ATLANTIS PRESS

Advances in Engineering Research (AER), volume 143

6th International Conference on Energy and Environmental Protection (ICEEP 2017)

Viscoelastic analysis for circular cylinders using the eigenvector expansion method

Weixiang Zhang[1, a], Li Chen[2,b]

[1]Nantong polytechnic college, Nantong 450052, PR China

[2] School of Civil Engineering and Architecture, Henan University of Technology, Zhengzhou 450052, PR China

[a]Weixiang_zh@163.com, [b]Lichen@163.com

Keywords: Hamiltonian system; cantilever-beam; eigenvector

Abstract. In this paper, we introduce the symplectic method into the quasi-static analysis of the viscoelastic circular cylinders, with the emphasis on the local effects. By using the of variable separation method, all the general eigenvectors of the problem are obtained analytically. The traditional Saint-Venant problems and local effect problems can described by combinations of the eigenvectors. Numerical results show the local effects due to the displacement constraints.

Introduction

Recently, a great amount of computational techniques can be found in the research of viscoelastic problems [1]. However, it is difficult to find analytical solutions because of the complexity of the constitutive relations. Therefore, the numerical method is taken into account, especially the finite element method, which iswidely applied in the analysis of viscoelastic problems, and has been proved to be an effective method [2]. The other important tool is the boundary element method. Compared with the finite element method, the boundary element method can reduce the dimension of the problem and provide an attractive idea for the viscoelastic research [3]. Based on di□erential constitutive relations, Mesquita and Coda [4] provided the important algebraic equations and presented a method for the treatment of two dimensional coupling problems between the finite element method and the boundary element method by discussing Kelvin and Boltzmann models in a two-dimensional boundary element atmosphere.

In this paper, the symplectic method is further applied to viscoelastic circular cylinder problems. On the basis of the variable separation method and the Laplace intagerl, all the fundamental eigenvectors of the governing equations are obtained in the Laplace domain directly. Because the eigenvectors are expressed in concise analytical forms, their corresponding expressions in the time domain can be obtained easily. Thus the eigenvector expansion method can be applied directly in the eigenvector space of the time domain, and the iterative application of Laplace transform is not needed. Moreover, the boundary conditions, such as displacement conditions, stress conditions and mixed conditions of displacement and stress, can be conveniently described by the fundamental variables, since the generalized displacements and stresses are used as fundamental variables. As the application of the method, some typical numerical examples are presented to describe the stresses and deformations of the viscoelastic hollow circular cylinder under certain boundary conditions. The results demonstrate that local effects due to displacement constraints are significant but only confined to a local region near the end. Because of the viscoelastic property of the material, the displacements exhibit the creep phenomenon when the cylinder subject to external force.

Governing equations in the symplectic system

ConsiderA homogeneous isotropic viscoelastic hollow circular cylinder. Let the origin of the cylindrical coordinates $(r,q\,,\,z)$ be located at the center of the cylinder, with the z axis pointing the longitudinal direction. We suppose that the behavior of the material is governed by the standard viscoelastic model., which consists of a spring and Kelvin model connected in series, where the spring is used to represent the instantaneous elastic response and the Kelvin model to represent the delayed elastic response. The bulk behavior of the model is supposed purely elastic, which results in a constant modulus K. Thus, the material is characterized by two shear moduli G_1 and G_2, one viscosity coefficient h, and one bulk modulus K. In order to derive the final governing equations of the symplectic system, the function of the strain energy density in the Laplace domain is written as

$$L = \frac{\bar{E}\bar{v}r}{2(1+\bar{v})(1-2\bar{v})}\left(\frac{\partial \bar{u}}{\partial r}+\frac{\bar{u}}{r}+\dot{\bar{w}}\right)^2 + \frac{\bar{E}r}{2(1+\bar{v})}\left[\left(\frac{\partial \bar{u}}{\partial r}\right)^2+\frac{\bar{u}^2}{r^2}+\dot{\bar{w}}^2+\frac{1}{2}\left(\frac{\partial \bar{w}}{\partial r}+\dot{\bar{u}}\right)^2\right] \tag{1}$$

in which a dot on a variable represents its partial differential. To convert the Lagrangian description to Hamiltonian, write the displacement variables in vector form $\bar{\mathbf{q}}=\{\bar{u},\ \bar{w}\}^{\mathrm{T}}$. The symplectic system method requires the dual vector $\bar{\mathbf{p}}$ be obtained by direct differentiation of the function of the strain energy density

