpha^R + \tilde{\gamma}_\alpha^R \tilde{c}_\alpha^L) d\alpha = \frac{1790}{3},$$

$$\frac{1}{2} \int_0^1 (\tilde{D}_\alpha^L \tilde{c}_\alpha^R + \tilde{D}_\alpha^R \tilde{c}_\alpha^L) d\alpha = \frac{29900}{3}.$$

The results of the optimal expected solutions under three non-cooperative games are showed in Tables 3 and 4.

Table 3. Expected values of the equilibrium prices under three non-cooperative games

	m	w	p	θ
MS	4.367	18.777	23.144	3.348
RS	7.507	15.264	22.770	3.989
VN	6.159	16.201	22.360	4.692

Table 4. Expected values of the equilibrium profits under three non-cooperative games

	$E[\Pi_R]$	$E[\Pi_M]$	$E[\Pi_{SC}]$
MS	762.976	1386.471	2149.447
RS	1567.936	859.074	2427.010
VN	1517.424	1165.755	2683.179

From Tables 3 and 4, we can find that

(1)The optimal green innovation’s level θ is the highest in the VN case when the actors have equal bargaining power. The MS case provides the lowest level of green innovation this is because under this case the full costs of greening are afforded by manufacturer.

(2)The wholesale price w under the MS case is the highest, followed by VN and then RS cases, this is because that the manufacturer incurs green costs and has the dominant power in pricing of the green product. The profit margin of the retailer m is highest in the RS case this is because in this case manufacturer charges a low wholesale price w . Under MS case, The retail price p is the highest, followed by RS and then VN cases.

(3)The retailer makes the smallest profit under MS case, and the largest under RS case. On the other hand, the manufacturer’s fuzzy expected profits are in the reverse order. It shows that the more power the actor has the more fuzzy expected profits he can derive. That is, the retailer’s fuzzy expected profit is the largest when the retailer is the leader, and the manufacturer’s largest when the manufacturer is the leader in the channel.

(4)The integrated system obtains his largest expected profit in the VN case when no actor is a channel leader. However, the manufacturer or the retailer has an incentive to be a leader this is because as a leader he can obtain more fuzzy expected profit.

(5) Under VN case, the retail price p is the lowest, and the level of green innovation θ is highest. For the customers, this means VN case is a preferred policy.

5.2. Discussion 2

In this subsection, we discuss the impacts of the fuzziness of parameters $\tilde{\beta}$ and $\tilde{\gamma}$ on prices, level of green innovation and expected profits under MS, RS and VN cases. Decreasing the fuzzy degree of the parameters $\tilde{\beta}$, $\tilde{\gamma}$ and observing their impacts. We use the same values of other parameters as in Discussion 1. Tables 5, 6, 7 and 8 give the solutions as follows.

Table 5. The change of the expected value of the prices for three fuzzy models with the fuzzy degree of parameter $\tilde{\beta}$

	$\tilde{\beta}$	m	w	p	θ
MS	(35, 40, 45)	4.367	18.777	23.144	3.348
	(36, 40, 44)	4.368	18.770	23.139	3.343
	(37, 40, 43)	4.370	18.764	23.134	3.338
	(38, 40, 42)	4.371	18.758	23.128	3.332
	(39, 40, 41)	4.372	18.751	23.123	3.327
RS	(35, 40, 45)	7.507	15.264	22.770	3.989
	(36, 40, 44)	7.508	15.257	22.765	3.984
	(37, 40, 43)	7.510	15.249	22.760	3.979
	(38, 40, 42)	7.512	15.242	22.754	3.973
	(39, 40, 41)	7.514	15.235	22.749	3.968
VN	(35, 40, 45)	6.159	16.201	22.360	4.692
	(36, 40, 44)	6.161	16.194	22.355	4.687
	(37, 40, 43)	6.162	16.187	22.349	4.682
	(38, 40, 42)	6.164	16.180	22.344	4.677
	(39, 40, 41)	6.165	16.174	22.339	4.672

