Research on Merging Pattern after Toll Based on Simulation

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Abstract. We establish a fan-in system (including combination area design, merging pattern and so on) which can best prevent accidents, improve throughput and minimize the cost. To achieve our goal, first cellular automaton is used to stimulate the actual toll plaza and establish some kinds of combination area with different shapes and sizes so that we can estimate the cost of construction. In this process, we also use Nagel-Schreckenberg (NS) vehicle-following model and select 3 merging patterns to determine the movements of our "cars". By doing this, we get sufficient data from simulation, such as average velocity of moving cars, sharp braking frequency, throughput, etc. Finally, we establish social cost analysis model to obtain a relatively objective evaluation. We calculate the economic cost of accident per year, the time cost per year measured by money and the construction fee of the whole plaza shared by each year. Thus, we can compare different solutions and reach the optimization. The best solution is the symmetrical narrowing shape, Merging as soon as possible. To test our solution, we put it in different conditions including various traffic density, self-driving cars. We also find out the divergent influence of different kinds of tollbooth. One more interesting results is that by applying basic optimizing model and some simple simulations. we can at last find the best proportion of ECT, AT and MTC under certain circumstances.

Introduction

Turnpike is widely used throughout the world, which is an efficient way to compensate for highway construction and investment of maintenance. Nowadays, many governments use turnpike to balance financial revenue and expenditure. Management of transportation demand is carried out by price leverage in Singapore and London, to make the best of finite road resources and maximize economic benefit. However, traditional Manual Toll Collection (MTC) has many disadvantages such as large occupation of land, car owners' waste of time, more exhaust emissions and so on. People are finding ways to solve these problems such as building Automated Tollbooths (AT), Electronic Toll Collection (ETC). Design of toll plaza, including shape, size and merging pattern, is also closely connected with reducing accident rate, increasing traffic capacity and minimizing road construction fee.

Assumptions in this article

1. The number of tollbooths is 12 and the lane after fan-in process is 4.
2. The road is flat so that vehicles will not brake because of barriers on the ground.
3. Every tollbooth can only deal with one vehicle at one time.
4. Every driver wants to pass through the combination area as soon as possible under the safe condition.
5. Traffic in highway satisfies the assumption of Poisson distribution in a short period. [1]
6. Every vehicle has the same size.

7. Ignore effects by weather condition.
8. The preceding vehicle cannot be affected by vehicles right behind it.

Model 1 traffic flow model

The traffic flow model focuses on the fan-in process, which is based on the two areas as below.

Consider a toll having L lanes and B tollbooths ($B > L$) in each direction.

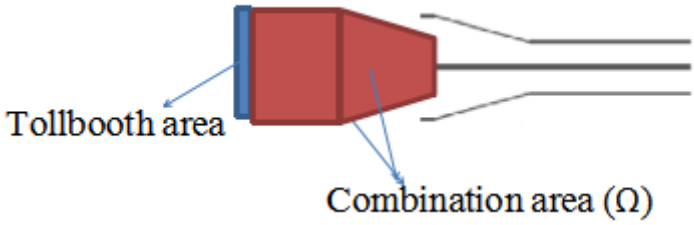

Fig. 1. Areas of toll

This process mainly contains three procedures:

Generate traffic flow and pass through the tollbooth.

Follow the preceding vehicle in combination area (Ω).

Vehicles on the side lanes combine to the target traffic flow when road becomes narrow.

We discuss each of the procedure separately as below.

Generate traffic flow and pass through the tollbooth.

The process which vehicles are waiting to pay behind the tollbooth is a queuing model. Based on the real situation, we consider the M/G/K queuing model. [2] The arriving of vehicles accords with Poisson distribution and the charging time accords with Normal distribution in each of the tollbooth. The probability density function of time interval can be written as

$$p(x) = \lambda e^{-\lambda x} + \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right).$$

Follow the preceding vehicle in combination area (Ω).

Vehicles which move on in combination area (Ω) are affected by driving conditions of preceding vehicles. Therefore, we use Nagel-Schreckenberg (NS) vehicle-following model [3] to simulate real situation.

Vehicles in the side lane combine to the target traffic flow when road becomes narrow.

Although vehicles have different merging patterns, the combination process is the same after they determine to make the decision to change the lane. When the combination vehicle enters target lane, distance between combination vehicle and later vehicle begins to decrease. According to NS vehicle-following model, later vehicle maybe start to decelerate and the deceleration value is affected by speed of combination vehicle and their distance. Those combination vehicles behind can make the same deceleration decision in order until the whole system reaches a stable state.

Cost analysis evaluation

The evaluation of these programs covers accident prevention, throughput and construction fee. We now use social cost analysis to obtain a relatively objective evaluation.

