

# Discussion on Teaching of Calculus Limit

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**Abstract**—Calculus, including differential and integral, is a major course in college, which can make students study and calculate many problems. Throughout teaching of calculus, limit concept plays an important role, which has been discussed in this paper in order to help students studying mathematics and cultivating their mathematical thinking ability.

**Keywords**—calculus; limit; teaching

## I. INTRODUCTION

As the basic content of calculus learning, limit plays an important role in the teaching of calculus, which should be focused on by teachers. However, in university mathematics teaching now, some teachers just take a simple teaching for limit because of its basic characteristics, so that the importance and difficulty of limit have been ignored. Since students often need to accept a variety of symbols, a new concept after entering university, it is not easy to understand the limit concept for them, therefore, detailed explain of limit should be taken by teachers so that students can lay a solid foundation.

## II. KEY POINT IN LIMIT TEACHING

Before teaching limit concept, teachers should be clear of key points in students learning so as to enable students truly grasping the concept of limit through teaching. Furthermore, since mathematical concepts are relatively independent, during mathematics teaching, the teacher usually teaches basic definition firstly, and then gives some exercises to help students analyze and explore the typical exercises, so as to complete teaching. However, in learning of limit concept, we will find that because students learn the concept of limit in first time, it is difficult to understand the concept of limit which includes some new symbols of “ $\infty$ ”, “ $\delta$ ”, “ $\varepsilon$ ”, “ $\Delta$ ”, “ $\rightarrow$ ”, as well as a variety of dialectical views which are easy to make students concept confusion. Therefore, in teaching of limit concept, the key points of the concept should be sorted out so as to help students learn the foundation of calculus.

## III. TEACHING DISCUSSION OF LIMIT CONCEPT IN CALCULUS

### A. History of Limit

In 17th century, Newton and Leibnitz respectively defined limit concept, and respectively established differential calculus and integral calculus, which has brought great changes to mathematical, and then is further improved by Cauchy and Weierstrass so that calculus became strict. As a kind of mathematical tool, calculus plays an important role in life and work of human beings, especially in engineering application.

Dialectical thought is the main content of calculus concept of which the main studying object are functions. There is a dependence relationship of variables between them which includes three elements of value domain, definition domain and laws, of which the definition domain and laws play key roles. Based on limit concept, the calculus knowledge system is essentially work from amount changes to qualitative changes, and includes dialectical thought of amount changes and qualitative changes, finite and infinite dialectical mathematical thought. Therefore, as the basic concept of calculus teaching, limit concept should be explained detailed by teachers. In this paper, through explaining calculus history so as to help students understand preliminarily calculus, and develop learning mathematics interest.

### B. Gradually Introduce Limit Concept

Due to the abstraction and the new content of the limit concept, in teaching the definition of limit, teachers can use the progressive method introduced with sequence, to introduce the concept of limit when teachers should list various types of sequences to help students clearly find the limit characteristics of the sequences, such as:

$$\left\{ \frac{n}{n+1} \right\} : \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \dots, \frac{n}{n+1}, \dots;$$

$$\left\{ \frac{1+(-1)^n}{2} \right\} : 0, 1, 0, 1, \dots, \frac{1+(-1)^n}{2}, \dots;$$

$$\left\{ \frac{2n+(-1)^n}{2} \right\} : 3, \frac{3}{2}, \frac{7}{3}, \frac{7}{4}, \frac{11}{5}, \frac{11}{6}, \dots, \frac{2n+(-1)^n}{2}, \dots;$$

From the problems above, we find that introducing infinite sequences can make students intuitively find the change law of the sequences and have a preliminary understand, so as to motivate students to explore the limit concept, and then naturally introduce the definition of limit.

### C. Teaching in Step by Step

After introducing the definition of limit, puts forward some practical problems to give rise to make a thorough inquiry of students. For example, an infinite sequence: a predetermined length of rope has been cut the half length of the rope every day, then, how much the rope length remained after n days? According to the subject, it is a geometric progression as follows:

$$\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \dots, \frac{1}{2^n}, \dots;$$

In this problem, no matter how many days, the remaining length of rope is close to zero infinitely instead of zero. In this case, teachers can make students realize the importance of sequence for limit, so as to enlighten and guide students to think and gradually give definition of sequence limit through definition of limit. The definition of sequence limit: provide a sequence  $\{u_n\}$  and a constant A, if for any given positive  $\varepsilon$ , there is a positive integer N which make  $|u_n - A| < \varepsilon$  valid for all  $n > N$ , then, which is called that when  $n \rightarrow \infty$ ,  $\{u_n\}$  is the limit of A and converges to A, denoted by  $\lim_{n \rightarrow \infty} u_n = A$  or  $u_n \rightarrow A (n \rightarrow \infty)$ ; if  $u_n$  has not limit,  $\{u_n\}$  divergent to  $(n \rightarrow \infty)$ .