Table 6. The change of the expected value of the profits for three fuzzy models with the fuzzy degree of parameter $\tilde{\beta}$

	$\tilde{\beta}$	$E[\Pi_R]$	$E[\Pi_M]$	$E[\Pi_{SC}]$
MS	(35, 40, 45)	762.976	1386.471	2149.447
	(36, 40, 44)	763.346	1378.757	2142.104
	(37, 40, 43)	763.717	1371.045	2134.762
	(38, 40, 42)	764.088	1363.335	2127.423
	(39, 40, 41)	764.459	1355.626	2120.085
RS	(35, 40, 45)	1567.936	859.074	2427.010
	(36, 40, 44)	1568.698	851.104	2419.802
	(37, 40, 43)	1569.459	843.136	2412.595
	(38, 40, 42)	1570.221	835.169	2405.391
	(39, 40, 41)	1570.984	827.204	2398.188
VN	(35, 40, 45)	1517.424	1165.755	2683.179
	(36, 40, 44)	1518.161	1157.934	2676.095
	(37, 40, 43)	1518.898	1150.114	2669.013
	(38, 40, 42)	1519.636	1142.297	2661.933
	(39, 40, 41)	1520.373	1134.481	2654.854

Table 7. The change of the expected value of the prices for three fuzzy models with the fuzzy degree of parameter $\tilde{\gamma}$

	$\tilde{\gamma}$	m	w	p	θ
MS	(25, 30, 35)	4.367	18.777	23.144	3.348
	(26, 30, 34)	4.366	18.773	23.138	3.339
	(27, 30, 33)	4.364	18.769	23.133	3.329
	(28, 30, 32)	4.362	18.766	23.128	3.319
	(29, 30, 31)	4.360	18.762	23.122	3.310
RS	(25, 30, 35)	7.507	15.264	22.770	3.989
	(26, 30, 34)	7.503	15.261	22.765	3.979
	(27, 30, 33)	7.500	15.259	22.760	3.969
	(28, 30, 32)	7.497	15.257	22.754	3.959
	(29, 30, 31)	7.494	15.255	22.749	3.950
VN	(25, 30, 35)	6.159	16.201	22.360	4.692
	(26, 30, 34)	6.157	16.198	22.355	4.682
	(27, 30, 33)	6.154	16.196	22.350	4.672
	(28, 30, 32)	6.151	16.193	22.345	4.662
	(29, 30, 31)	6.149	16.191	22.340	4.651

Table 8. The change of the expected value of the profit for three fuzzy models with the fuzzy degree of parameter $\tilde{\gamma}$

	$\tilde{\gamma}$	$E[\Pi_R]$	$E[\Pi_M]$	$E[\Pi_{SC}]$
MS	(25, 30, 35)	762.976	1386.471	2149.447
	(26, 30, 34)	762.341	1385.356	2147.697
	(27, 30, 33)	761.706	1384.245	2145.951
	(28, 30, 32)	761.071	1383.137	2144.208
	(29, 30, 31)	760.437	1382.032	2142.469
RS	(25, 30, 35)	1567.936	859.074	2427.010
	(26, 30, 34)	1566.631	858.399	2425.030
	(27, 30, 33)	1565.326	857.726	2423.052
	(28, 30, 32)	1564.022	857.057	2421.079
	(29, 30, 31)	1562.718	856.391	2419.109
VN	(25, 30, 35)	1517.424	1165.755	2412.595
	(26, 30, 34)	1516.161	1164.824	2680.985
	(27, 30, 33)	1514.898	1163.896	2678.794
	(28, 30, 32)	1513.636	1162.972	2676.608
	(29, 30, 31)	1512.374	1162.050	2674.425

From Tables 5, 6, 7 and 8, we can find that

(6) When the fuzziness of the parameter $\tilde{\beta}$ decreases, the profit margin m and expected profit of retailer increase slightly in the three game cases. Therefore, the retailer should seek as low fuzziness of parameter $\tilde{\beta}$ as possible.