Accident prevention

Accident-prone areas are those which traffic flow has large density, short following distance and a big amount of lane changing times. Therefore, EI are defined as lane changing times, deceleration times and traffic flow density. According to actual data of accident happening times, we use regression analysis to determine correlation between EI and accident happening times, then accident rate can be forecasted.

Throughput

Because the average speed in combination area (Ω) is less than on highway, we can calculate time cost and switch it to money cost using GDP per capita. Finally, we get the economic cost of throughput C_2 as

$$C_2 = \left(\frac{1}{w_k} - \frac{1}{w^*}\right) \cdot w \cdot \frac{GDP}{N_p \cdot H}.$$

Where w_k is the throughput of case k, w^* is the throughput of highway, w is the average of vehicles getting through the turnpike per year, N_p is the population of the area, H is the average working hours per year.

Construction fee

The construction fee of toll mainly refers to land cost, road construction fee and facility cost. We can calculate the economic cost of construction fee C_3 according to different cases of land area, and the money of building tollbooth in different charge mode.

Design of combination area and computer simulation

Then, we use MATLAB to establish cellular automaton (CA) model and simulate the driving regulation above.

Plaza Shape

We consider two types of shapes, narrowing symmetrically and narrowing on one side.

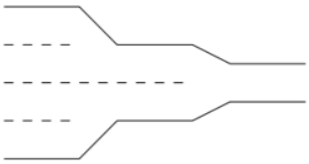

Fig. 2. Shape 1

Fig. 3. Shape 2

Then we use computer to simulate shape 1 (symmetrically narrowing) and shape 2 (narrowing on one side), and other parameters are same as before.

Size

Firstly, we define the concept of Gradient Rate, which is used to measure the angle lean-state of combination area.

Gradient Rate: $R_g = L_{lane} \cdot \frac{B-L}{2L_{toll}}$, where L_{lane} is the width of lane.

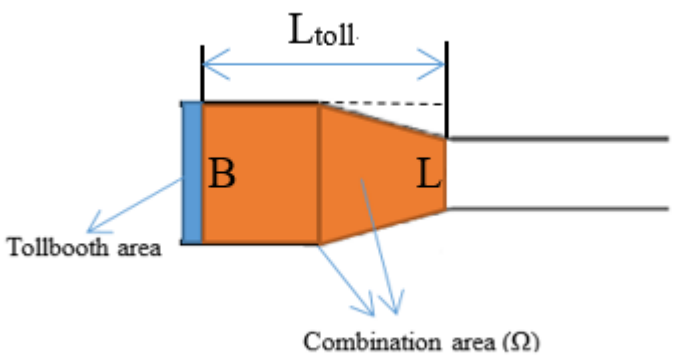

Fig. 4. Gradient Rate

Since B and L are parameters, we change the size of combination area mainly through adjusting the length of L_{lane}, which also means adjusting the Gradient Rate.

We then use computer to simulate, and set different Gradient Rate in order to change the size of combination area. Larger size will cause better accident prevention and traffic capacity. Meanwhile, construction fee will increase owing to larger size. We will do the quantitative comparison later.

Merging pattern

Merginag pattern is a driving regulation that how the vehicles change lane on the gradually narrowing lanes when driving in the combination area, especially involving whether to merge when satisfying the feasible merging condition (FMC). Then we discuss the following 3 merging patterns respectively.

Now, We divide the merge patterns into the following three types.

Merge as soon as possible. Every vehicle immediately merges when it satisfies FMC.

Combine randomly.

Step 1. Set t as 1 second.

Step 2. Judge whether the vehicle satisfies FMC.

Step 3. If yes, move on to step 4. Else, move on to step 5.

Step 4. Randomly determine whether merge or not. If yes, execute merging process. Else, continue to move on for 1 second. Move on to step 6.

Step 5. Judge whether reach the end of lane. If yes, stop and wait for 1 second. Else, continue to move on for 1 second. Move on to step 6.

Step 6. Determine whether t equals to N. If yes, end the process. Else, return to Step 2, and add 1 second to t.

Merge at the end of lane.

Every vehicle judges if it satisfies FMC when it reaches the end of lane. If yes, then merge immediately, otherwise stop immediately.

The simulating conditions:

Chart.1. Simulation condition

merging pattern	λ	self-driving cars	ETC	AT	MTC	tollbooth	Lanes	plaza length (cellular)
as soon as possible	0.8	0	0	0	100%	12	4	100

Changes one variable in different merging patterns each time and run the simulation. Randomly merging pattern is obviously better than the other two patterns because average velocity is the greatest, the number of vehicles in combination area and sharp braking frequency is the lowest. Therefore, we can basically conclude that randomly merging pattern is the best among three patterns.