$$\bar{\mathbf{p}} = \frac{\partial L}{\partial \dot{\bar{\mathbf{q}}}} = \begin{Bmatrix} \dfrac{\bar{E}r}{2(1+\bar{v})}\left(\dot{\bar{u}}+\dfrac{\partial \bar{w}}{\partial r}\right) \\ \dfrac{\bar{E}\bar{v}r}{(1+\bar{v})(1-2\bar{v})}\left(\dfrac{\partial \bar{u}}{\partial r}+\dfrac{\bar{u}}{r}+\dot{\bar{w}}\right)+\dfrac{\bar{E}r}{1+\bar{v}}\dot{\bar{w}} \end{Bmatrix} = \begin{Bmatrix} r\bar{t}_{rz} \\ r\bar{s}_z \end{Bmatrix}$$

(2)

Using the principle of minimum total potential energy, we get

$$\dot{\bar{\mathbf{Y}}} = \mathbf{H}\bar{\mathbf{Y}} \tag{3}$$

where $\bar{\mathbf{Y}} = \{\bar{u},\ \bar{w},\ \bar{p}_1,\ \bar{p}_2\}^{\mathrm{T}}$, and $\mathbf{H}$ is the Hamiltonian operator matrix. the eigenequation in the symplectic system is obtained as

$$\mathbf{H}\bar{y}(r) = k\bar{y}(r) \tag{4}$$

where k is the eigenvalue, and $\bar{y}$ is the corresponding eigenvector.

Nonzero eigenvectors

For axial symmetric case, zero eigenvectors only consist of solutions of the rigid body motion and the simple tension along z direction, and they can't reveal the local effects. In this section, Nonzero eigenvectors corresponding to the local effects, which decay exponentially in the distance from the edges according to the Sain-Venant principle, will be discussed. Consider Eq. (4), we have

$$(\mathbf{H}-k\mathbf{I})\bar{y} = 0 \quad (k \neq 0) \tag{5}$$

The displacement components of the general solution of Eq. (5) can be expressed as

$$\begin{aligned} \bar{U} &= R_r - \frac{1}{4(1-\bar{v})}\frac{d}{dr}(R_0 + rR_r) \\ \bar{W} &= -\frac{k}{4(1-\bar{v})}(R_0 + rR_r) \end{aligned} \tag{6}$$

in which the functions R_r and R_0 satisfy

$$\begin{aligned}&(r^2d^2+rd+r^2k^2)R_0=0\\&(r^2d^2+rd+r^2k^2-1)R_r=0\end{aligned}\tag{7}$$

respectively, where $d=d/dr$, $d^2=d^2/dr^2$. The solutions are

$$\begin{aligned}&R_0=c_1J_0(kr)+c_2Y_0(kr)\\&R_r=c_3J_1(kr)+c_4Y_1(kr)\end{aligned}\tag{8}$$

where c_1, c_2, c_3 and c_4 are integral constants, and $J_n(kr)$ and $Y_n(kr)$ are Bessel functions and Neumann functions, respectively. Substituting Eq. (8) into Eqs. (6), we get the nonzero eigenvector

$$\begin{aligned}&\bar{U}=a_{11}c_1+a_{12}c_2+a_{13}c_3+a_{14}c_4\\&\bar{W}=a_{21}c_1+a_{22}c_2+a_{23}c_3+a_{24}c_4\\&\bar{P}_1=a_{31}c_1+a_{32}c_2+a_{33}c_3+a_{34}c_4\\&\bar{P}_2=a_{41}c_1+a_{42}c_2+a_{43}c_3+a_{44}c_4\end{aligned}\tag{9}$$

where a_{ij} are constants, and $J_n=J_n(kr)$, $Y_n=Y_n(kr)$. As an example, we discuss the stress lateral boundary conditions:

$$\begin{aligned}&\bar{s}_r=\bar{s}_r^a,\quad \bar{t}_{rz}=\bar{t}_{rz}^a\quad (r=a)\\&\bar{s}_r=\bar{s}_r^b,\quad \bar{t}_{rz}=\bar{t}_{rz}^b\quad (r=b)\end{aligned}\tag{10}$$

