# Misalignment Angle Identification Method of Photoelectric Sensor for Floated Inertial Platform 

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#### Abstract

A method of attitude measurement using general commercial photoelectric sensor is proposed for the floated inertial platform. The installation error of the photoelectric sensor is analyzed and modeled, and the weighted recursive least squares algorithm is proposed to identify the misalignment angle. The results show that the photoelectric attitude measurement method can realize the high precise attitude measurement of the shell, and the designed weighted recursive least squares algorithm can identify the misalignment angle of photoelectric sensor effectively and accurately.


## 1. Introduction

Floated inertial platform (also known as advanced inertial reference sphere) is a frameless inertial platform ${ }^{[1]}$. The hydrostatic effect is used to stabilize the platform instead of the gimbal system of traditional inertial platform so that it can provide a high accuracy inertia reference for the carrier. The floated inertial platform was equipped in the Peacekeeper intercontinental ballistic missile (ICBM), which possessed a circular error probability of 90 m to 130 m under the flying range of $11,000 \mathrm{~km}^{[2]}$. Thus, the Peacekeeper installing the floated inertial platform is the most precise ICBM using the pure inertial navigation.

The main structure of the floating inertial platform is shown in Figure 1.The inertial devices are installed in the sphere. The gap between the sphere and the shell is filled with the liquid. The sphere is supported in the shell under the hydrostatic support of the suspension pad. The shell is fixed in the carrier. The sphere keeps stable in the inertial space and provides the inertial reference for the carrier. The attitude of the carrier can be obtained by the attitude measurement of the shell relative to the sphere.

The sphere and the shell are separated by the liquid. So it is necessary to utilize a non-contact attitude measurement method. At present, the main attitude measurement methods are classified as follows:

1) Acoustic ranging attitude measurement method: In the late seventies of the last century, Stephen ${ }^{[3]}$ proposed an acoustic ranging attitude measurement method. This method uses an acoustic transmitter mounted on the outer surface of the sphere to emit a signal and uses the acoustic receiver mounted on the shell to receive the signal. The distance between the transmitter and receiver is calculated so as to solve the attitude of the shell. However, the acoustic wave produces the complex echoes during the spread process, which results in the difficulty of processing the signal and data.
2) Electromagnetic induction attitude measurement method: Fan Weiguang et al ${ }^{[4]}$. introduced the electromagnetic induction attitude measurement method of the floated inertial platform. This attitude measuring system consists of three drive bands perpendicular to each other on the sphere, a signal receiving band on the shell and the associated electronic circuit. The drive band generates excitation with a continuous sinusoidal pulse current, and the receiving band is an inductive
receiver which senses electrical signals produced by electromagnetic induction. But this method is easy to be disturbed by electromagnetic interference, for example, the motor, pump and other devices installed in the sphere will produce electromagnetic interference in their work process. This will affect the measurement processing and then influence the accuracy of attitude measurement.
3) Capacitive attitude measurement method: Zhou Kaishi et al ${ }^{[5]}$. proposed a method for measuring the attitude of the shell using a ring capacitance sensor, and Hu Yuemin ${ }^{[6]}$ carried out theoretical derivation and simulation analysis of the method. Similar to the electromagnetic induction attitude measurement method, this method also possesses three drive band and one receiving band. However, a flat capacitor is formed between the drive band and the receiving band instead of electromagnetic induction. When the shell rotates relative to the sphere, the attitude information can be obtained by indirectly measuring the capacitance of the capacitor. However, the shortcomings of this system are the huge difficulty and high cost of manufacturing the accurate attitude band with exquisite metal stripes.


In order to overcome the shortcomings of the above methods, this paper presents an attitude measurement method using general commercial photoelectric sensors, that is, photoelectric attitude measurement method. The photoelectric attitude measurement method has the characteristics, such as high precision, high reliability ${ }^{[7]}$, small volume, light weight ${ }^{[8]}$ and fast response, and conforms to the requirements of the floated inertial platform. Since the photoelectric sensor is mounted on the sphere, the assembly errors are inevitable. In this paper, the principle of error generation is analyzed based on the proposed photoelectric attitude measurement method and the identification method of the misalignment angle is proposed.