In order to examine the reliability of data, we repeat the simulation for ten times of each variable, and get Standard Deviation (SD) and mean value. The table below gives the Coefficient of Variation (define as SD divided by mean value, which can reflect the discrete degree of data) of each variable. As can be seen, except the *lane shift ratio*, other five variables have small Coefficient of Variation, which means that our data is centralized and reliable.

Table. 1 Coefficient of Variation of each variable

Average time	Average velocity	Throughput	Sharp braking frequency	Traffic flow density	Lane shift ratio
[0.12%,0.18%]	[0.14%,0.26%]	[0.63%,0.69%]	[0.53%,0.64%]	[0.62%,0.73%]	[5.02%,5.98%]

Random combine

We have discussed the effect of shape, size and merging pattern in combination area to each variable separately. According to our analysis, we come to some conclusion as below.

Symmetrically narrowing shape is obviously better than narrowing on one side, when fixing size and merging pattern

Randomly merging pattern has a slight advantage than merging as soon as possible.

Traffic capacity and accident prevention increase when Gradient Rate increase, but the economic cost is not sure for the increasing of construction area.

Based on these conclusion, we select shape 1, Gradient Rate as 0.1 and 0.2, merging pattern as merging as soon as possible and randomly merging. We then select another common design, which is shape 2, Gradient Rate 0.1 and randomly merging. Use these 5 cases to calculate social cost and find a better solution.

We use forecast model made by Hong Yang in his article Assessing the Safety Effects of Removing Highway Mainline Barrier Toll Plazas, to forecast accident happening times. After finding the construction fee of highway and comparing construction fee among different projects and area, we can finally calculate that the construction is $1,500 per square meter. The price of MTC varies from area to area, we calculate as an average value of $20,000.

Based on the data above and the simulated data in our traffic flow model, we solve the cost analysis model and get the social cost as the following table.

Table. 2 Total cost per year of 5 cases

	Case 1	Case 2	Case 3	Case 4	Case 5
Shape type	1	1	1	1	2
Gradient Rate	0.1	0.2	0.1	0.2	0.1
Merging pattern	Randomly	Randomly	As soon as possible	As soon as possible	randomly
Total cost/year	$947,200	$1,069,400	$986,900	$789,600	$1,328,760

We can find from the table that case 4 has the lowest cost. According to brief analysis, we can find that although merging pattern is better than merging as soon as possible, but the difference is slight, and size can be design smaller in randomly merging pattern. We can conclude that the result above is reasonable. We have to admit that although using quantification method, the accuracy is still to be decided because the range of data in the reference is too large. So in our article, we only give an order instead of comparing value.

Tests and applications

Light and heavy traffic

In order to describe the performance of vehicle in different traffic flow situation, we select different value of arriving density when simulation. When we analysis the shape， size and merge pattern ,we have consider the light and heavy traffic , and our solution performs better than other four designs no matter in light traffic or heavy traffic.

Add autonomous (self-driving) vehicles to the traffic flow

Self-driving vehicles can avoid making inaccurate judgement of surrounding situation because of drivers' fatigue driving, and they can also improve road capacity and patency macroscopically. Owing to the short responding time and accurate judgement of road condition, we can make the assumption below when adding self-driving vehicles into the traffic flow system.

The Safety Distance between self-driving vehicle and preceding vehicle is shorter.

Self-driving vehicles do not have randomly slowing down process.

The lane shift ratio of self-driving vehicles does not affect safety of system.

Coordination or communication among self-driving vehicles are not taken into account. In other words, from the perspective of one self-driving vehicle, all other vehicles are seen as the same moving barriers.

After simulation, the results could be summarized as following, when more autonomous vehicles are added to the traffic mix, the average passing time of each vehicle is lower, and the average velocity increase obviously. The sharp breaking frequency is lower in more autonomous vehicles system, In conclusion, adding more autonomous vehicles can improve traffic capacity in merging area effectively, and can also allow great throughput and reduce accident happening times. In addition, adding more autonomous vehicles perform better in heavy traffic.

Proportions of conventional tollbooths, automated tollbooths, and electronic toll collection booths

There are three types of common tollbooths in use: conventional tollbooths (also known as Manual Toll collection (MTC)), automated tollbooths (AT) and electronic toll collection (ETC, also known as EZ-pass in New Jersey).

These three types have a great difference in traffic capacity and construction fee. According to references, we obtain specific data of these tollbooths as below.

