Kinematic Analysis and Simulation of an Uncoupled Translational Parallel Mechanism

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Abstract: A novel uncoupled translational parallel mechanism (UTPM) with three degrees of freedom (3-DOFs) is presented, which is composed of a moving platform connected to a fixed base by three identical kinematic chains. Firstly, kinematic output property of the moving platform is analyzed. Then, both the forward and the inverse analytical solutions of the kinematic problems are discussed in detail. The velocity Jacobian is a 3×3 diagonal matrix, which implies that there exists a one-to-one corresponding relation between the inputs and the outputs of the parallel mechanism. Finally, the kinematic simulation is carried out by MATLAB and Pro/E software. Simulation results show that the theoretical analysis is correct and feasible.

1.   Introduction

Parallel mechanism (PM), which comprises of a moving platform (MP) connected to a fixed base (FB) by several serial open-single kinematic chains, is the essential part of robot technology and has been extensively studied by many scholars during past decades[1-2]. Contrasted with 6-DOFs PM, the limited-DOF PM possesses many advantages in terms of simpler mechanical structure and control system, easier kinematics and dynamic analysis. Due to those superiorities, the limited-DOF PM owns the expansive application prospect in advanced manufacturing industry and robot industry[3-4].

Translational parallel mechanism (TPM) with 3-DOFs has been concerned specially, which has widely applications in the fields of industrial robots, machine tools, medical robots, and so on. Although the kinematic coupling brings about some merits in terms of higher rigidity and stronger load capacity, it also produces some problems on kinematics analysis and design of control system. Uncoupled parallel mechanism (UPM), however, possesses the special property that one input of the actuator only effect one output of the platform. Therefore, it has been highlights in mechanism field[5-6], recently. Kong and Gosselin[7] proposed a 3-CRR UTPM. Li[8] presented a kind of decoupled TPMs. Zhang[9] synthesized a type of 3-DOFs UTPMs based on reciprocal screw theory.

In this paper, a novel 3-DOFs UTPM is presented, in which the MP is linked to the FB by three serial open-single kinematic chains. Other contents are arranged as follows: firstly, a brief description of architectural design of the PM are performed, Secondly, the kinematic output property of the MP is discussed, thirdly, both the forward and the inverse kinematic problems are analyzed, finally, the simulation of this PM was carried out by MATLAB and Pro/E software, which concludes that the theoretic analysis is correct and verifies that the TPM is uncoupling.

2. Design of the parallel mechanism

Structure of the TPM proposed here is shown in Figure 1. It is composed of three kinematic chains with identical architecture. For the ith chain, it is regarded as $R_{i1} \mathbin{/\!/} R_{i2} \perp R_{i3} \mathbin{/\!/} (\perp P_{i4} \perp) R_{i5}$ (i=1, 2, 3) in sequence from the base to the platform. Here R, P denote the revolute and the prismatic joints, respectively, the symbol "$/\!/$" and "$\perp$" represent the axes of the neighboring joints being parallel or perpendicular to each other. The assembly conditions of the chains are that the axes of R_{i1} joints are normal to each other and intersected together. Similarly, the axes of R_{i5} joints must be also satisfied the same condition. Origin O of the global frame $Oxyz$ lies on the intersection point of the axes of R_{i1} joints. The axes of R_{11}, R_{21} and R_{31}, joints are superimposed with three axes of the global frame, respectively.
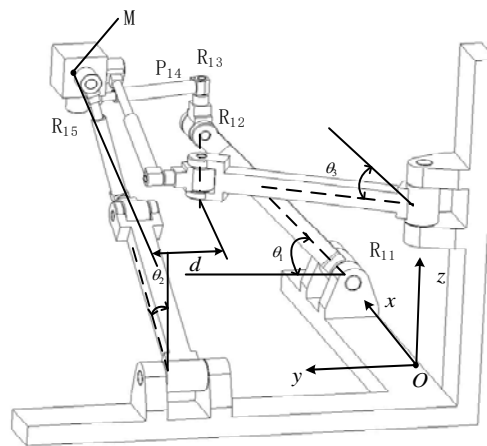

Figure 1 The structure diagram of the UTPM

3. Mobility analysis

The output motion of the MP for the PM is determined by the constraints provided by all of kinematic chains. In other words, it is the intersection set of output motion characteristic matrixes of all kinematic chains and can be expressed as

$$M_p = \bigcap_{i=1}^{n} M_{li} \tag{1}$$

where M_p denotes the output motion characteristic matrix of the MP, M_{li} is output motion characteristic matrixes associated with the ith kinematic chain, n expresses the mount of the kinematic chains.

