

# Multi-targets ISAR Imaging Technology based on Robust Principal Component Analysis

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**Abstract**—In multi-targets inverse synthetic aperture radar imaging, range profiles of multi-targets with different motion are coupled, so traditional Range-Doppler imaging algorithm is failure. A new imaging technology based on low-rank decomposition is proposed in this paper. After translational compensation and range compression, multi-targets signal can be decomposed into a low-rank part and a sparse part by Robust Principal Component Analysis. Then imaging processing is applied in multiple signals respectively. Simulation results verify the validity of the proposed method.

**Keywords**-Multi-targets; Inverse Synthetic Aperture Radar; Low-rank; Robust Principal Component Analysis

## I. INTRODUCTION

Inverse synthetic aperture radar (ISAR) imaging is an important method in radar target identification and classification, and is widely used for military and civilian purpose. 2-D high resolution imaging can be obtained via transmitting signal with large bandwidth and forming large effective rotational angle. In modern battle circumstance and aviation management, there usually exists the situation that more than one target emerges in the same antenna beam. Due to the difference of the translational motion, range profiles of each target wrapped together. Hence it is impossible to estimate and compensate translation motion of each target simultaneously. Traditional imaging method can't be applied directly in multi-target imaging. In this paper, a new imaging technology based on Robust Principal Component Analysis (RPCA) [1] is proposed to apply in multi-target ISAR imaging.

The remainder of this paper is organized as follows. Section 2 introduces the imaging geometry and signal model. Section 3 introduces the basic theory of RPCA. The detailed steps of the proposed multi-target imaging technology based on RPCA are presented in section 3. Simulation is given in section 4. Finally, section 5 draws the conclusion.

## II. MULTI-TARGETS SIGNAL

Without loss of generality, we suppose that multi-targets are two planes with different velocity. Imaging geometry is shown as fig.1. Target coordinate system O-XY is fixed on one plane. The dash line in fig.1 is the line of sight from radar center to the origin of target coordinate system. The origin is also the reference point.

Inverse Synthetic Aperture Radar (ISAR) transmits Linear Frequency Modulation (LFM) signal. Suppose that range estimation of reference point is accurate. Then after

de-chirping, the imaging of the plane 1 is simplified to a turntable imaging. The echo of the plane 1 can be expressed as

$$s_1(\kappa_n, \theta_m) = \sum_{k=1}^K \sigma_{1k} \exp[-j 2\pi \kappa_n (x_{1k} \cos \theta_m + y_{1k} \cos \theta_m)] \quad (1)$$

where  $\kappa_n$  is the sample of wave number, and  $\theta_m$  is the sample of observed angle between LOS and X axis.  $\sigma_{1k}$ ,  $(x_{1k}, y_{1k})$  are complex amplitude and location of the k-th scattering center on the plane 1. K is the number of scattering centers. Simultaneously, the echo of the plane2 after de-chirping can be expressed as

$$s_2(\kappa_n, \theta_m) = \sum_{k=1}^P \sigma_{2k} \exp[-j 2\pi \kappa_n (x_{2k} \cos \theta_m + y_{2k} \cos \theta_m + R_m)] \quad (2)$$

where  $\sigma_{2k}$ ,  $(x_{2k}, y_{2k})$  are complex amplitude and location of the k-th scattering center on the plane 2. P is the number of scattering centers.  $R_m$  is range migration due to the velocity difference between two planes. In this paper, three-dimensional maneuvering is not considered. So, multi-targets ISAR echo can be written as

$$s(\kappa_n, \theta_m) = s_1(\kappa_n, \theta_m) + s_2(\kappa_n, \theta_m) \quad (3)$$