## 2. Attitude Calculation Model

### 2.1. Installation Position, Measurement Principle and Coordinate System Definition

In order to describe the mounting position of the photoelectric sensors, some coordinate systems are established as follows.
(1) Sphere coordinate system ( $\mathrm{C}-x_{c} y_{c} z_{c}$ ): This coordinate system, as shown in Figure 2, is fixed on the sphere. Its origin $C$ is situated at the geometrical center of the sphere. The coordinate axes are parallel to the sensing directions of the three gyroscopes, which are installed in the sphere orthogonally and constitute a right-hand system.
(2) Photoelectric coordinate system ( $\mathrm{P}_{i} x_{i} y_{i} z_{i}$ ): In Figure 2, eight suspension pads are mounted at the center of the eight octants of the sphere coordinate system. The axis of each suspension pad coincides with a radius of the sphere. Coordinate system $P_{i} x_{i} y_{i} z_{i}$ is the measurement coordinate
system of the $i$-th photoelectric sensor. Point $\mathrm{P}_{\mathrm{i}}$ is the origin, which is located at the intersection point of the axis of the $i$-th suspension pad and the shell (the shell is not drawn). $\boldsymbol{R}_{c i}$ is the vector from point $C$ to point $\mathrm{P}_{\mathrm{i}}$. Axis $\mathrm{P}_{i} z_{i}$ points to the origin C of the sphere coordinate system. Supposing that the unit vector of axis $\mathrm{P}_{i} z_{i}$ is $\varepsilon_{i}$, axis $\mathrm{P}_{i} y_{i}$ points to the same direction of the vector $\boldsymbol{R}_{c i} \times \boldsymbol{\varepsilon}_{i}$. Axis $\mathrm{P}_{i} x_{i}$, axis $\mathrm{P}_{i} y_{i}$ and axis $\mathrm{P}_{i} z_{i}$ constitute a right-hand system.

The photoelectric sensor can only be sensitive to the movement of the object in the plane perpendicular to the optical axis, and the movement of the shell parallel to the optical axis can not be measured. Therefore, the photoelectric sensor measurement data is two-dimensional. Regardless of various errors, 2.2 and 2.3 will design the photoelectric attitude calculation model.

### 2.2. Rotation quaternion Calculation Method for a Single Rotation of the Shell

Supposing that the time step is sufficiently small, the rotation of the shell at each time step can be approximated by turning the shell around a fixed axis with a certain angle. Assuming that the shell rotates $\phi^{n}$ (rotation angle) around the axis $\boldsymbol{\eta}_{c}^{n}=\left[\begin{array}{ll}n c x \\ n & \eta_{c y}^{n} \\ \eta_{c z}^{n}\end{array}\right]$ (unit vector in the sphere coordinate system) in time step $n$, a single rotation process is shown in Figure 3.

In each time step, $\boldsymbol{\eta}_{c}^{n}$ is a constant vector. The output of the $i$-th photoelectric sensor at time $n$ is set as $\left[\begin{array}{lll}X_{i}^{n} & Y_{i}^{n} & 0\end{array}\right]^{\mathrm{T}}$ and the output of the $i$-th photoelectric sensor at time $n-1$ is set as $\left[\begin{array}{lll}X_{i}^{n-1} & Y_{i}^{n-1} & 0\end{array}\right]^{\mathrm{T}}$. Thus, the arc length of the shell measured by the $i$-th photoelectric sensor from time $n-1$ to time $n$ (time step $n$ ) can be written as follows.

$$
\begin{equation*}
l_{i}^{n}=\sqrt{\left(X_{i}^{n}-X_{i}^{n-1}\right)^{2}+\left(Y_{i}^{n}-Y_{i}^{n-1}\right)^{2}} \tag{1}
\end{equation*}
$$



Figure 3 A single rotation of the shell
The components of the arc length measured in the $i$-th photoelectric coordinate system are expressed as

$$
\boldsymbol{I}_{\mathrm{p} i}^{n}=-\left[\begin{array}{lll}
X_{i}^{n}-X_{i}^{n-1} & Y_{i}^{n}-Y_{i}^{n-1} & 0 \tag{2}
\end{array}\right]^{\mathrm{T}}
$$