Based on Figure 1, the output motion characteristic matrix of the UTPM is obtained, and have

$$M_p = \begin{bmatrix} x & y & z \\ \hat{x} & \bullet & \hat{z} \end{bmatrix} \cap \begin{bmatrix} x & y & z \\ \hat{x} & \hat{y} & \bullet \end{bmatrix} \cap \begin{bmatrix} x & y & z \\ \bullet & \hat{y} & \hat{z} \end{bmatrix} = \begin{bmatrix} x & y & z \\ \bullet & \bullet & \bullet \end{bmatrix} \tag{2}$$

in which the symbol '$\bullet$' expresses the constant in the motion characteristic matrix. From Eq.(2), we conclude that the MP only possesses three translational DOFs along the axes of the global frame.

4. Kinematics analysis

4.1. Position Analysis

Position analysis is to determine the relationship between the input displacements of the actuated

joints and the output displacements of the platform. As for the mechanism shown in Figure 1, to select the centroid M of the MP as research object, its coordinate is defined as M($x_\text{M}, y_\text{M}, z_\text{M}$) with respect to the frame *Oxyz*. *d* is the distance between the axis of R_{i2} and the line passing through point M and parallel to the corresponding axis of the frame. *l* is the length of the link connected with R_{i1} and R_{i2} joints.

4.1.1. Forward position analysis

The forward position analysis is to compute the position coordinate M($x_\text{M}, y_\text{M}, z_\text{M}$) of the MP when all generalized input angular displacements θ_i ($i=1,2,3$) of the actuators are given. According to the structure of mechanism, it is very convenient to find that

$$x_\text{M} = l\sin\theta_2 + d \tag{3}$$

$$y_\text{M} = l\sin\theta_3 + d \tag{4}$$

$$z_\text{M} = l\sin\theta_1 + d \tag{5}$$

4.1.2. Inverse position analysis

The inverse position analysis is to find the input angular displacements θ_i of the actuators when the position coordinate ($x_\text{M}, y_\text{M}, z_\text{M}$) of the point M are given. On the basis of Eqs.(3)-(5), yields

$$\theta_1 = \sin^{-1}\left[(z_\text{M} - d)/l\right] \tag{6}$$

$$\theta_2 = \sin^{-1}\left[(x_\text{M} - d)/l\right] \tag{7}$$

$$\theta_3 = \sin^{-1}\left[(y_\text{M} - d)/l\right] \tag{8}$$

4.2. Velocity Analysis

Differentiating Eqs.(3)-(5) with respect to time *t* and rewriting them as matrix form, the direct velocity vector equation can be obtained as follows

$$\begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = \begin{bmatrix} 0 & l\cos\theta_2 & 0 \\ 0 & 0 & l\cos\theta_3 \\ l\cos\theta_1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_{11} \\ \dot{q}_{21} \\ \dot{q}_{31} \end{bmatrix} \tag{9}$$

where $[v_x, v_y, v_z]^\text{T}$ and $[\dot{q}_{11}, \dot{q}_{21}, \dot{q}_{31}]^\text{T}$ represent the output velocity vector of the MP and the input angular velocity vector of the actuators, respectively. Clearly, the Jacobian is equivalent to a diagonal matrix. Therefore, there exists a one-to-one corresponding relationship between the inputs and the outputs. That is to say, one output of the platform is only controlled by one input of the actuators and the mechanism shows uncoupled performance throughout the workspace.

4.3. Acceleration analysis

According to the method of the second order influence coefficient[10], the acceleration equation can be written as

$$a_\text{M}^\text{T} = \dot{q} H_q^{\text{M}} \dot{q}^\text{T} + G_q^{\text{M}} \ddot{q}^\text{T} \tag{10}$$

where $a_\text{M} = [a_x, a_y, a_z]$ denotes the acceleration vector of the PM, $\ddot{q} = [\dot{q}_{11}, \dot{q}_{12}, \dot{q}_{13}]$ represents the angular acceleration vector of the actuators, G_q^{M} is the first influence coefficient matrix of the velocity equation, i.e., Jacobian matrix, H_q^{M} is the second order influence coefficient matrix, and have

$$H_q^{\text{M}} = \begin{bmatrix} H_{11} & H_{12} & H_{13} \\ H_{21} & H_{22} & H_{23} \\ H_{31} & H_{32} & H_{33} \end{bmatrix} \tag{11}$$

in which all of elements in matrix can be calculated, and have, $H_{11} = [-l\sin\theta_1 \quad 0 \quad 0]^\text{T}$,

$H_{12}=H_{21}=[0 \quad 0 \quad 0]^{\mathrm{T}}$, $H_{22}=[0 \quad -l\sin\theta_2 \quad 0]^{\mathrm{T}}$, $H_{13}=H_{31}=[0 \quad 0 \quad 0]^{\mathrm{T}}$, $H_{33}=[0 \quad 0 \quad -l\sin\theta_3]^{\mathrm{T}}$, $H_{23}=H_{32}=[0 \quad 0 \quad 0]^{\mathrm{T}}$.

