

# The Model of Toll Station Planing

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**Abstract**—When designing barrier tolls, the shape, size and merging pattern need to be taken into consideration. Besides, the safety codes and standard of accident prevention and throughout capacity must be met while the development and operating costs should be kept as low as possible. Six major factors impacting the toll plaza performance were discussed, including the corresponding relationship between the amount of main lanes in single direction and tollbooth, the critical length of toll plaza meeting safety codes and standards, the critical width of toll plaza, the critical size of transition region, the development cost of toll plaza and its throughput capacity. We set up a multiple index analysis system after calculating those six factors to analyze the pros and cons of the existing toll plaza, which could provide scientific basis for the new design. Moreover, difference motivated principle was applied to determine the weight vector, to comprehensively evaluate the impact of each parameter. To determine the optimal design, we designed and compared three control schemes. The evaluation system was applied to compare the pros and cons of each scheme based on the actual situations, and the optimal scheme was designed through combining the strengths of all three. By investigating the adjustment strategy and corresponding performance of the toll plaza of the different scheme in the case of light and heavy traffic flow, the case when the proportion of automatic driving vehicles increases, and the case under different charging modes, the feasibility and rationality of the optimal scheme was tested. In the process of inspection, we used the queuing theory model to determine the maximum capacity of the toll plaza under the manual toll mode, and used the integer programming model to solve the optimal proportional relation of the three tolling models.

**Keywords-** multi-index comprehensive analysis system; difference driving principle; queuing theory model; integer programming model; weight vector

## I. INTRODUCTION

A barrier toll is a row of tollbooths placed across the highway, perpendicular to the direction of traffic flow. When exiting the tollbooths in a barrier toll, vehicles must “fan in” from the large number of tollbooth egress lanes to the smaller number of regular travel lanes. However, with the development of modern semi-automatic toll station, electronic toll station technology and the increase of vehicles, the traditional toll station design mode can not meet the requirements of modern transportation development, and optimization and innovation of toll station design urgently need to be solved.

## II. ASSUMPTIONS AND JUSTIFICATION

A. It is assumed that the error between the theoretical value and the actual value can be neglected by the approximate method to calculate the area of the toll plaza.

B. Assuming that the various factors that affect the performance of the toll plaza, the role of other secondary factors other than the main factors considered in the model can be ignored.

C. It is assumed that the effects of weather, climate change, season and local economic development on the toll plaza throughput can be ignored.

## III. MODEL

In the design of a barrier toll plaza, the shape, size and merge pattern must be taken into consider. And we have to make sure that the trunk road can not be interfered, the accident rate keeps in low level, minimize the operating cost (including the cost of pre-construction and normal operation) and optimize the comprehensive benefits.

Usually, the amount of lane of travel a toll highway has in each direction is given. If the number of tollbooths is too small, it's pretty easy to cause traffic jam, although it saves the cost of construction and operating. However, an unreasonable large number of tollbooths will result in excessive land resource occupation, and the costs in the latter part of the operation will be far beyond reach. Therefore, in the case where the number of conventional road lanes is constant, we need to determine a reasonable number of tollbooths.

### A. The Investment in Constructing of Tollbooth [1]

The cost of the tollbooth in daily operation includes the cost of payment for employee, the cost of the operating and fixing the infrastructure and the cost of basic purchase. The investment in the operation goes right into the maintenance of the facilities and the electric charge. Basic acquisition costs refer to the fee of the necessary fuel and work items related. The specific calculation is as follows:

$$C_{\text{toll}} = C_r + C_s + C_g = \frac{C \bullet (r + n) + M + W + G}{30}$$

$C_{\text{toll}}$  — the total cost of toll plaza (\$/Day.)

$C_r$  — the cost of employment (\$/Day.)

$C_s$  —— the cost of running equipment (\$ / Day.)

$C_g$  —— daily basic cost (\$ / Day.)

$n$  —— the amount of tollbooths

$r$  —— the amount of staff

$M$  —— the average electric fee of the toll plaza in each month (\$ / Mon.)

$W$  —— the cost of equipment repairs in each month (\$ / Mon.)

$C$  —— the average salary of each staff in each month

#### B. The Cost of Queuing

Queuing cost refers to the cost due to frequent starting, braking and idling fuel consumption, when the motor vehicle going ahead into the toll plaza. In this sense, we set the queuing cost as a reference to determine the amount of toll stations. We divided ordinary vehicles into coaches (using gasoline as fuel) and trucks (using diesel as fuel). In the reference [1], In fact, however, the vehicle fuel consumption not only depends on the above three cases, but also is affected by COTY, displacement and many other uncertain factors. If we check on these factors one by one, the complexity of the model will be unnecessarily increased. Therefore, we may wish to approximate those different fuel costs of vehicle into an equivalent vehicle. The fuel consumption is as followed:

$$P_i = k \bullet (P_{car} + P_{tra}) \bullet L + 0.23 \bullet k \bullet (B - L),$$

$P_i$  —— the cost of queuing per car

$P_{car}$  —— the average fuel consumption of each coach (ml / h)

$P_{tra}$  —— the average fuel consumption of each truck

$k$  —— the coefficient of proportionality

(from large sample data training)

#### C. The Correspondence of Optimum Solution

Assuming a certain period of time traffic volume is  $q$  (every car/h), the total cost is  $Z$ . The process of determining the optimal L-B correspondence is to minimize the total cost. In specific cases, the number of lanes of the conventional road  $L$  is given by the realistic situation. The constraints is given ,as  $B > L$ .

$$\min Z = q \bullet P_i + C_{toll}$$

$$\text{S. t. } B > L$$

Considering the local situation, we can determine the B-L optimal correspondence. This can provide a reliable reference for the analysis and construction of toll plaza.