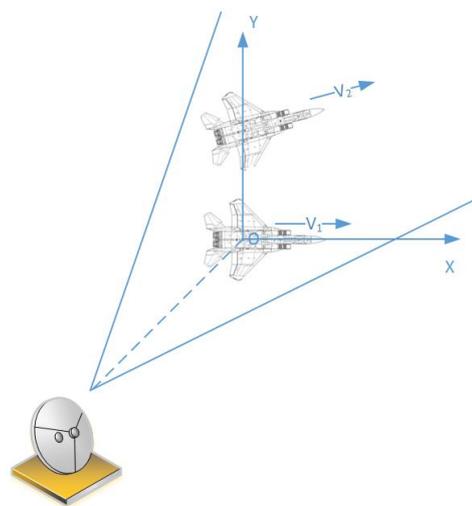


Figure 1. The imaging geometry of multi-targets ISAR

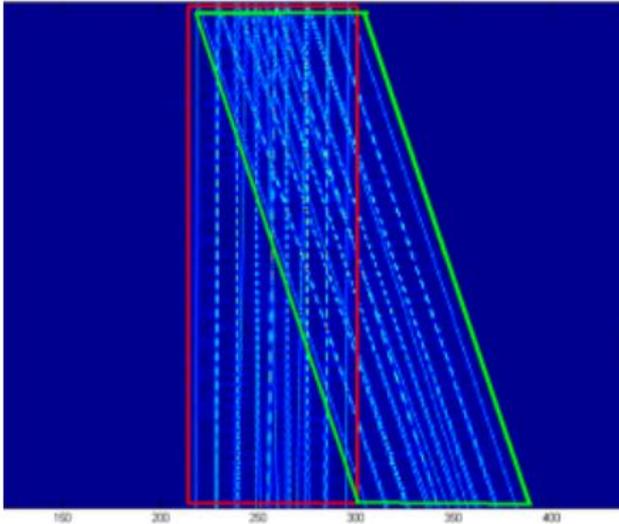


Figure 2. Multi-targets echo after range compression. The red box part is the echo of plane 1, The green box part is the echo of plane 2

### III. ROBUST PRINCIPLE COMPONENT ANALYSIS

The RPCA method, introduced and analyzed in [1], applied to a matrix  $M \in \mathbb{R}^{m \times n}$  that is sums of a low rank matrix  $L_0$  and a sparse matrix  $S_0$ . It solves a convex optimization problem called principle component pursuit (PCP):

$$\min_{L, S \in \mathbb{R}^{m \times n}} \|L\|_* + \eta \|S\|_1, \text{ subject to } L + S = M \quad (4)$$

with  $\eta = 1/\sqrt{\max\{m, n\}}$ . Here  $\|L\|_*$  is the nuclear norm, i.e. the sum of the singular values of  $L$ , and  $\|S\|_1$  is the matrix 1-norm of  $S$ . The optimization can be done for any matrix, but it is important that if  $M = L_0 + S_0$ , with  $L_0$  low rank and  $S_0$  sparse and high rank, then the principle component pursuit recovers exactly  $L_0$  and  $S_0$ . The analysis in [2] gives sufficient conditions under which the

decomposition is exact. These conditions are bounds on the rank of  $L_0$  and the number of non-zero entries in the high rank matrix  $S_0$ . The decomposition can be achieved for a much larger class of matrices than those fulfilling the assumptions of the theorems in [2].

### IV. MULTI-TARGETS IMAGING BASED ON RPCA

According to section 2, multi-targets ISAR echo can be written as eq. 3. After range compression, eq.3 can be given by

$$S(\kappa_n, \theta_m) = S_1(\kappa_n, \theta_m) + S_2(\kappa_n, \theta_m) \quad (5)$$

As shown in fig. 2, It appears that  $S_1(f_n, \theta_m)$  which corresponds to turntable target is a matrix with almost parallel columns, so we expect it to be low rank. The data matrix  $S_2(f_n, \theta_m)$  which corresponds to translation target is given by the sloped curves in data plane. The range compression does not remove the range migration  $R_m$ , and this is why we see this sloped curves. Consequently,  $S_2(f_n, \theta_m)$  is a higher rank matrix, but it is sparse in data plane.

It is obviously shown that similarly to original signal  $S$ , the real and imaginary parts of the signal also have the low-rank and sparse properties respectively. Because the PCP can only deal with real matrix,  $S$  is divided in to real and imaginary part matrices and processed them separately according to eq.4. Then we combine with the real part and imaginary parts of the extracted sparse matrix to form the sparse complex matrix  $\hat{S}_2$ , which only contains translation target. Similarly, the low rank complex matrix  $\hat{S}_1$  can be also obtained, which only contains turntable target. In this paper, we use ALM algorithm [3] to solve PCP problem. At last, the image of  $\hat{S}_1$  can be gotten by two dimensional IFFT transform, and the image of  $\hat{S}_2$  can be gotten by traditional range-doppler algorithm [4]. The following figure shows the processing of multi-target imaging based on RPCA.