Table. 3 Comparison of different tollbooths

Comparison of different tollbooths				
type	traffic capacity (pcu/h)	toll time (s)	construction fee ($)	exit speed （m/s）
MTC	350	10	20,000	0
AT	650	6	40,000	0
ETC	1500	3	150,000	6

we use MATLAB to simulate all the tollbooths are ETC, or AT or MTC respectively. We fix other variables, including choosing shape as symmetrically narrowing (i.e. shape 1) and merging pattern as merging as soon as possible, setting gradient rate as 0.1 and all the vehicles are manual driving vehicles.

From the simulation results, ETC tollbooths can improve traffic capacity in combination area effectively, and can also allow great throughput and reduce accident happening times. In addition, ETC tollbooths perform extremely better in heavy traffic.

However, it is impossible for every vehicle to allocate induction device of ETC, so that we should analyze how the proportion of three types of tollbooths affects our model and figure out the propitiate number of three types of tollbooths. We use basic optimization method as follows:

Optimization model

According to the reference, 70% of cars in New Jersey have installed EZ-pass system. Suppose the rest of cars choose Automated tollbooths (AT) and Manual Toll Collection (MTC) tollbooths with the same possibility, i.e. 15% choose AT and 15% choose MTC.

From queuing theory and some basic optimization skills, the problem can be written in the following form:

Objective function:

minimize

$$t_{ETC}\cdot\left(\sum_{i=1}^{\left[\frac{0.7N_{car}}{N_{ETC}}\right]} i+\mathrm{mod}(0.7N_{car},N_{ETC})\cdot\left(\left[\frac{0.7N_{car}}{N_{ETC}}\right]+1\right)\right)+t_{AT}\cdot\left(\sum_{j=1}^{\left[\frac{0.15N_{car}}{N_{AT}}\right]} j+\mathrm{mod}(0.15N_{car},N_{AT})\cdot\left(\left[\frac{0.15N_{car}}{N_{AT}}\right]+1\right)\right)+$$

$$t_{MTC}\cdot\left(\sum_{k=1}^{\left[\frac{0.15N_{car}}{N_{MTC}}\right]} k+\mathrm{mod}(0.15N_{car},N_{MTC})\cdot\left(\left[\frac{0.15N_{car}}{N_{MTC}}\right]+1\right)\right)$$

Functional constraints:

$$\begin{cases} N_{ETC}+N_{AT}+N_{MTC}=B;\dfrac{N_{ETC}}{t_{ETC}}\geq 0.7\lambda \\ \dfrac{N_{AT}}{t_{AT}}\geq 0.15\lambda;\dfrac{N_{MTC}}{t_{MTC}}\geq 0.15\lambda \end{cases},$$

Set constraints:

$$N_{ETC},N_{AT},N_{MTC}\in\mathbb{N}^{+}$$

When set N_{car} as 100, in order to minimize the objective function, we use MATLAB to calculate and find the proportions of ETC, AT and MTC at different value of λ. Generally, λ is less than 1. The solution of this optimization problem should be (9, 1, 2) or (10, 1, 1). Furthermore, we can set 9 ETC tollbooths, 1 AT tollbooth and 1 MTC tollbooth. The additional tollbooth should be ETC/MTC tollbooth, and when heavy traffic flow, i.e. λ is more than 0.5, we use it as MTC to relief congestion in MTC, otherwise use it as ETC.

Conclusions

First, based on the fact that 70% of the drivers in New Jersey have EZ-pass, we establish an optimization model to minimize toll time. We assume a common case that 12 tollbooths are opening and 4 main lanes on highway. Then we find that the proportion of 9 ETC tollbooths, 1 AT tollbooth and 2 MTC tollbooths is the most efficient when early and evening peak. At other time when the traffic is relatively light, one MTC tollbooth can be switched to ETC to minimize average toll time.

Second, elect a Sign Board on the fan-in side of toll plaza. Actually, we have studied three different types of merging patterns, finding that merging as soon as possible and merging randomly are better than merging at the end of lane. It shows that the average velocity is low, the number of vehicles in toll plaza is large and sharp braking frequency is high. That is to say, merging at the end of lanes can easily cause congestion, reduce efficiency and increase accident risk.

Third, the narrowing of road when fanning-in should be gradually, but not suddenly. According to our computer simulation, gradually narrowing road has lower sharp braking frequency, less number of vehicles in combination area and less lane changing times. That is to say, gradually narrowing road is safer than suddenly narrowing road. Although road construction fee could be higher,]that safety should be put on the first position.

Finally, EZ-pass system is a low-speed ETC, and vehicles still need to slow down a lot to pass the tollbooths. However, high-speed ETC is spreading out nowadays, which allows vehicles to pass through the tollbooths at 60 *km/h*. Purchasing some high-speed ETC and switching the old devices can have greater efficiency.

Reference

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