The negative sign results from the fact that the measurement direction of the photoelectric sensor is always opposite to the actual movement direction of the measured object. When $\boldsymbol{I}_{\mathrm{p} i}^{n}$ is converted to the sphere coordinate system, $\boldsymbol{I}_{\mathrm{p} i}^{n}$ can be expressed as $\boldsymbol{I}_{c i}^{n}$. The transformation is shown as follows

$$
\begin{equation*}
\left.\boldsymbol{I}_{c i}^{n}=\boldsymbol{M}_{\mathrm{Pi} 2 \boldsymbol{l}} l_{\mathrm{P} i}^{n}, \quad \boldsymbol{I}_{\mathrm{P} i}^{n}=\left[\boldsymbol{M}_{\mathrm{Pi} i 2 C}\right]^{\mathrm{T}}\right]_{c i}^{n} \tag{3}
\end{equation*}
$$

where $\boldsymbol{M}_{P i C C}$ is the transformation matrix from the photoelectric coordinate system to the sphere coordinate system.

The rotation angle vector of the shell in time step $n$ is denoted as $\phi^{n}$, which possess a magnitude of $\phi^{n}$ and a direction of $\boldsymbol{\eta}_{c}^{n}$. $\boldsymbol{\phi}^{n}$ satisfies the following relation.

$$
\begin{equation*}
\phi^{n} \times \boldsymbol{R}_{c i}=\boldsymbol{M}_{P i 2 C} I_{P i}^{n} \tag{4}
\end{equation*}
$$

In this equation, $\boldsymbol{R}_{c i}$ and $\boldsymbol{M}_{\mathrm{PiPC}}$ are known according to the assembly, and $\boldsymbol{I}_{\mathrm{Pi}}^{n}$ can be measured
by the photoelectric sensors. This equation can be written as a form of matrix multiplication.

$$
\begin{equation*}
\left[\boldsymbol{R}_{c i}^{n} \times\right] \phi^{n}=-\boldsymbol{M}_{P i 2 C} I_{P i}^{n} \tag{5}
\end{equation*}
$$

In equation (5),

$$
\left[\boldsymbol{R}_{c i}^{n} \times\right]=\left[\begin{array}{ccc}
0 & -R_{c i x}^{n} & R_{c i y}^{n}  \tag{6}\\
R_{c i z}^{n} & 0 & -R_{c i x}^{n} \\
-R_{c i y}^{n} & R_{c i x}^{n} & 0
\end{array}\right]
$$

and $\phi^{n}$ can be calculated by the least square method. The solution is

$$
\begin{equation*}
\boldsymbol{\phi}^{n}=\left(\left[\boldsymbol{R}_{c i}^{n} \times\right]^{\mathrm{T}}\left[\boldsymbol{R}_{c i}^{n} \times\right]\right)^{-1}\left[\boldsymbol{R}_{c i}^{n} \times\right]^{\mathrm{T}}\left[-\boldsymbol{M}_{P i 2 C} \boldsymbol{I}_{P i}^{n}\right] \tag{7}
\end{equation*}
$$

where $i=1,2, \cdots, 8$. Thus, $\boldsymbol{\eta}_{\mathrm{c}}^{\mathrm{n}}$ can be obtained by standardizing $\boldsymbol{\phi}^{n}$. The magnitude of vector $\boldsymbol{\phi}^{n}$ is the rotation angle $\phi^{n}$.

After calculating the rotation axis $\boldsymbol{\eta}_{\mathrm{c}}^{\mathrm{n}}$ and the rotation angle $\phi^{n}$, the rotation quaternion $\boldsymbol{q}_{\text {shell }}^{n}$ of the shell rotating from time $n-1$ to time $n$ is obtained ${ }^{[9]}$.

$$
\boldsymbol{q}_{\text {shell }}^{n-1-n}=\left[\begin{array}{ll}
\cos \left(\phi^{n} / 2\right) & \boldsymbol{\eta}_{\mathrm{c}}^{\mathrm{n}} \cdot \sin \left(\phi^{n} / 2\right) \tag{8}
\end{array}\right]
$$

### 2.3. Method for Calculating Attitude of the Shell

In the previous section, the rotation quaternion calculation of a single rotation for the shell is completed. In order to obtain the attitude of the shell relative to the sphere at any time, the rotation quaternion from time 0 to time $n$ (any time) should be calculated. Then, the attitude of the shell can be obtained by transforming the rotation quaternion to Euler angle. Assuming that the shell coordinate system at the initial time (time 0 ) coincides with the sphere coordinate system, the rotation quaternion of the shell in each time can be obtained by using the measurement data.