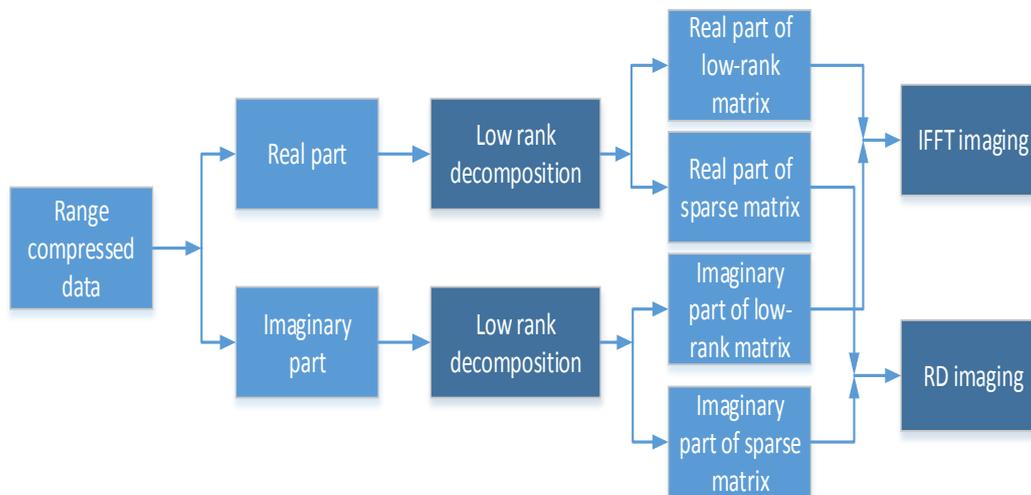
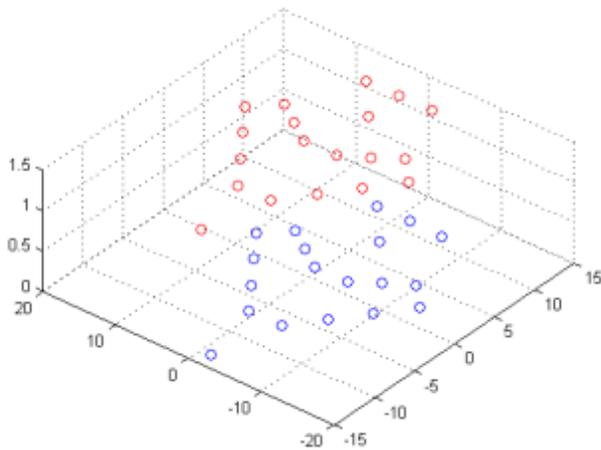


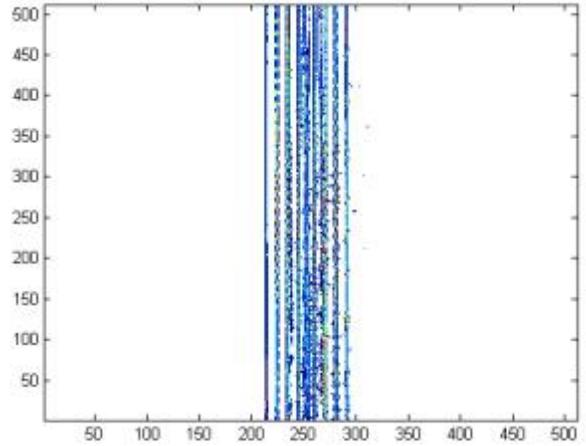
Figure 3. The processing of multi-target imaging based on RPCA