To homogenize boundary conditions (10), let

$$\bar{\mathbf{Y}}^{**}=\{\bar{u}^{**},\ \bar{w}^{**},\ \bar{p}_1^{**},\ \bar{p}_2^{**}\}^{\mathrm{T}}\tag{11}$$

where $\bar{u}^{**}=(1-\bar{v})(\bar{s}_r^a a^2-\bar{s}_r^b b^2)/(\bar{E}b^2-\bar{E}a^2)r+(1+\bar{v})(\bar{s}_r^a-\bar{s}_r^b)a^2b^2/(\bar{E}b^2r-\bar{E}a^2r)$, $\bar{w}^{**}=\bar{p}_2^{**}=0$, and $\bar{p}_1^{**}=r(b-r)\bar{t}_{rz}^a/(b-a)+r(a-r)\bar{t}_{rz}^b/(a-b)$. Then the new governing equation is established as

$$\bar{\mathbf{Y}}^{*}=\bar{\mathbf{Y}}-\bar{\mathbf{Y}}^{**}\tag{12}$$

The governing equation is changed into nonhomogeneous one

$$\dot{\bar{\mathbf{Y}}}^{*}=\mathbf{H}\bar{\mathbf{Y}}^{*}+\mathbf{g}^{*}\tag{13}$$

Numerical calculations

The geometrical data and the parameter are selected as : $a/b=0.4$, $l/b=0.8$, $G_1=2G_2=K$. The end conditions are $w=u=0$ $(z=-l)$ and $s_z=p_0$, $t_{rz}=0$ $(z=l)$. According to Saint-Venant principle, the effect of displacement constraints is confined near the end $z=-l$ and decreases rapidly with the distance from the end. The point of view can be well explained by Fig. 1 and Fig. 2. The values of the figures are nearly constants near the end $z=l$, which demonstrate that zero eigenvectors provide the approximate solution. However, this approximation is not accurate enough near the clamped end $z = -l$, where the local effects are shown.

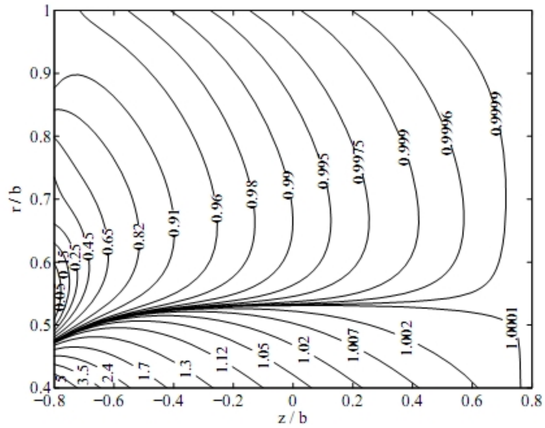

Fig. 1 Stress s_z / p_0 distribution

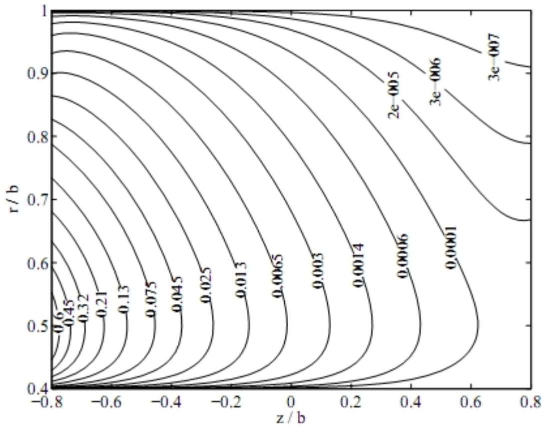

Fig. 2 Stress t_{rz} / p_0 distribution

Conclusion

On the basis of the adjoint symplectic relationships between the eigenvectors, the solution of the governing equation in the symplectic system can be expressed by the combination of eigenvectors. Because the eigenvectors in the time domain can be obtained easily by using the analytical inverse Laplace transform, the eigenvector expansion method can be applied in the time domain directly. Numerical results demonstrate that local effects appear in the region near the displacement constraints and decay rapidly with the distance from the boundary.

References

[1] L.Q. Chen, H. Chen, C.W. Lim: Int. J. Eng. Sci Vol. 46 (2008), p. 976

[2] M. Bottoni, C. Mazzotti, and M. Savoia: Int. J. Solids Struct Vol. 45 (2008), p. 770

[3] A.D. Mesquita and H.B. Coda:Appli. Math. Model Vol. 31 (2007), p. 1149

[4] A.D. Mesquita, H.B. Coda: Comput. Methods Appl. Mech. Engrg Vol.192 (2007), p.1911