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Application of Kalman Filter Model Based on Hyperbolic Curve Model in the Deformation forecast

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Abstract: The hyperbolic curve model is erected, the least square method is used to obtain parameters of the hyperbolic curve model, parameters of the hyperbolic curve model are regarded as state vectors to contain dynamic noises to erect Kalman filter model based on the hyperbolic curve model, on the basis of Kalman filter model based on the hyperbolic curve model, settlement amounts of the building are forecasted. Because parameters of Kalman filter model change continuously in the process of Kalman filter, the ability that Kalman filter model suit the observation data is increased, and the fitting error of the model is lessened. An example of calculation shows that the forecast error is small, and the forecast effect is better to use Kalman filter model based on the hyperbolic curve model to forecast settlement amounts of the building .

Introduction

In order to guarantee the safety of buildings, the settlement observation must be done termly to obtain the settlement observation data, on the basis of the settlement observation data, the forecast model that the fitting precision and forecast precision is high can be erected. The model parameters are looked as fixed values in the hyperbolic curve model, the ability that the model suit the observation data is decreased, the fitting forecast is big, the forecast effect is not very well. In order to raise the fitting precision and forecast effect, model parameters of hyperbolic curve model are looked as state vectors to contain dynamic noises, Kalman filter model based on hyperbolic curve model is erected to forecast settlement amounts of the building. An example of calculation shows that the forecast error is small, and the forecast effect is better.

Hyperbolic curve model

Hyperbolic curve model can be written as[1]

$$\frac{t}{S_t - S_0} = a + bt \tag{1}$$

Where S_0 is the initial settlement value of the building, t is the observation time, a and b are model parameters of hyperbolic curve model, S_t is the settlement value of the building at time t.

Let $y = \frac{t}{S_t - S_0}$, (1) can be written as

$$y = a + bt \tag{2}$$

On the basis of the settlement observation series, model parameters a and b can be obtained by means of least square method, the settlement value of the building can be forecasted.

Kalman filter model

The state equation and observation equation of the discrete linear system to Kalman filter model are[2]-[10]

$$X_{k+1} = \Phi_{k+1,k} X_k + \Omega_k \tag{3}$$

$$L_{k+1} = B_{k+1} X_{k+1} + \Delta_{k+1} \tag{4}$$

Where X_k is the state vector at the time t_k, L_k is the observation vector at the time t_k, $\Phi_{k+1,k}$ is the state transfer matrix at the time t_k to t_{k+1}, B_{k+1} is the observation matrix at the time t_{k+1}, Ω_k is the dynamic noise at the time t_k, Δ_k is the observation noise at the time t_k.

The random model of Kalman filter method are[2]-[10]

$$E(\Omega_k) = 0,\ E(\Delta_k) = 0,\ \mathrm{cov}(\Omega_k, \Omega_j) = D_\Omega(k)\delta_{kj},\ \mathrm{cov}(\Delta_k, \Delta_j) = D_\Delta(k)\delta_{kj},\ \mathrm{cov}(\Omega_k, \Delta_j) = 0,$$
$$E(X_0) = \mu_X(0) = X(0/0),\ \mathrm{var}(X_0) = D_X(0),\ \mathrm{cov}(X_0, \Omega_k) = 0,\ \mathrm{cov}(X_0, \Delta_k) = 0 \tag{5}$$

If $j = k$, $\delta_{kj} = 1$, if $j \neq k$, $\delta_{kj} = 0$

Where $E(\Omega_k)$ is the mathematical expectation of Ω_k, $E(\Delta_k)$ is the mathematical expectation of Δ_k, $\mathrm{cov}(\Omega_k, \Omega_j)$ is the covariance of Ω_k and Ω_j, $D_\Omega(k)$ is the variance of Ω_k, $\mathrm{cov}(\Delta_k, \Delta_j)$ is the covariance of Δ_k and Δ_j, $D_\Delta(k)$ is the variance of Δ_k, $\mathrm{cov}(\Omega_k, \Delta_j)$ is the covariance of Ω_k and Δ_j, $E(X_0)$ is the mathematical expectation of X_0, $\mathrm{var}(X_0)$ is the variance of X_0, $\mathrm{cov}(X_0, \Omega_k)$ is the covariance of X_0 and Ω_k, $\mathrm{cov}(X_0, \Delta_k)$ is the covariance of X_0 and Δ_k.