**V. SIMULATION**

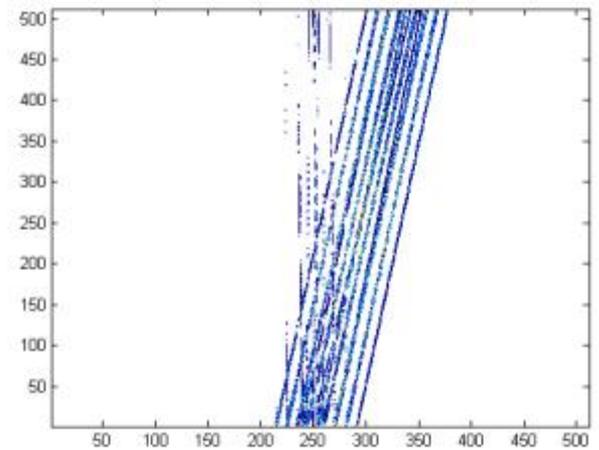
In this section, we use a simulation to show the validity of proposed method. Parameters of simulation are set as following: carrier frequency is 8GHz, and bandwidth is 1GHz. PRF is 100Hz, and sample frequency is 10Mhz. the sample numbers of fast time and slow time equal to 512. Two planes are modeled as fig.4 (a). The distance between the planes and radar is 20Km. Suppose that the flight directions of the planes are same. The speed of the plane 1 is 200m/s, and the speed of the plane 2 is 205m/s. The simulation result is shown in fig. 4. From fig.4 (b), we can see the plane 1 because of its turntable model, but image of the plane 2 is defocused because of its higher speed. The low rank part and the sparse part of echo are gotten by low rank decomposition as shown in fig.4 (c) and (d). Then the image of the plane 1 is obtained by IFFT imaging on low rank part as shown in fig.4 (e), and the image of the plane 2 is obtained by range-doppler imaging on sparse part as shown in fig.4 (f).



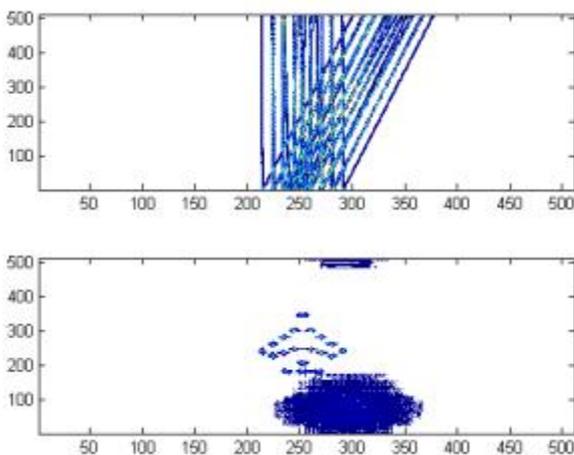
(a) Geometrical model of two planes



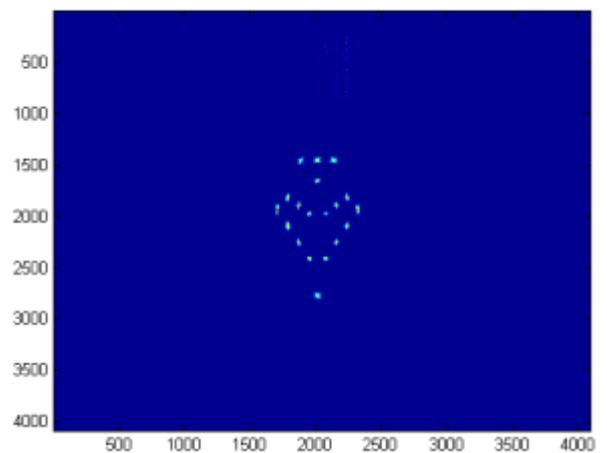
(c) low rank part after low-rank decomposition



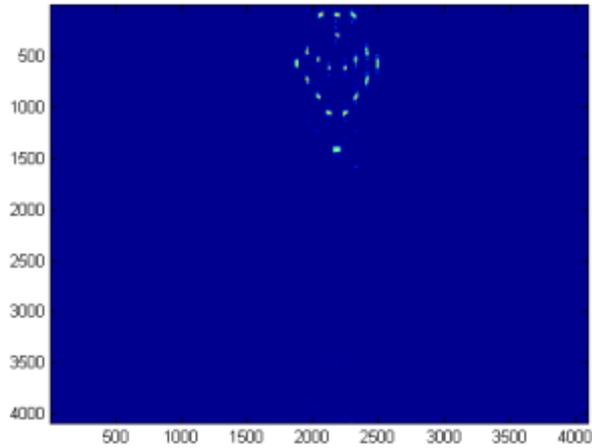
(d) sparse part after low-rank decomposition



(b) Directly IFFT imaging result



(e) IFFT imaging of low rank part



(f) RD imaging of sparse part

Figure 4. Simulation result of multi-target imaging base on RPCA

## VI. SUMMARY

In this paper, multi-target ISAR imaging technology base on RPCA is proposed to decompose compressed multi-target echo to a low rank part and a sparse part. Then IFFT imaging is applied on the low rank part data to obtain image of one target, and RD imaging is applied on the sparse part data to obtain image of another target. Simulation is shown the validity of proposed technology.

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