

Magnetic Field Analysis On A Cylindrical Matrix Of HGMS Through Conform Mapping

*Mei Yuan**, *Dengyuan Mo[†]*, *Huiqing Peng**, *Di Wu**,

**School of Resources and Environmental Engineering, Wuhan University of technology, Wuhan 430070, China*

[†]Wuhan YuanHaiBoChuang Science and Technology Co., Ltd., Wuhan 430070, China

Keywords: Cylindrical Matrix, High-Gradient Magnetic Separation, Conform Mapping, Schwarz-Christoffel mapping.

Abstract

Magnetic matrix plays a decisive role in the capture efficiency of high-gradient magnetic separation (HGMS) system. This paper took a parallel arranged cylindrical matrix for example, and put forward using conform mapping to calculate the magnetic field distribution in the matrix. After established the simplified two-dimensional analytical model, this paper promoted the procedure of magnetic field analysis through conform mapping, and then analyzed the magnetic field strength and magnetic traction force that derived from this method. The results were consistent with previous studies that mainly adopted Finite Element Method. The method also can be used to determine hydrodynamic drag force that affects the capture efficiency mainly. The study will contribute to further explore the capture mechanism of magnetic particles in high-gradient magnetic separation.

1 Introduction

Magnetic matrix is the core component of high-gradient magnetic separation (HGMS) system and plays a decisive role in its operation [1]. Special cross-section matrix may have better magnetic characteristics and present better performance. When analysing the magnetic particle motion trajectory in HGMS, the particle would be determined as a result of the competition of the main two forces, magnetic force (\vec{F}_m) and hydrodynamic drag force (\vec{F}_d) [2]. And the balance of forces which describe the particle motion is given by :

$$\vec{F}_i = \vec{F}_m + \vec{F}_d + \vec{F}_g \quad (1)$$

Where, \vec{F}_i is the inertial force, \vec{F}_g is the buoyant Force (usually it was neglected when the particle small enough). In order to trace the trajectory of particles and find how magnetic matrix affect the capture efficiency of HGMS, magnetic field analysis on the matrix plays an important role. Formally, it was solved according to the particles trajectory equation and aggregation theory on single-wire mainly, or indirect calculated by Finite Element Modeling (FEM) method. Eisentrger (2014) has calculated the magnetic force (\vec{F}_m) exerted on a particle due to a single magnetic cylinder, which was expressed in plane-polar coordinates [3]. Abbasov (2016)

solved the magnetic force exerted on the particle through components expressed in the cylindrical coordinates [2]. In the above, mass magnetization of single wire must be solved firstly. Hayashi (2010) has simulated the magnetic field and the fluid using FEM and calculated the particle trajectory by solving the equation for a rectangular wire shape [4]. Lindner (2013) derived magnetic forces from FEM and first embedded it in a computational fluid dynamics simulation, the simulation showed the highest deposit at the end of the wire and yield an acceptable agreement with experiments. [5]. Smythe (1950) investigated the electric field around an elliptic dielectric and derived the complex potential functions of the electric field [6]. Based on the functions derived by him, Zheng (2016) obtained

the magnetic force acting on a particle in the elliptic coordinate system [1]. Since magnetic force (\vec{F}_m) is a vector, and strength of magnetic is interactions within the matrices, the theory focus on single-wire and FEM method may not be enough to reveal the particle capture mechanism. Vector finite element method (VFEM) maybe is a choice for the solution.

In this paper, conform mapping, especially Schwarz-Christoffel mapping (abbr. SC mapping) is proposed to analyse the multi-wire magnetic field such as the cylindrical matrix. Since conformal mapping keeps orthogonal invariance over the conservation, it can simplify the solution of boundary value problems. The most famous application of conformal mapping is to Laplace's equation, which includes standard problems in theory and applications on an electromagnetic field (EMF) [7]. Connell (2008) has compared advantages and disadvantages between SC mapping method and boundary element method (BEM) in motor design, and indicated that SC mapping tends to help 2-dimension parameter analysis quickly [8].

2 Conformal mapping and its application in magnetic field analysis

2.1 Content of SC Mapping

The SC mapping and its variations yield formulas from standard regions to the interiors or exteriors of possibly unbounded polygons. Set a_i ($i \geq 3, i = 1, 2, 3, \dots, i, \dots, n-1, n$) as a series of points that distributed on the real axis of the upper-half plane in complex plane ε , and n -sided ($n \geq 3$) polygon P is

a single-connected domain in complex plane Z with vertices and inner angles as z_i and a_i respectively, here $\theta_i \in (0, \pi) \cup (\pi, 2\pi)$. Then, an univalent mapping described in equation(5) can shift the upper-half of plane \mathcal{E} into the polygon p , correspondingly a_i were converted as z_i .

$$f(z) = K \int_0^\varepsilon \prod_{i=1}^{n-1} (x - a_i)^{\frac{\theta_i}{\pi} - 1} dx + C. \tag{2}$$

(Where a_i are the preimages of the vertices z_i , K and C are undetermined constants.)