From time 0 to time 1 (first time step), the shell rotates $\phi^{1}$ around the axis $\boldsymbol{\eta}_{c}^{1}$, and the rotation quaternion $\boldsymbol{q}_{\text {shell }}^{1}$ can be expressed as follows according to equation (8).

$$
\begin{equation*}
\boldsymbol{q}_{\text {shell }}^{0-1}=\left[\cos \left(\phi^{1} / 2\right) \quad \boldsymbol{\eta}_{\mathrm{c}}^{1} \cdot \sin \left(\phi^{1} / 2\right)\right] \tag{9}
\end{equation*}
$$

Assume that $\boldsymbol{q}_{\text {shell }}^{0-i}$ represents the rotation quaternion of the shell coordinate system rotating relative to the sphere coordinate system from time 0 to time i. $\boldsymbol{q}_{\text {Shell }^{0}-2}$ can be calculated according to the rotation quaternion multiplication.

$$
\begin{equation*}
\boldsymbol{q}_{\text {Stell }}^{0-2}=\boldsymbol{q}_{\text {shelel }}^{0-1} \otimes \boldsymbol{q}_{\text {shell }}^{1-2} \tag{10}
\end{equation*}
$$

From time 1 to time 2 (2nd time step), the shell rotates angle $\phi^{2}$ around the axis $\boldsymbol{\eta}_{c}^{2}$. In order to constructing the rotation quaternion in the second time step, it is necessary to project the rotation axis vector $\boldsymbol{\eta}_{\mathrm{c}}^{2}$ to the shell coordinate system of time 1 according to the rotation quaternion multiplication. Assuming that $\boldsymbol{\eta}_{\mathrm{s}}^{2}$ is the projection of $\boldsymbol{\eta}_{\mathrm{c}}^{2}$ in the shell coordinate system of time 1 , the rotation quaternion $\boldsymbol{q}_{\text {shell }}^{1-2}$ can be expressed as

$$
\boldsymbol{q}_{\text {Stell }}^{1-2}=\left[\begin{array}{ll}
\cos \left(\phi^{2} / 2\right) & \boldsymbol{\eta}_{\mathrm{s}}^{2} \cdot \sin \left(\phi^{2} / 2\right) \tag{11}
\end{array}\right]
$$

where $\boldsymbol{\eta}_{\mathrm{s}}^{2}$ can be obtained using the following equation according to the quaternion multiplication.

$$
\begin{equation*}
\boldsymbol{\eta}_{\mathrm{s}}^{2}=\left(\boldsymbol{q}_{\text {shell }}^{0-1}\right)^{*} \otimes \boldsymbol{\eta}_{\mathrm{c}}^{2} \otimes \boldsymbol{q}_{\text {shell }}^{0-1} \tag{12}
\end{equation*}
$$

Generally,

$$
\begin{equation*}
\boldsymbol{q}_{\text {shell }}^{0-n}=\boldsymbol{q}_{\text {sheell }}^{0-n-1} \otimes \boldsymbol{q}_{\text {shell }}^{n-1}{ }^{n-1} \tag{13}
\end{equation*}
$$

The shell rotates angle $\phi^{n}$ around the axis $\boldsymbol{\eta}_{\mathrm{c}}^{\mathrm{n}}$ from time $n-1$ to time $n$ ( $n$-th time step). The rotation quaternion $\boldsymbol{q}_{\text {shell }}^{n-1}$ n can be expressed as

$$
\boldsymbol{q}_{\text {shell }}^{n-1-n}=\left[\begin{array}{ll}
\cos \left(\phi^{n} / 2\right) & \boldsymbol{\eta}_{\mathrm{s}}^{\mathrm{n}} \cdot \sin \left(\phi^{n} / 2\right) \tag{14}
\end{array}\right]
$$

where $\boldsymbol{\eta}_{\mathrm{s}}^{n}$ can be calculated using the following equation.

$$
\begin{equation*}
\boldsymbol{\eta}_{\mathrm{s}}^{\mathrm{n}}=\left(\boldsymbol{q}_{\text {shell }}^{0-n-1}\right)^{*} \otimes \boldsymbol{\eta}_{\mathrm{c}}^{\mathrm{n}} \otimes \boldsymbol{q}_{\text {shell }}^{0-\mathrm{n}-1} \tag{15}
\end{equation*}
$$

Thus, the attitude of the shell at time $n$ can be calculated by transforming the rotation quaternion $\boldsymbol{q}_{\text {shhell }}^{0-n}$ to Euler angle according to the Z-Y-X reverse order.