According to above definition: ①  $\varepsilon$  is given randomly, only  $\varepsilon > 0$ , the inequality of  $|u_n - A| < \varepsilon$  can express  $u_n$  infinitely close to A; ② N is related to  $\varepsilon$ , denoted as  $N(\varepsilon)$ , which is given with selection of  $\varepsilon$ , and not unique; ③ there is only  $n \rightarrow \infty, u_n \rightarrow A$  in the definition, without method of calculation for A; ④ geometrical meaning of the definition: Randomly provide  $N(A, \varepsilon)$ , then there is a N which makes all of  $u_{N+1}, u_{N+2}, \dots$ , located within  $N(A, \varepsilon)$ . The definition of function limit can be introduced as above.

D. Strengthen Understanding of Limit Concept through Comparison

About calculus textbooks, both domestics and abroad there are many, which makes teachers can widely found different versions of the textbooks to compare, so that students can strengthen understanding of limit thought, negation of negation, quantitative and qualitative. Through studying of limitation concept, can help students can be improved the ability of dialectical thinking.

E. Understanding by Graphic Mode

Using diagrams in the process of teaching calculus limit can give full play to intuitive and vivid image and help to further strengthen understanding of limit concept, as well as arouse students' interest in learning calculus. The graphic method can make abstruse mathematics knowledge easier for students to understand and help cultivating students' interest in mathematics learning. This method can not only improve students' understanding of limit so as to fully understand the abstract mathematical definition and formula of limit, but also lead to students interested in exploring calculus. In teaching of calculus limit, whether sequence limit or function limit, the graphic method can play an effective role in helping students. As shown in following: Fig.1 is a diagram of sequence limit definition, Fig. 2 is a diagram of the definition of function limit, both which can help students to master the abstract concept and lay a more solid foundation of calculus.

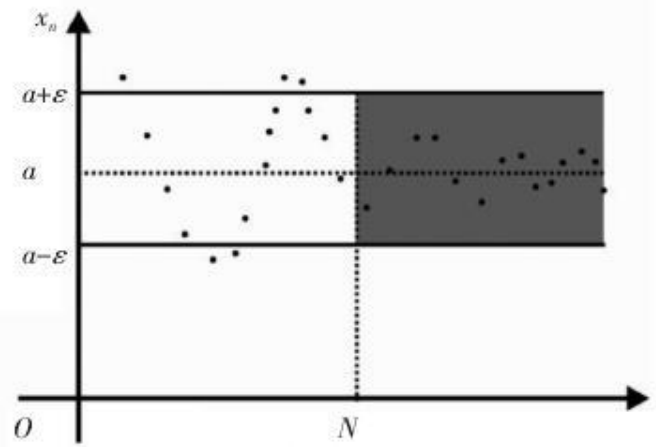


Fig. 1. Diagram of sequence limit definition.

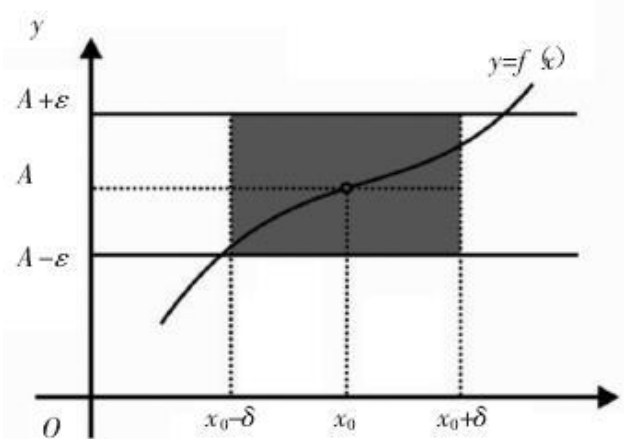


Fig. 2. Diagram of function limit definition.

F. Extension and Application of Limit Concept in Advanced Mathematics

As the center of calculus, the limit has been widely used in advanced mathematics learning which is closely linked to other content of advanced mathematics, which should be taken attention to by teachers. For example, after teaching, expand the application of limit thought in advanced mathematics, so as to motivate students further explore in limit to help other content of advanced mathematics.

IV. CONCLUSION

Since limit plays an important role in advanced mathematics, therefore teachers must pay enough attention to the specific teaching of limit thinking. In this paper, through discussing the teaching calculus limit thought, put forward a teaching method of calculus limit to help students make detailed grasp of limit concept and cultivate students' interest in learning mathematics, so that students can lay a solid foundation in learning calculus.

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