Equation(2) doesn't have a fixed expression, which let the evaluation of integrals to be difficulty, especially for complex polygons. The Schwarz-Christoffel Toolbox for MATLAB may serve to increase awareness that the SC mapping is a practical reality [7].

2.2 The application of Conformal Mapping in EMF

Generally, the magnetic force exerted on the particle is expressed in an expanded form as [9]:

$$\vec{F}_m = \mu_0 \kappa V H \nabla H \tag{3}$$

Where, μ_0 is magnetic constant ($= 4 \pi \times 10^{-7} \text{H/m}$), V and κ refer to the volume and magnetic susceptibility of the particle respectively, H and ∇H are the magnetic field strength and gradient of the magnetic field. All of the values require proper substantiation.

Formally, Laplace's equation is expressed as equation(4).

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0 \tag{4}$$

In EMF, harmonic functions $\Phi(x,y)$ can be understood as electric (or magnetic) potential at point (x,y) , its partial derivatives $\partial\phi$ used to represent the magnetic field intensity H of the electric(or magnetic) field. And the second order partial derivative $\partial^2\phi$ represent the gradient ∇H of which [10]. Thus, electric (magnetic) potential on the border of certain area, or normal component of electric (magnetic) field intensity and electric (magnetic) potential outside the area can be determined. Once H and ∇H have been determined ,the magnetic force \vec{F}_m could be derived.

Besides of the above, curves defined by equation(4) expressed as equipotential line.

3 Procedure of magnetic field analysis on the cylindrical matrix

3.1 An Two-dimensional Model of the Cylindrical Matrix

Due to the advantages of high reliability, easy combination and seldom blockage, the cylindrical matrix is commonly used in most HGMS system. Its diameter, configuration mode and filling rate are the main factors influencing separation efficiency of HGMS. This paper focus on a typical cylindrical matrix, also called as parallel arranged rod array. Fig.1

describe a two-dimensional simplified model of the matrix, which is a large square array of long parallel cylinders of radius R with a constant magnetization perpendicular to their axes. The arrow points the magnetization of the cylinders by the action of a strong, uniform outer magnetic field. The smallest distance between cylinders in the x- and y-direction is denoted by $2d$, so that the period of the array in both directions is $2L = 2(R + d)$. Using symmetry across the y-axis, we may reduce the computational domain to half the periodic cell, i.e. $0 \leq x \leq L, -L \leq y \leq L$.

To facilitate compares and analyzes with others, this paper set parameters of the Cylindrical Matrix for: $R = 25\text{nm}$, and $2d = 70\text{nm}$, the background magnetic field intensity at $2.3 \times 10^8 \text{Am}^{-1}$ [11].

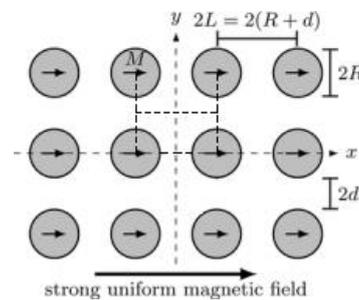


Fig. 1 an two-dimensional model of the Cylindrical Matrix. The dotted box indicates the computational domain

Since symmetry principle has been widely used in EMF, so the internal magnetic field can be divided into a series of small units as the dotted box which possessed the same properties. And the analysis on single unit may reveal the whole magnetic field distribution of the cylindrical matrix. This model captures the important physics of the system and represents a worst-case scenario.

3.2 Process of the Conform Mapping

When conformal mapping was applied to solve the EMF boundary problem, the process contains two steps as :

(a) mapping the given area into a simpler area (e.g. rectangle), then deduce the harmonic functions Ψ that meet the "transformation" boundary conditions.

(b) mapping the harmonic functions Ψ back to the original area inversely, and derivate the harmonic functions $\Phi(x,y)$ that satisfy specific boundary condition.

The process on a single unit can be divided into three steps. For the first, it must determine the oriental field, boundary conditions on plane Z , construct the initial polygon P . It may contain piecewise smooth curves, which may be replaced by a connection of tangents on certain points.

Secondly, SC mapping should be performed, which transform the vertices of the initial polygon P to be a series of points on the real axis of ε -plane. And polygon P was mapped as the upper-half plane of it.

At next, Jacobi conformal mapping was adopted to shift the upper-half plane of ε -plane into the inter of rectangle

P' on δ -plane. And points on the real axis of ε -plane was mapping onto the edge of rectangle P' . When rectangle P' was meshed as a network of squares, the magnetic lines of flux and equipotential lines can be represent straightforward as the orthogonal lines inside it.

