Estimation of Minimum Number of Urban Taxis

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Abstract. Our basic model has two parts: to find the relationship between the waiting time and the rate of the vacant taxis arrive, and to find the relationship between the passenger flow volume and the rate of the taxi arriving in the area. In an extended model, we take the Scheduling efficiency into consideration. Finally, we discuss the different methods of taxi ride prices.

1. Introduction

Taxi as public transport supplement, plays

A significant role in the transport system. With its convenient, comfortable, fast features, there are parts of populations relies on taxicabs. Nevertheless, some customers are not so pleased with the quality of services by the taxi companies [1, 2]. Passengers always wait for a long time to take a taxi, while the taxis often drive with no passenger. Taxi transportation demand and supply is a pair of interrelated and indivisible concept. Therefore, a balance of supply and demand taxi industry is of importance for the optimized allocation of resources. On the other hand, there are many other factors related to the waiting time. We should take all of these into consideration to improve the situation.

In addition, the price of services also causes the complaints at some time. So drafting the method of taxi ride prices is important in a way. The major methods are zone-to-zone prices and meter-based prices nowadays.

The unlicensed companies currently control about 1/7 of all cabs in the city. The governors need to attach great importance to the regulations against unlicensed companies. However, the consequences of strengthening the regulations against unlicensed companies will be a mystery that needs to be explored.

2. Assumptions

We make the following basic assumptions in order to simplify the problem. Each of our assumptions is justified and is consistent.

1. We assume that passenger flow volume is only associated with the location and different times of day. We ignore the impact of the tourist season to simplify our model. since our research is needed in the day of the taxi number, so you can ignore this.

2. We consider is a taxi one-on-one service, regardless of the carpool. Because carpooling number is difficult to determine, and is secondary for model. we neglect this effect In order to obtain better explanation of the model for the taxi number required.

3. We assume that arrived in a certain area of the probability of a taxi obeying Poisson distribution. We think that "taxi arrived in an area" this incident is random, and the possibility of the Incidence is associated with the current passenger flow volume.

4. We assume that the taxi people are obeying that first come first served, so there is no cut in line caused by congestion.

3. Basic Model

3.1 Model Overview.

The model has two parts: to find the relationship between the waiting time and the rate vacant taxis arrive in the area, and to find how the passenger flow volume varies.

For the first, we focus on the rate of the taxi arriving in the area and the time. The rate is related to the different times of the day. We assume it is a piecewise constant function. So, we can obtain the arriving interval of vacant taxis follows an exponential distribution.

Next, we calculate the mathematical expectation of single passenger's waiting time. We take the arriving interval of vacant taxis into consideration and make a reasonable assumption. Then we have the probability distribution of waiting time. Still, we can get the mathematical expectation of the waiting time.

We estimate the rate at interval j by using maximum likelihood estimation.

To derive an expression for the total number of cabs needed, we introduce the vacancy rate. We analyzed the change of the passenger flow volume varies. Then we can get the total number of cabs needed based on the change and the vacancy.

The model parameters are shown in Table 1.

Table 1. Model parameters

Parameter	Meaning
t_w	Waiting time (the time a passenger needs to wait for a taxi service)
t	different times of the day
λ	the rate of the taxi arriving in the area
μ	the Sample mean of the time intervals
$f(t_0)$	the Probability density of passengers turn up
Δt	arriving interval of vacant taxis
T	the total number of cabs needed to serve the area
p_i	the vacancy rate at different times of the day
k_i	the passenger flow volume
S	Customer satisfaction
D	Scheduling efficiency
$E(t_w)$	average waiting time

3.2 Step One.

We assume the numbers of taxicabs arriving in the area follow a Poisson distribution. That is to say, the time intervals that connected taxicabs arrive in the area follow an exponential distribution. However, it is not suitable. The reality of the taxi operation is related to the time. From the Poisson distribution, the numbers of empty taxicabs will become lager as the time getting later. Apparently, it is not to the fact.

Recent reports show that the quantities of the taxicabs arriving in the area follow a non-time-homogeneous. The rate (λ) of the taxi arriving in the area is different for distinguish of time. It is changed over time (t), so we derive a function for it.

$$\lambda = f(t) \tag{1}$$

We assume it is a piecewise constant function. In a certain period of time, the rate is constant. That is:

$$f(t) = \lambda_i, t_i \le t \le t_{i+1} \tag{2}$$

After this period of time, the arriving interval of vacant taxis follows an exponential distribution.

$$p_i(\Delta t) = \lambda_i e^{-\lambda_i \Delta t} \tag{3}$$

3.3 Step Two.

We can calculate the mathematical expectation of single passenger's waiting time based on the probability distribution of the arriving interval of vacant taxis. Taking the time interval j of the day as

an example, we assume the arriving interval of vacant taxis follow an exponential distribution. Now, we assume the last taxi pass by at time t and the passenger come taking $f(t_0)$ as probability density at time $t + t_0$.