(7) The wholesale price w , retail price p , level of green innovation θ , expected profit for manufacturer and whole

supply chain drop slightly in three cases, as the fuzziness of parameter $\tilde{\beta}$ falls.

(8)When the fuzziness of the parameter $\tilde{\gamma}$ decreases, the profit margin m , prices w and p , level of green innovation θ and profits for supply chain members will all increase.

5.3. Discussion 3

Thirdly, we present the results of the α -optimistic values and α -pessimistic values under three decentralized channel decisions shown in Tables 9, 10, 11 and 12, respectively. In Tables 9, 10, 11 and 12, the 2th, 8th and 14th rows show the optimal equilibrium decisions under three cases in crisp environment at $\alpha=1$.

Table 9. α -optimistic value of the equilibrium prices under three cases with respect to α

	α	m	w	p	θ
MS	1.00	4.364	18.727	23.091	3.273
	0.95	4.487	18.924	23.411	3.436
	0.90	4.614	19.128	23.742	3.608
	0.85	4.745	19.339	24.084	3.790
	0.80	4.879	19.559	24.438	3.981
	0.75	5.019	19.787	24.806	4.182
	RS	1.00	7.500	15.217	22.717
0.95		7.667	15.360	23.026	4.142
0.90		7.835	15.512	23.347	4.389
0.85		8.005	15.675	23.680	4.652
0.80		8.177	15.850	24.027	4.936
0.75		8.351	16.038	24.389	5.240
VN		1.00	6.154	16.154	22.308
	0.95	6.343	16.293	22.637	4.858
	0.90	6.540	16.440	22.979	5.114
	0.85	6.743	16.593	23.336	5.386
	0.80	6.954	16.754	23.709	5.673
	0.75	7.174	16.924	24.099	5.979

Table 10. α -optimistic value of the equilibrium profits under three cases with respect to α

	α	$(\Pi_R)_\alpha^R$	$(\Pi_M)_\alpha^R$	$(\Pi_{SC})_\alpha^R$
MS	1.00	761.653	1309.091	2070.744
	0.95	800.296	1367.387	2167.683
	0.90	840.898	1427.903	2268.801
	0.85	883.602	1490.754	2374.357
	0.80	928.565	1556.068	2484.633
	0.75	975.963	1623.981	2599.943
	RS	1.00	1565.217	782.609
0.95		1648.542	824.271	2472.713
0.90		1736.704	868.352	2605.056
0.85		1830.141	915.071	2745.212
0.80		1929.353	964.677	2894.029
0.75		2034.907	1017.453	3052.360
VN		1.00	1514.793	1088.757
	0.95	1599.432	1133.361	2732.793
	0.90	1689.237	1179.204	2868.440
	0.85	1784.665	1226.302	3010.967
	0.80	1886.233	1274.671	3160.903
	0.75	1994.521	1324.319	3318.840

Table 11. α -pessimistic value of the equilibrium prices under three cases with respect to α

	α	m	w	p	θ
MS	1.00	4.364	18.727	23.091	3.273
	0.95	4.244	18.537	22.781	3.117
	0.90	4.127	18.354	22.480	2.969
	0.85	4.103	18.176	22.189	2.828
	0.80	3.902	18.004	21.906	2.694
	0.75	3.794	17.838	21.631	2.566
	RS	1.00	7.500	15.217	22.717
0.95		7.335	15.084	22.420	3.698
0.90		7.172	14.960	22.132	3.496
0.85		7.011	14.842	21.853	3.307
0.80		6.851	14.732	21.584	3.130
0.75		6.693	14.629	21.322	2.962
VN		1.00	6.154	16.154	22.308
	0.95	5.971	16.021	21.992	4.386
	0.90	5.794	15.894	21.687	4.169
	0.85	5.622	15.772	21.394	3.963
	0.80	5.456	15.656	21.111	3.767
	0.75	5.294	15.544	20.838	3.581