#### D. Accident Prevention Index

In the design of toll plaza, we need to be in accordance with the provisions of the national general design specifications.

The actual toll plaza is often a "row" (almost all of the stations are in a row), and shape of it is not a regular geometric figure. In order to describe the performance of toll plaza simply and reasonably, we will deal with the toll plaza.

There are  $L$  lanes in a single direction before entering the toll booth. According to the regulation [2], the width of single lane is 3.6m,which means the total width  $D_1$  of the lanes is :

$$D_1 \geq 3.6L$$

In order to avoid the vehicle cut rub or other traffic accidents, we need to well organize the width of tollbooth, lanes near the booth, outer overtaking lane and emergency lane.

TABLE I REFERENCE VALUE

	$d_1$	$d_2$	$d_3$	$d_4$
size	3.0m ~ 3.5m	3.0m ~ 3.5m	2.2m	4.0m ~ 4.5m

$d_1$  —— the standard width of toll channel

$d_2$  —— the width of outer overtaking lane

$d_3$  —— the width of tollbooth ,

$d_4$  —— the width of emergency lane

Take all the data into consideration, the width in total need to meet the requirement :

$$D_2 \geq 2 \cdot d_2 + (B - 2) d_1 + B d_3 + d_4$$

The merging situation does happened when vehicle pull into toll plaza from the regular lane. That means when it comes to the programming of toll plaza, the secure limit must be well-

organized. Thus the buffer area can be planed legitimately. The technique index are as followed[2]:

TABLE II TECHNIQUE INDEX OF TOLL PLAZA

		Arithmetic velocity of driving(KM/h)		120	100	80	60	40
Minimize of curve radius R(m)	Convex	General value	2000	1500	1100	500	250	
		Limit value	1500	1000	700	350	200	
Minimize of upright curve radius(m)	Convex	General value	45000	25000	12000	6000	2000	
	Concave	Limit value	23000	15000	6000	3000	1500	
		General value	16000	12000	8000	4000	3000	
		Limit value	12000	8000	4000	2000	1500	

In this part we need to make sure the ratio  $\varepsilon$  of the transition region width S and length  $L_1$ .

$$\varepsilon = S / L_1 = (D_2 - D_1) / L_1 \leq \frac{1}{3}$$

$S$  — the width of the transition region

$L_1$  — the length of the transition region

Therefore, we can approximate the expression of the transition area A according to the geometric relationship:

$$A \geq SL_1 + D_1 L_1 + d_4 L_1 = \frac{(D_2 - D_1)^2}{\varepsilon} + \frac{D_1(D_2 - D_1)}{\varepsilon} + d_4 L_1 = \frac{D_2(D_2 - D_1)}{\varepsilon} + d_4 L_1$$

#### E. The Critical Value of Total Length the Toll Plaza

To ensure that there is sufficient safety distance  $L_0$  between the two cars, we assume that the vehicle travels at maximum speed, although this is rare. Then we can ignore the driver's reaction time, and approximate the safety distance between the two cars.

$$L_0 \geq \frac{V_{MAX}^2}{2a}$$

$a$  — the acceleration of vehicle braking

$L_0$  — the minimum safety distance

between two vehicles

Among them, the recommended values for the mains and ramps under the general specification are as follows:

TABLE III FEATURE TABLE OF THE TRANSITION REGION IN THE TOLL PLAZA

recommended value	mains	c	$\varepsilon$
	ramps	100m 50m	1/6~1/7 1/4~1/5

In this case, the total length of the value in the toll plaza should be met:

$$L_2 = \frac{l}{2} + c \geq 2l_1 + L_0 + c$$

The total length of the toll plaza:

$$L_T \geq l_1 + 2l_1 + L_0 + c =$$

$$\varepsilon(D_2 - D_1) + 2l_1 + L_0 + c$$

#### IV. MODEL CHECKING

The design scheme can change the confluence mode, so that it can test the rationality of the confluence mode of the design scheme by using its performance under the condition of small vehicle flow and large vehicle flow. Whether the design of the restricted passage of the different types of vehicles is reasonable, the test can be carried out by examining the influence of the number of the autopilot vehicles on the traffic capacity when the proportion of the automatic driving vehicles changes; The design rationality of the charging model of the program can be tested by examining whether the input cost is optimal when the traffic capacity is not affected under different traffic flows.