We can have the probability distribution of waiting time (t_w).

$$p(t_w) = \frac{\int f(t_0)\lambda_j e^{-\lambda_j(t_w+t_0)}dt_0}{\iint f(t_0)\lambda_j e^{-\lambda_j(t_w+t_0)}dt_w t_0} = \frac{\lambda_j e^{-\lambda_j t_w}\int f(t_0)\lambda_j e^{-\lambda_j t_0}dt_0}{\int f(t_0)\lambda_j e^{-\lambda_j t_0}dt_0 \int \lambda_j e^{-\lambda_j t_w}dt_w} = \lambda_j e^{-\lambda_j t_w} \tag{4}$$

Therefore, the probability distribution of the waiting time must follow an exponential distribution. The mathematical expectation of the waiting time is:

$$E(t_w) = \int p(t_w)dt_w = \int \lambda_j e^{-\lambda_j t_w} t_w dt_w = \frac{1}{\lambda_j} \tag{5}$$

The rate (λ_j) of the time interval j of the day is unknown, we can obtain it based on the historical data. We estimate it by using maximum likelihood estimation.

$$\hat{\lambda} = \arg\max \hat{l}(\lambda \mid \Delta t_1,...,\Delta t_n) = \arg\max_\lambda \lambda^n e^{-\lambda \sum \Delta t_i} = \sum t_i / n = \frac{1}{\mu} \tag{6}$$

3.4 Step Three.

We establish an objective function based on the situation. The total number (T) of cabs needed to serve the "Greater Mythical" area is related to the passenger flow volume (k_i) and the vacancy rate (p_i). We have

$$T = \frac{k_i}{p_i} \tag{7}$$

3.5 Step Four.

We analyzed the relationship between the passenger flow volume and the rate of the taxi arriving in the area, we have

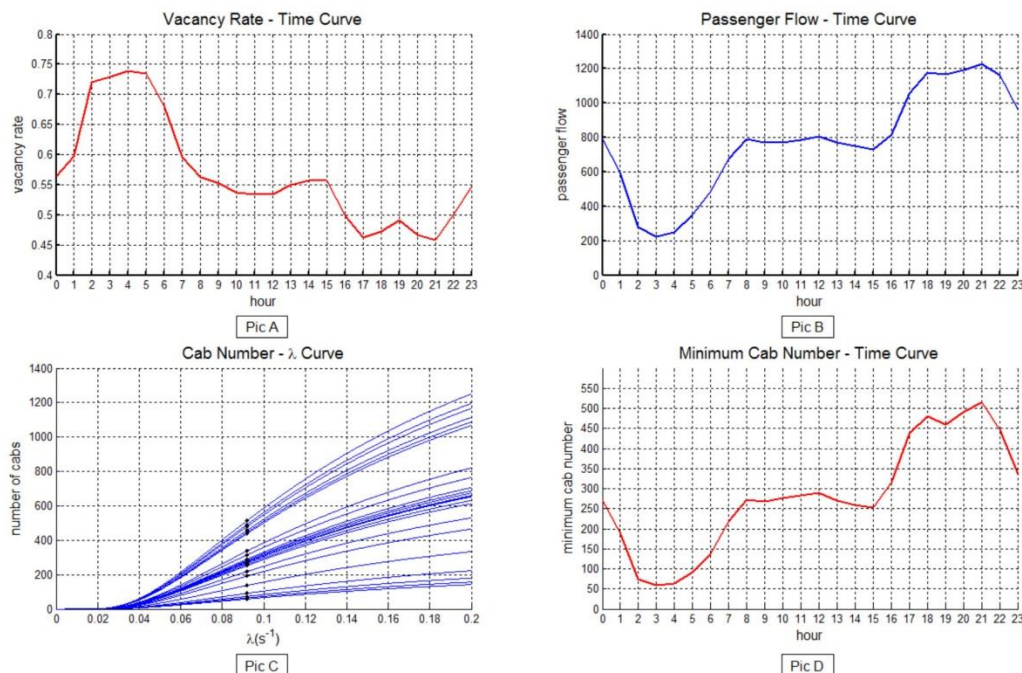$$k_i = e^{-a\frac{1}{\lambda_j}} k_0 \tag{8}$$

Fig. 1 Result of the main parameters at different times

4. Numerical Computation

We get data from Archiving Wireless Data—epfl /mobility dataset (v. 2009-02-24). This dataset contains mobility traces of taxi cabs in San Francisco, USA. It contains GPA coordinates of approximately 500 taxies collected over 30 days in the San Francisco Bay Area. This archive contains file '_cabs.txt' with the list of all cabs and for each cab its mobility trace in a separate ASCII file, e.g. 'new_abboip.txt'. The format of each mobility trace file is the following – each line contains [latitude, longitude, occupancy, time], e.g.: [37.75134 -122.39488 0 1213084687], where latitude and longitude are in decimal degrees, occupancy shows if a cab has a fare (1 = occupied, 0 = free) and time is in UNIX epoch format.

We get the GPS trace data from the file corresponding to each taxi. Then we convert UNIX time in San Francisco. The statics are collected by the hour.

Will be recorded in the number of records in the empty, the number of records will be included in the all, the number of cars will be included in the flow. The statistics on all vehicles, the total load records at all hours, record the total, boarding times (i.e., the total flow of passengers). By dividing the number of hours of empty records into the corresponding total number of records that have to be within the hours of the taxi empty. By the time of waiting for the car two constraints (Get the constraint on the taxi arrival rate and the range of the taxi arrival rate. The hour for the number of the taxi and the taxi arrived at the speed of image and according to taxi to arrive at a range of rates to obtain the minimum value of the hour for the number of the taxi, for each hour for the minimum number of taxis in the maximum obtain the solution.

5. Conclusion

We can find the waiting time is related to the rate of the vacant arriving in the area. The correlation of them is negative. Still, the mathematical expectation of the waiting time is inversely proportional to the rate of the taxi arriving.

The rate is constant in a certain period of time. At the end of this period, the rate is changed over time.

We establish the relationship between the passenger flow volume and the rate of the taxi arriving.

The total number of cabs is associated with the passenger flow volume and the vacancy rate. Then we can obtain the total number of cabs needed.

According to the statistics, we find the vacancy rate is from 40 percent to 70 percent. And we get the total number of cabs needed is 514.

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