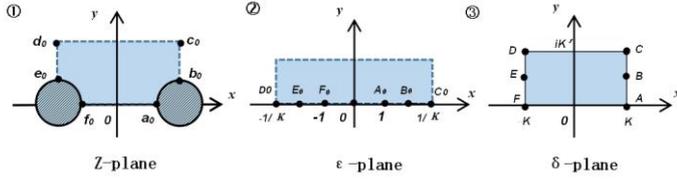


Fig. 2 procedure of the conform mapping on cylindrical matrix.

After a series of inverse operations, each cross-over points of the meshing grid could be mapped as the intersection of flux and equipotential lines inside the magnetic field. The accuracy of the calculation is determined by the density of the mesh grid. The inverse operations obey the rulers described in the following.

3.3 Magnetic Field Analysis on the Cylindrical Matrix

Set the magnetic potential function on δ -plane as W , which is easy to be acquired. According to definition of complex potential function, magnetic field strength H within the initial polygon can be derived as :

$$H = \frac{dW}{dZ} = \frac{dW}{d\delta} \cdot \frac{d\delta}{d\varepsilon} \cdot \frac{d\varepsilon}{dZ} \quad (5)$$

Z, δ , and ε refer to the coordinate of the point on different planes.

The gradient of magnetic field, viz, maximum speed variation of magnetic field strength, can be derived by the following as :

$$\nabla H = \frac{\partial H}{\partial r} = \frac{d^2W}{dZ^2} = \frac{d}{d\varepsilon} \left(\frac{dW}{dZ} \right) \quad (6)$$

r is directing to the variation of magnetic field.

So, the magnetic traction force \vec{F} can be exerted by :

$$\vec{F} = H \frac{\partial H}{\partial r} = \frac{dW}{dZ} \cdot \frac{d^2W}{dZ^2} \quad (7)$$

and magnetic force \vec{F}_m can be deduced as :

$$\vec{F}_m = \mu_0 K V \cdot \frac{dW}{dZ} \cdot \frac{d^2W}{dZ^2} \quad (8)$$

The direction of \vec{F}_m , just the same as \vec{F} , points to the maximum speed variation of magnetic field strength, the value is:

$$\text{Arg} \left(H \frac{\partial H}{\partial r} \right) = \text{Arg} \frac{dW}{dZ} - \text{Arg} \frac{d^2W}{dZ^2} \quad (9)$$

Actually, above process is rather complex. Nevertheless, functions in SC Toolkit can be invoked to solve the problem, which runs under Matlab environment and gives a relatively commonly use about the calculation.

4 Result and discussion

4.1 Magnetic Field Strength

The unhomogenized magnetic field in the single unit of cylindrical matrix was shown in Fig.3 as a contour map. The result was consistent with the solution drawn from FEM [11].

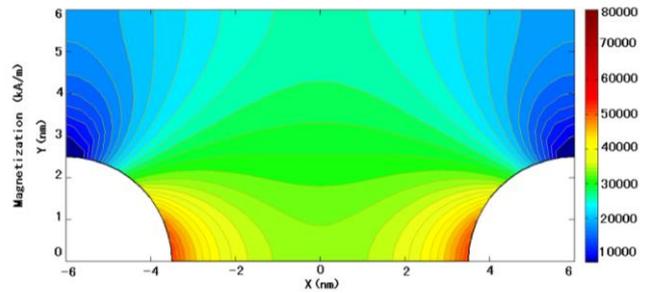


Fig. 3 a contour map of magnetic field strength.

Fig. 3 indicates the distribution of magnetic field strength is symmetrical with the y-axis of the polygon p . The colors indicate the field strength; there is a maximum in the horizontal field direction and a minimum perpendicular to the field direction.

4.2 Magnetic Traction Force

For demonstration purposes, this paper only finds the solution of magnetic traction force in a single unit, which was denoted in Fig. 4. Within it, the value of magnetic traction force was represented as a hypsometric map, The resulting direction of the force is shown by the arrows on certain points.

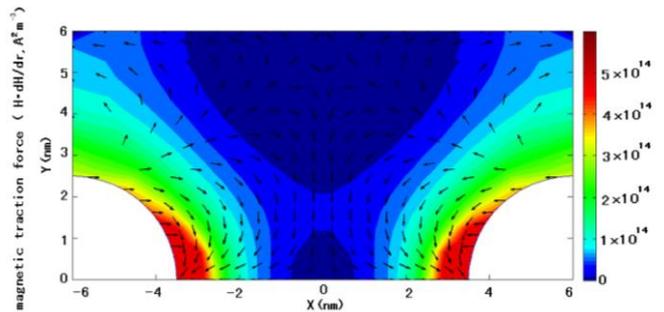


Fig. 4 the contour and vectors of magnetic traction force.