## 3. Identification Method for Misalignment Angle of the Photoelectric Sensor

### 3.1. Misalignment Angle Analysis

In the installation process of the photoelectric sensor, there will always be installation errors in position and angle. Therefore, it is necessary to model and correct these errors.

The installation of the photoelectric sensor is based on the suspension pad which has a high installation accuracy, so the position error of the photoelectric sensor can be ignored. The inner cavity plane of the suspension pad also provides an accurate mounting plane for the photoelectric sensor so that the $z_{i}$-axis of the photoelectric sensor can be on the same line with the axis of the suspension pad. However, it is not possible to ensure that the $x$-axis and $y$-axis are consistent with their designed direction. So the installation error can be simplified to a misalignment angle rotating around the $z_{i}$-axis.

The designed coordinate transformation matrix from the sphere coordinate system to the photoelectric coordinate system is denoted as $\boldsymbol{M}_{C 2 P i}=\left[\boldsymbol{M}_{P i C}\right]^{\mathrm{T}}$, and the actual coordinate transformation matrix is denoted as $\hat{\boldsymbol{M}}_{\text {C2pi }}$. Only considering the influence of the misalignment angle, the coordinate transformation matrix satisfies the following relation.

$$
\begin{equation*}
\hat{\boldsymbol{M}}_{C 2 P i}=\boldsymbol{M}_{i}^{\prime} \boldsymbol{M}_{C 2 P i} \tag{16}
\end{equation*}
$$

In this equation, $\boldsymbol{M}_{i}^{\prime}$ is the coordinate transformation matrix from the designed photoelectric coordinate system to the actual photoelectric coordinate system, which can be expressed as follows.

$$
\boldsymbol{M}_{i}^{\prime}=\left[\begin{array}{ccc}
\cos \theta_{i} & \sin \theta_{i} & 0  \tag{17}\\
-\sin \theta_{i} & \cos \theta_{i} & 0 \\
0 & 0 & 1
\end{array}\right]
$$

In this equation, $\theta_{i}$ is the misalignment angle of the $i$-th photoelectric sensor.

### 3.2. Weighted Recursive Least Squares Method for Identifying the Misalignment Angle

$\hat{\boldsymbol{I}}_{\mathrm{p} i}^{n}$ represents the measurement considering the influence of the misalignment angle. $\boldsymbol{I}_{p i}^{n}$ represents the measurement ignoring the influence of the misalignment angle. $\hat{\boldsymbol{I}}_{\mathrm{p} i}^{n}$ and $\boldsymbol{I}_{P i}^{n}$ satisfy the following equation.

$$
\hat{\boldsymbol{I}}_{\mathrm{Pi}}^{n}=\mathbf{M}_{i}^{\prime} \boldsymbol{I}_{P i}^{n}=\left[\begin{array}{ccc}
\cos \theta_{i} & \sin \theta_{i} & 0  \tag{18}\\
-\sin \theta_{i} & \cos \theta_{i} & 0 \\
0 & 0 & 1
\end{array}\right] \boldsymbol{I}_{P i}^{n}
$$

Let $a_{i}=\sin \theta_{i}, \quad b_{i}=\cos \theta_{i}$. Changing the multiplication order of the equation (18), it can be transformed as follows.

$$
\left[\begin{array}{l}
\hat{l}_{\text {pix }}^{n}  \tag{19}\\
i_{\text {piij }}^{n} \\
\hat{l}_{\text {pii }}^{n}
\end{array}\right]=\left[\begin{array}{ccc}
l_{\text {piy }}^{n} & l_{\text {pix }}^{n} & 0 \\
-l_{p_{\text {pix }}}^{n} & l_{\text {piy }}^{n} & 0 \\
0 & 0 & l_{\text {piz }}^{n}
\end{array}\right]\left[\begin{array}{c}
a_{i} \\
b_{i} \\
1
\end{array}\right]
$$

The photoelectric sensor can only be sensitive to the movement of the object in the plane perpendicular to the optical axis, and the movement of the shell parallel to the optical axis can not be measured. Therefore, the photoelectric sensor measurement data is two-dimensional ${ }^{[10]}$. So $\hat{l}_{\mathrm{piz}}^{n}$ and $l_{\text {piz }}^{n}$ both are zero. Then equation (19) can be simplified as follows.