On the basis of the state equation and observation equation and random model, Kalman filter equations are obtained[2]-[10]

$$X(k/k) = X(k/k-1) + J_k[L_k - B_k X(k/k-1)]$$
$$D_X(k/k) = [I - J_k B_k] D_X(k/k-1) \tag{6}$$

Where I is a unit matrix, and

$$X(k/k-1) = \Phi_{k,k-1} X(k-1/k-1)$$
$$D_X(k/k-1) = \Phi_{k,k-1} D_X(k-1/k-1) \Phi_{k,k-1}^T + D_\Omega(k-1)$$
$$J_k = D_X(k/k-1) B_k^T [B_k D_X(k/k-1) B_k^T + D_\Delta(k)]^{-1} \tag{7}$$

Kalman filter model based on hyperbolic curve model

In order to improve the fitting precision of hyperbolic curve model, model parameters of hyperbolic curve model a and b are looked as the state vector to contain dynamic noises , Kalman filter method is used to filter to obtain the best estimated value of the state vector, the settlement value of the building can be forecasted, we can erect the following model

$$y_k = a + bt_k + \Delta_k \tag{8}$$

Where a and b are model parameters of hyperbolic curve model, t_k is the observation time, Δ_k is the observation noise at the observation time t_k, $y_k = \dfrac{t_k}{S_{t_k} - S_0}$.

Let $L_k = y_k$, $B_k = \begin{bmatrix} 1 & t_k \end{bmatrix}$, $X_k = \begin{bmatrix} a \\ b \end{bmatrix}$, (8) can be written as

$$L_k = B_k X_k + \Delta_k \tag{9}$$

(9) is the observation equation.

In order to do Kalman filter, X_k is looked as the state vector to contain the dynamic noise, we have

$$X_{k+1} = X_k + \Omega_k \tag{10}$$

(10) can be written as

$$X_{k+1} = \Phi_{k+1,k} X_k + \Omega_k \tag{11}$$

Where $\Phi_{k+1,k}$ is a unit matrix.

On the basis of (9) and (11), the state equation and observation equation of Kalman filter model can be

Obtained.

$$X_{k+1} = \Phi_{k+1,k} X_k + \Omega_k \tag{12}$$

$$L_{k+1} = B_{k+1} X_{k+1} + \Delta_{k+1} \tag{13}$$

On the basis of (5), (6), (7), (12),(13), filter values of a and b can be obtained, fitting values of Kalman filter model can be obtained, and fitting values of settlement amounts can be obtained.

Example of the calculation

The settlement data of the monitoring point JD6 of the building are calculated, on the basis of the settlement data, $D_{\Delta}(k)$ =±1mm.

Let $X(0/0) = \begin{bmatrix} 0.51351 \\ 0.02669 \end{bmatrix}$, where 0.51351 and 0.02669 are calculated values of parameter a and parameter b in the hyperbolic curve model, the initial value of the state vector and dynamic noise are regarded as irrelevance, we have

$D_X(0) = D_X(0/0) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $D_{\Omega}(k) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, some computed results are listed in Table 1.

Table 1. Settlement observation values and their computed results of the monitoring point JD6 of the building

Observation time (year-month)	Observation values (mm)	Fitting values of model 1 (mm)	Residual errors of model 1 (mm)	Fitting values of model 2 (mm)	Residual errors of model 2 (mm)
2016-2	12.22	11.831	-0.389	12.130	-0.090
2016-3	13.78	13.508	-0.272	13.839	0.059
2016-4	14.11	15.034	0.924	14.192	0.082
2016-5	16.34	16.429	0.089	16.668	0.328
2016-6	17.65	17.708	0.058	17.640	-0.010
2016-7	18.12	18.886	0.766	18.134	0.014
2016-8	20.33	19.975	-0.355	20.301	-0.029
2016-9	21.01	20.983	-0.027	21.014	0.004
2016-10	22.17	21.920	-0.250	22.166	-0.004
2016-11	23.38	22.793	-0.587	23.376	-0.004

Model 1 is hyperbolic curve model, model 2 is Kalman filter model based on hyperbolic curve model, residual errors mean that fitted values of models subtract observation values

Table 1 shows that residual errors of hyperbolic curve model are great, the greatest residual error is 0.924 mm, the smallest residual error is -0.027 mm. Residual errors of Kalman filter model based on hyperbolic curve model are smaller than 0.33 mm, and the greatest residual error is 0.328 mm, the smallest residual error is 0.004 mm, the fitting precision of Kalman filter model based on hyperbolic curve model is higher.

The forecasted settlement value of the monitoring point JD6 of the building of hyperbolic curve model is 23.608 mm on December, 2016, the settlement observation value on December, 2016 is 24.03 mm, the forecast error is -0.422 mm, the forecast error is greater.

The forecasted settlement value of the monitoring point JD6 of the building of Kalman filter model based on hyperbolic curve model is 24.211 mm on December, 2016, the settlement observation value on December, 2016 is 24.03 mm, the forecast error is 0.181 mm, the forecast error is smaller, the forecast effect is better.

Conclusions

The paper looked parameters of hyperbolic curve model as the state vector to erect Kalman filter model based on hyperbolic curve model, the model is used to forecast the settlement value of the building. The calculation result show that Kalman filter model based on hyperbolic curve model is good in the suitability, the model precision and deformation forecast precision is high, the forecast effect is better.

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