##### A. Adjustment of the Confluence Mode

Confluence refers to the vehicles from different roads or directions meet to the same road, differences in confluence will affect the speed and density of vehicles, thus affecting the throughput of the toll plaza. We defined the throughput of the entire toll plaza as  $Q$ : the speed of traffic before the confluence as  $V_F$ , the traffic density at the toll plaza as  $\rho$ . We do notice that the convergence angle  $\theta$  ( $0 \leq \theta \leq 90^\circ$ ) at the edge of the transition zone is inversely proportional to the velocity  $V_B$  after convergence. Here we use a coefficient  $K$  (coefficient influenced by velocity) to represent the relationship between the real speed  $V_B$  of the vehicle after convergence and the average vehicle velocity  $V_F$  before convergence. The average speed of the vehicle  $V_F$  before convergence is related to the distance  $L_2$  between the center line of the toll plaza and the transition zone.

$$V_F = \begin{cases} \sqrt{2 \cdot a \cdot L_2}, L_2 < \frac{V_{MAX}^2}{2a} \\ V_{MAX}, L_2 \geq \frac{V_{MAX}^2}{2a} \end{cases}$$

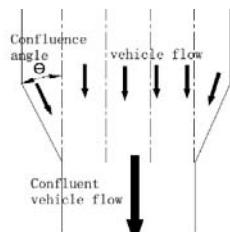


FIGURE I MERGING PATTERN

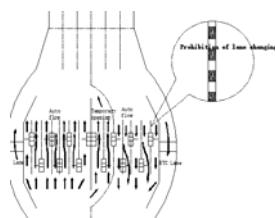


FIGURE II OBSTACLE ZONE

$$V_B = K \cdot V_F$$

In the above equation,  $K$  is a variable related to  $\theta$

$$K = \delta \frac{1}{\tan \theta + \delta}$$

$\delta$ -Motor braking parameters from the reference [6] know

$$\text{Throughput } Q = V_B \cdot \rho$$

$$Q = \delta \frac{1}{\tan \theta + \delta} V_B \cdot \rho$$

It can be seen from the model that the running situation of the toll plaza transition area will affect the driving condition of the vehicle, thus affecting the throughput. We designed the program can control the toll plaza fan-out area roadblock (limit lane) length to achieve the control of the motor vehicle course.

(1)When the traffic volume is small, shorten the length of the roadblock in the sector-in area, and make the motor vehicles select the lane in the sector-in area by themselves, to improve the utilization rate of the tollbooth, shorten the length of the queuing vehicle and improve the comprehensive utilization rate of the toll plaza.

(2)When the traffic volume is so large that traffic jams, extend the length of the toll plaza roadblocks, and forbid motor vehicles from changing lanes, to ensure that any lane speed is not affected by other adjacent lane.

(3)When the difference between the two directions' traffic volume of the obstructed tollbooth is large, open the "cross-gate" to guide the congestion side of the traffic flow through the gate

into the opposite direction of the tollbooth, whilst restrict the smaller traffic flow by prohibiting its entry into the occupied tollbooth, to achieve a composite adjustment.

### B. Conventional Tollbooth

We establish M / G / K queuing theory model [8] to calculate toll channel capacity.

The M / G / K model is as follows:

Vehicle Waiting Time Expected Value

$$W_q = \frac{D(S+G) + [E(S+G)]^2}{2E(S+G)[K-\lambda E(S+G)]} \left[ 1 + \sum_{i=0}^{K-1} \frac{(K-1)! [K-\lambda E(S+G)]}{i! [\lambda E(S+G)]^{K-i}} \right]$$

Vehicle Staying Time Expected Value

$$W = E[S + G] + W_q$$

Vehicle Queue Length Expected Value

$$L_q = \frac{\lambda D(S+G) + \lambda [E(S+G)]^2}{2E(S+G)[K-\lambda E(S+G)]} \left[ 1 + \sum_{i=0}^{K-1} \frac{(K-1)! [K-\lambda E(S+G)]}{i! [\lambda E(S+G)]^{K-i}} \right]$$

Here:

$$E[S + G] = E[S] + E[G] = E[V]$$

$$D[S + G] = D[S] + D[G] = D[V]$$

In the above formula:

$\lambda$  is the tollbooth average arrival rate of vehicles expected value, vch / s;

$K$  is the toll channel's opening quantity;

$E[S] --$  Average charge time, (s);

$E[G] --$  Average departure time, (s);

$D[S] --$  Service time variance;

$D[G] --$  Departure time variance;

$E[V] --$  Charging time expectation;

$D[V] --$  Charge time variance.

According to the M / G / K queuing theory model, we can calculate the maximum number of vehicles which can be handled under the condition of different toll channel number and

different queue length by using the expectation and variance of charging time and departure time of the tollbooth. The results are shown in the following table:

TABLE IV MAXIMUM SERVICE VOLUME OF TOOL-BY-WEIGHT

Number of open channels	Queue Length(Vehicles)					
	1	2	3	4	5	6
1	87	89	92	96	101	101
2	177	179	181	184	188	193
3	261	268	272	277	289	302
4	348	353	362	372	383	388
5	435	444	451	460	476	478
6	522	530	540	551	563	573

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