Substituting all known parameters into Eq.(10), acceleration of the point M will be gotten. After transforming the form of Eq.(10), when the inverse of G_q^{M} exists, the inverse acceleration equation is obtained as well, and

$$\ddot{q}^{\mathrm{T}} = G_q^{\mathrm{M}-1}(a_{\mathrm{M}}^{\mathrm{T}} - \dot{q}H_q^{\mathrm{M}}\dot{q}^{\mathrm{T}}) \tag{12}$$

5. Kinematics simulation

Pro/E software possesses well performance in modeling and simulation of the virtual prototype. Firstly, we use it to establish the 3-D model of the virtual prototype as shown in Fig.1. Then , adding three angular velocity equations to the servo motors, respectively. The kinematic expressions of the actuated joints are as follows, $\dot{q}_{11}=10\sin(\pi t/10)$, $\dot{q}_{12}=7.5\sin(\pi t/10)$ and $\dot{q}_{13}=12.5\sin(\pi t/10)$. At the same time, we adapt that the initial parameters of the actuated joints are selected as $\theta_1=\theta_2=\theta_3=\pi/4$. Moreover, other structural parameters are also given, such as $d=122.605\text{mm}$, $l=204.680\text{mm}$. Simulation results are described in Fig.2, in which (a), (b) and (c) respectively present the position, velocity and acceleration curves.
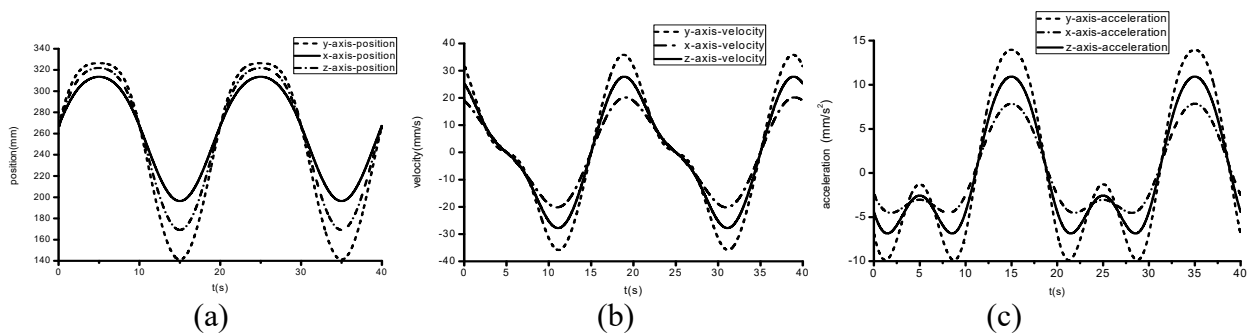

(a)   (b)   (c)

Figure 2 Kinematic curves carried out by Pro/E

Similarly, all kinematic curves of point M are also obtained by using MATLAB software, according to the theoretical mathematic models derived above, see Figure 3.
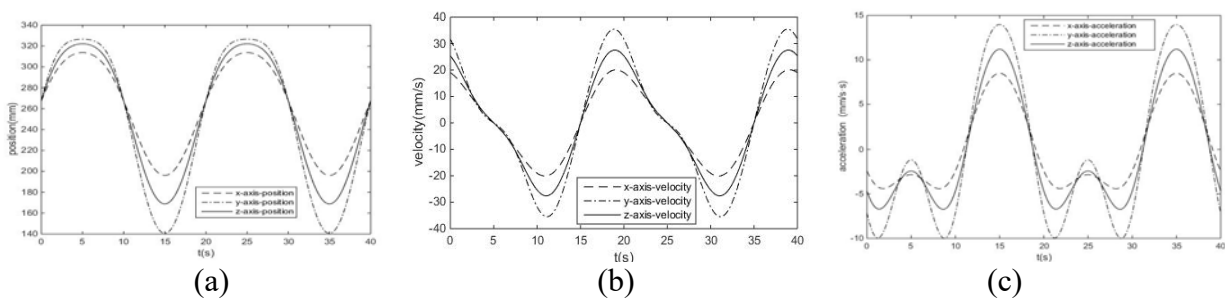

(a)   (b)   (c)

Figure 3 Kinematic curves carried out by MATLAB

To compare with Figure 3 and Figure 2, it manifests that the kinematic curves carried out by Pro-E and MATLAB software are identical respectively. Therefore, it concludes that the theoretical mathematic models established in this paper are correct.

6. Conclusion

This paper presented a novel UTPM, in which three kinematic chains are arranged in the space with orthogonal characteristic. Firstly, the kinematic output property is obtained. Secondly, the

relationship between input and output motion is achieved, which can achieve that the Jacobian matrix is diagonal. Thirdly, simulation carried out by Pro-E and MATLAB software, it proves the correctness of theoretical analysis, meanwhile, it concludes that the TPM is uncoupling. The UTPM can solve the problem of the kinematics and dynamic analysis and the bigger accumulative error of the MP brought by the kinematic coupling. Simultaneously, it can reduce accumulative error of the MP and simplify the control system. Because of superiority brought by kinematic uncoupling, it possesses a grander prospect in application of automatic production line and intelligent manufacturing system.

Acknowledgments

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