Table 12. α -pessimistic value of the equilibrium profits under three cases with respect to α

	α	$(\Pi_R)_\alpha^L$	$(\Pi_M)_\alpha^L$	$(\Pi_{SC})_\alpha^L$
MS	1.00	761.653	1309.091	2070.744
	0.95	724.840	1252.907	1977.747
	0.90	689.740	1198.737	1888.477
	0.85	656.246	1146.490	1802.735
	0.80	624.261	1096.081	1720.342
	0.75	593.696	1047.434	1641.129
	RS	1.00	1565.217	782.609
0.95		1486.341	743.171	2229.512
0.90		1411.570	705.785	2117.355
0.85		1340.596	670.298	2010.895
0.80		1273.147	636.574	1909.721
0.75		1208.977	604.488	1813.465
VN		1.00	1514.793	1088.757
	0.95	1434.910	1045.374	2480.328
	0.90	1359.416	1003.189	2362.605
	0.85	1287.985	962.181	2250.165
	0.80	1220.320	922.327	2142.646
	0.75	1156.155	883.604	2039.759

From Tables 9, 10, 11 and 12, we can find that

(9) In the three different fuzzy models, the α -optimistic values of profit margin, prices, level of green innovation and profits for supply chain participants increase, as the confidence level α decreases. That is to say, if the confidence level of the supply chain participant's profits $\alpha=1$, then the supply chain participants derive their smallest profits. It means that the less responsible of supply chain participants to risk, the lower profits will be when they are risk preferable.

(10) In the three different fuzzy models, the α -pessimistic values of profit margin, prices, greening level and profits for supply chain actors increase, as the confidence level α increases. That is to say, if the confidence level of the supply chain participant's profits

$\alpha=1$, then the supply chain participants derive their largest profits. It means that the more responsible of supply chain participants to risk, the larger profits will be when they are risk averse.

In practice, the supply chain participants can alter the value of the parameter α to obtain the different equilibrium decisions under the different level of the supply chain participant's profits. The equilibrium decision reflects the different risk attitude of the supply chain participant to market demand uncertainty and different prediction of possibility level.

(11) It is interesting to compare the solutions of this paper with the work of Ghosh and Shah³. When $\alpha=1$, the solutions in this study will be the conventional solutions according to the method of Ghosh and Shah³. In Tables 9, 10, 11 and 12, the 2th, 8th and 14th rows show the optimal equilibrium decisions under three cases in crisp environment. Compared these optimal equilibrium decisions in crisp environment to those in fuzzy decision making environment showed in Table 3, we observe that our profit margin, wholesale price, retail price and level of green innovation are all higher than those in crisp results. The manufacturer's profit is lower, while the retailer's profit and supply chain system's profit are higher when they face fuzzy uncertainty. It means that the retailer could benefit from the fuzzy environment, while the manufacturer could suffer from this environment.

6. Conclusions

This paper deals with the coordination strategy in a m In this paper, we considered three different fuzzy models in green supply chain, in which supply chain participants pursued three different power balance scenarios. For each model, we provided the optimal equilibrium decisions of the manufacturer and the retailer when they were risk neutral, risk preferable and risk averse. We also found that under certain circumstances, the formulated fuzzy supply chain models can degenerate into crisp models. Besides, we show that the different power structures, the confidence level of the supply chain participant's profits and fuzzy degree of parameters affect the final optimal solutions.

The current fuzzy green supply chain models have some limitations. The fuzzy demand for supply chain is assumed as a linear function. Another limitation is that we only consider one manufacturer and one retailer in GSC. Further research can focus on analyzing some more

complicated greening policies in which some other kinds of demand functions or multiple supply chain participants exist. Still, we will develop the channel coordination in GSC under a fuzzy decision environment.

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