When the parameters $a_{i}$ and $b_{i}$ in the upper model are estimated, the error angle $\theta_{i}$ can be obtained. In this paper, a method is proposed which identify the misalignment angle using weighted recursive least squares.

Equation (20) can be transformed as

$$
\begin{equation*}
\boldsymbol{A}_{P_{i}}(n) \boldsymbol{x}_{P_{i}}+\varepsilon_{P i}(n)=\boldsymbol{b}_{P i}(n) \tag{21}
\end{equation*}
$$

where $\quad \boldsymbol{A}_{P i}(n)=\left[\begin{array}{cc}l_{\text {Piy }}^{n} & l_{\text {Pix }}^{n} \\ -l_{\text {Pix }}^{n} & l_{\text {Piy }}^{n}\end{array}\right], \quad \boldsymbol{x}_{P i}=\left[\begin{array}{lll}a & b\end{array}\right]^{\mathrm{T}}, \quad \boldsymbol{b}_{P i}(n)=\left[\begin{array}{ll}\tilde{l}_{\text {Pix }}^{n} & \tilde{l}_{\text {Piy }}^{n}\end{array}\right]^{\mathrm{T}} . \quad \boldsymbol{\varepsilon}_{P i}(n)=\left[\begin{array}{l}\varepsilon_{P i x}(n) \\ \varepsilon_{P i y}(n)\end{array}\right] \quad$ represents $\quad$ the measurement noise at time $n$.

Supposing that the number of data sets is $l$, the following equations can be formed:

$$
\left[\begin{array}{c}
A_{P i}(1)  \tag{22}\\
A_{P i}(2) \\
\vdots \\
A_{P i}(I)
\end{array}\right] \boldsymbol{x}_{P i}+\left[\begin{array}{c}
\varepsilon_{P i}(1) \\
\varepsilon_{P i}(2) \\
\vdots \\
\varepsilon_{P i}(I)
\end{array}\right]=\left[\begin{array}{c}
\boldsymbol{p}_{P i}(1) \\
\boldsymbol{b}_{P i}(2) \\
\vdots \\
\boldsymbol{b}_{P i}(l)
\end{array}\right]
$$

$\boldsymbol{x}_{P i}$ can be estimated by the weighted recursive least squares ${ }^{[11]}$. $\hat{\boldsymbol{x}}_{P i k}=\left[\begin{array}{lll}\hat{a}_{i k} & \hat{b}_{i k}\end{array}\right]^{\mathrm{T}}$ represents the estimated value at time $k$. Since the misalignment angle is very small, the misalignment angle of the $i$-th photoelectric sensor at time $k$ can be estimated by the following equation.

$$
\begin{equation*}
\hat{\theta}_{i k}=\arctan \left(\frac{\hat{a}_{i k}}{\hat{b}_{i k}}\right) \tag{23}
\end{equation*}
$$

## 4. Simulation Verification

### 4.1. Simulation Verification of Attitude Algorithm

The calculation period is set as 20 ms . The rotation axis $\boldsymbol{\eta}_{\mathrm{c}}^{\mathrm{n}}$ in each time step is given randomly. The magnitude of the rotation angle in each time step is given by the uniform distribution on the interval $\left(-0.573^{\circ}, 0.573^{\circ}\right)$. The measurement of the photoelectric sensor is the sum of the data from time 0 to the current time. The measurement of the shell in time step $n$ is obtained by subtracting the measurement at time $n-1$ from the measurement at time $n$. The measurement noise in each time step is set as Gaussian white noise.

Assuming that the resolution of the photoelectric sensor is 10000 DPI , the minimum distance that can be measured is $25.4 / 10000=0.00254 \mathrm{~mm}$. According to the $3 \sigma$ limit criterion of Gaussian white noise, the standard deviation of the measurement noise can be set as $\sigma=0.00254 / 3 \approx 0.0009 \mathrm{~mm}$. The designed Euler angle and the calculating Euler angle which is calculated by the attitude algorithm proposed in this paper are shown in Figure 4.