The figure indicates that magnetic traction force also is symmetrical with the y-axis of the polygon p . It reached maximum at the top of the rod (red part) and weakened towards the middle of the field. Part of the arrows point to the top and others point to the edge, which split the field as capture area and non-capture area. And it reveals that the effective capture area just in a small patch around the top. Since higher particle concentrations would cause chains to form that would be effective capture area may large than the result. This conclusion also verifies the agglomeration and simulation of particles on a wire under the magnetic field, which raised by Lindner [5].

4. Conclusion

When conform mapping was applied to analysis the magnetic field of the cylindrical atrix inside High-Gradient Magnetic Separator, the magnetic field strength and the magnetic force acting in the field could be determined precisely. Compared with Fem,it decreases the modelling process, reduces the computing time, and enhance the accuracy of solution. Further, when the process is encapsulated in mathematical software and possess with graphical user interface, the magnetic field analysis on multi wires of HGMS with different shaped may be an easy operation and would offer a good user experience.

Meanwhile,the method can also be used to analyze the problem on fluid fields, another main force that affects the capture efficiency ,i.e. hydrodynamic drag force(\vec{F}_d) could also be determined. Once the inertial force \vec{F}_i was defined, the separation efficiency of wires on various particals could be controlled and optimized.

References

- [1]Zheng X , Wang Y, Lu D. Effect of Matrix Shape on the Capture of Fine Weakly Magnetic Minerals in High- Gradient Magnetic Separation. *IEEE T. Magn.* , 52 (2016), 1-11.
- [2]Abbasov T , Ggebakan V, Karada T. Particle capture modeling for an axial magnetic filter with a bounded non-Newtonian flow field .*Powder Technol.*, 291(2016), 223-228.
- [3]Eisentrger A, Vella D, Griffiths I M .Particle capture efficiency in a multi-wire model for high gradient magnetic separation .*Appl. Phys. Lett.*,105(2014),033508 - 033508-4.
- [4]Hayashi S, Mishima F, Akiyama Y, Nishijima S. Development of high gradient magnetic separation system for a highly viscous fluid. *IEEE T.APPL. SUPERCON.*, 20(2010), 945948.
- [5]Lindner J, Menzel K, Nirschl H . Simulation of magnetic suspensions for HGMS using CFD, FEM and DEM modeling. *Comput. & Chem. Eng.*, 54(2013),111-121.
- [6]Smythe W R.Static and Dynamic Electricity, 2nd ed. New York, NY,USA: The Maple Press Company, 1950.
- [7]Driscoll T A and Trefethen L N.SchwarzChristoffel Mapping. Cambridge Monographs, vol. 8. Cambridge University Press, Cambridge, 2002.
- [8]Connell T C O. An Investigation of Boundary-based Field Analysis Meth-ods for Electric Machines: The Schwarz-Christoffel and Boundary Element Methods. Ph.D. thesis,University of Illinois at Urbana- Champaign, 2008.
- [9]Baik S K ,Ha D W,Kwon J M,Lee Y J,Ko R K.Magnetic force on a magnetic particle within a high gradient magnetic separator.*Physica C: Superconductivity*, 48(2013), 33-337.
- [10]Saff E B and Snider A D . Fundamentals of Complex Analysis with Applications to Engineering and Science (Third Edition). Prentice Hall, 2003.
- [11]Qiu W, Zheng Y, Haralampides K A .Study on a novel POM-based magnetic photocatalyst: Photocatalytic degradation and magnetic separation. *Chem.Eng. J.*,125 (2007) , 165-176.
- [12]Duschl-Maass M, Maass W, Hoffmann H, Friedlaender F J. Particle Accumulation on a Single Wire in Transverse HGMS Configuration. *Appl. Phys. A* , 42(1987),117-124.
- [13]Sandulyaka A, Sandulyak A, Belgacem F B M, Kiselev D. Special solu-tions for magnetic separation problems using force and energy conditions for ferro-particles capture. *J. Magn. Magn. Mater.* ,401(2016),902-905.
- [14]Zheng X , Wang Y, Lu D. Study on capture radius and efficiency of ne weakly magnetic minerals in high gradient magnetic eld. *Miner. Eng.*, 74(2015),79-85.
- [15]Chen F, Smith K A, Hatton T Alan. A dynamic buildup growth model for magnetic particle accumulation on single wires in high-gradient magnetic separation. *AIChE Journal*, 58(2012),2865-2874.
- [16]Zheng X , Wang Y, Lu D. A realistic description of influence of the magnetic field strength on high gradient magnetic separation. *Miner. Eng.* ,79(2015), 94-101.
- [17]Driscoll T A, The SchwarzChristoffel toolbox for MATLAB.<http://www.math.udel.edu/driscoll/SC/>