Figure 4 Comparison of designed attitude angle and calculating attitude angle
In Figure 4, the Euler angle is represented by pitch angle $\psi_{z}$, yaw angle $\psi_{y}$, roll angle $\psi_{x}$. It can be seen from Figure 4 that the calculating Euler angle and the designed Euler angle are very
close, and the calculation errors are shown in Figure 5. Figure 5 shows that the errors of the roll angle, yaw angle and pitch angle are not more than $0.04^{\circ}$.

### 4.2. Identification of Misalignment Angle

The designed misalignment angle is shown in Table 1. The misalignment angle identification is based on the attitude calculation method proposed in the previous paper. A series of measurement data are produced during the shell rotating relative to the sphere, which are the original data used to identify the misalignment angle. In the identification process, the inputs are the measurement ignoring the installation errors and the outputs are the measurement considering the misalignment angle. The misalignment angle of the photoelectric sensor 1 is shown in Figure 6, which is identified by the recursive least squares method.

Table 1 Designed misalignment angles

| Photoelectric sensor <br> number | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Designed <br> misalignment angles | $\theta_{1}=3.1^{\circ}$ | $\theta_{2}=3.2^{\circ}$ | $\theta_{3}=3.3^{\circ}$ | $\theta_{4}=3.4^{\circ}$ | $\theta_{5}=3.5^{\circ}$ | $\theta_{6}=3.6^{\circ}$ | $\theta_{7}=3.7^{\circ}$ | $\theta_{8}=3.8^{\circ}$ |



Figure 6 The identification process of misalignment angle of photoelectric sensor 1.
It can be seen from Figure 6 that each photoelectric sensor misalignment angle can be quickly identified within 100s. The misalignment angles of the other seven photoelectric sensors are identified in the same way, and the identification process is similar to that of Figure 6(not shown). The results of the misalignment angle identification of all photoelectric sensors are shown in Table2. It can be found that the relative errors of identification are less than $1 \%$.

Table2 Misalignment angle identification results of all photoelectric sensor

| Number | Identified parameters | Designed <br> value | Result of identification | Identification error | Relative error |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $\theta_{1}$ | $3.1^{\circ}$ | $3.117^{\circ}$ | $0.017^{\circ}$ | $0.54 \%$ |
| 2 | $\theta_{2}$ | $3.2^{\circ}$ | $3.202^{\circ}$ | $0.002^{\circ}$ | $0.06 \%$ |
| 3 | $\theta_{3}$ | $3.3^{\circ}$ | $3.311^{\circ}$ | $0.011^{\circ}$ | $0.33 \%$ |
| 4 | $\theta_{4}$ | $3.4^{\circ}$ | $3.398^{\circ}$ | $0.002^{\circ}$ | $0.05 \%$ |
| 5 | $\theta_{5}$ | $3.5^{\circ}$ | $3.492^{\circ}$ | $0.008^{\circ}$ | $0.22 \%$ |
| 6 | $\theta_{6}$ | $3.6^{\circ}$ | $3.604^{\circ}$ | $0.004^{\circ}$ | $0.11 \%$ |
| 7 | $\theta_{7}$ | $3.7^{\circ}$ | $3.706^{\circ}$ | $0.006^{\circ}$ | $0.16 \%$ |
| 8 | $\theta_{8}$ | $3.8^{\circ}$ | $3.781^{\circ}$ | $0.019^{\circ}$ | $0.50 \%$ |

## 5. Conclusion

In this paper, according to the non-contact characteristic between the sphere and shell of the floating inertial platform, an attitude measurement method is proposed using the general commercial photoelectric sensors. This method uses photoelectric sensors to measure the rotational arc length of the shell in each time step, and the rotation quaternion in this time step can be calculated. Finally, the rotation quaternion from initial time to current time can be calculated
according to the rotation quaternion multiplication. When the shell is randomly rotated, the error of the attitude calculation is not more than $0.04^{\circ}$. Thus, the attitude measurement method is verified. The installation error is analyzed and modeled in the actual working environment of the photoelectric sensor. The misalignment angles are identified by the weighted recursive least squares method. The identification results show that the relative error is less than $1 \%$, which testify that the identification method proposed in this paper has a good effect.

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