

## Research of Phase-Shift Angle on the Differential Delta Auto-Connected Transformer

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**Abstract.** Using the traditional phase-shift angle, the differential delta-connected transformer is not able to boost voltage. Broadening the phase-shift angle of the autotransformer can boost its output voltage. But the greater phase-shift angle will increase the equivalent kilovolt ampere (KVA) rating as well. This paper analyzes the effect of phase-shift angle on the input currents and the equivalent (KVA) rating of the differential delta-connected transformer. A novel phase-shift angle is proposed, with which the differential delta-connected transformer is suitable to be a step-up autotransformer, meanwhile, the equivalent KVA rating of the autotransformer is still smaller than the isolated transformer. Some simulation is carried out to verify the correctness of the analyses.

### Introduction

Multi-pulse rectifiers are widely applied in high power rectification for input line harmonic currents reduction [1-5]. The multi-pulse is referred to as the number of output voltage pulse of dc, which is more than 6 pulses during every single cycle in three-phase power supplied system. Multi-pulse converter can reduce the input harmonics of grid voltages effectively. Theoretically, The more pulses a converter produce, the better performance of the input line currents will be. 12, 18, 24, 30-pulse rectifiers are proposed generally [6-9]. The 12-pulse rectifier not only owns the simpler circuit than other multi-pulse converters, but also is able to satisfy the requirements of most electric equipment. Therefore, 12-pulse rectifier is extensively used in high power rectification.

Phase shift transformer is the key device of multi-pulse converters [10]. Multi-pulse converters use isolated transformers to achieve a certain phase difference between input voltages and output voltages. The energy transmits by electromagnetic interference completely. While the multi-pulse autotransformer uses autotransformers to transmit energy by both electromagnetic method and circuit connection, by which the volume of rectifier will be reduced largely [11-12]. Without the strict requirements of electric isolation, autotransformers are usually used in multi-pulse rectifier to save volumes and reduce input mains harmonics. In general, phase transformer is used to generate several sets of phase-shift voltages, not to boost or lower voltages [13-14].

In application, output voltages need to be boost sometimes. This paper proposes a step-up differential delta-connected transformer by using a novel phase-shift angle. Under the novel phase-shift angle, the total harmonic distortion of input line currents is almost the same as the converter under the traditional phase-shift angle and the equivalent KVA rating of autotransformer is about 63%, which is still smaller than isolated transformer used in multi-pulse rectifier.

### 1 12-pulse Rectifier with the Differential Delta-Connected Transformer

Figure 1 shows the diagram of the 12-pulse rectifier with differential delta-connected transformer.

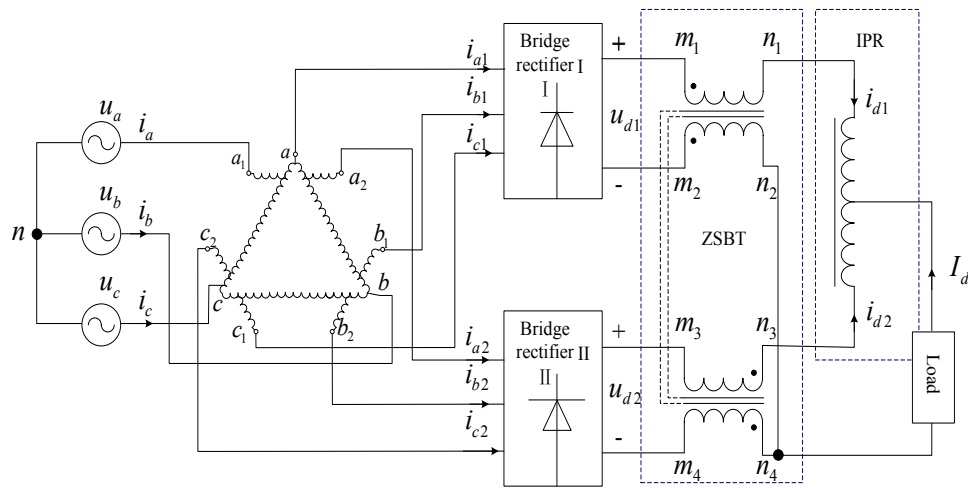


Fig. 1 Diagram of the 12-pulse rectifier with differential delta-connected transformer bridge

In figure 1, the relationship between the traditional phase-shift angle and the number of Diode Bridge is shown as

$$\varphi = \frac{\pi}{3N} \tag{1}$$

Where  $\varphi$  is the phase-shift angle of transformer and  $N$  is the number of diode bridge respectively. In figure 1, there are two diode bridges, thus the traditional 12-pulse rectifier phase-shift angle should be  $\pi / 6$ .

Assume input three-phase voltages as

$$\begin{cases} u_a = U_m \sin(\omega t) \\ u_b = U_m \sin\left(\omega t - \frac{2\pi}{3}\right) \\ u_c = U_m \sin\left(\omega t + \frac{2\pi}{3}\right) \end{cases} \tag{2}$$

Where  $U_m$  is the amplitude of the input phase voltage of differential delta-connected transformer.

Under large inductive load, the output currents of differential delta-connected transformer is derived as

$$\begin{cases} i_{a1} = \sum_{k=1,3,5,\dots}^{\infty} \frac{2I_d}{I_r \pi} \cos \frac{k\pi}{6} \sin k(\omega t + \alpha) \\ i_{b1} = \sum_{k=1,3,5,\dots}^{\infty} \frac{2I_d}{I_r \pi} \cos \frac{k\pi}{6} \sin k\left(\omega t + \alpha - \frac{2\pi}{3}\right) \\ i_{c1} = \sum_{k=1,3,5,\dots}^{\infty} \frac{2I_d}{I_r \pi} \cos \frac{k\pi}{6} \sin k\left(\omega t + \alpha + \frac{2\pi}{3}\right) \end{cases} \tag{3}$$

$$\begin{cases} i_{a2} = \sum_{k=1,3,5,\dots}^{\infty} \frac{2I_d}{T_{\pi}} \cos \frac{k\pi}{6} \sin k(\omega t - \alpha) \\ i_{b2} = \sum_{k=1,3,5,\dots}^{\infty} \frac{2I_d}{T_{\pi}} \cos \frac{k\pi}{6} \sin k\left(\omega t - \alpha - \frac{2\pi}{3}\right) \\ i_{c2} = \sum_{k=1,3,5,\dots}^{\infty} \frac{2I_d}{T_{\pi}} \cos \frac{k\pi}{6} \sin k\left(\omega t - \alpha + \frac{2\pi}{3}\right) \end{cases} \quad (4)$$

Where  $\alpha$  is half of the phase-shift angle  $\varphi$ .

## 2 Effect of Phase-Shift Angle on Input Line Currents

Figure 2 shows the winding configuration of differential delta-connected transformer.

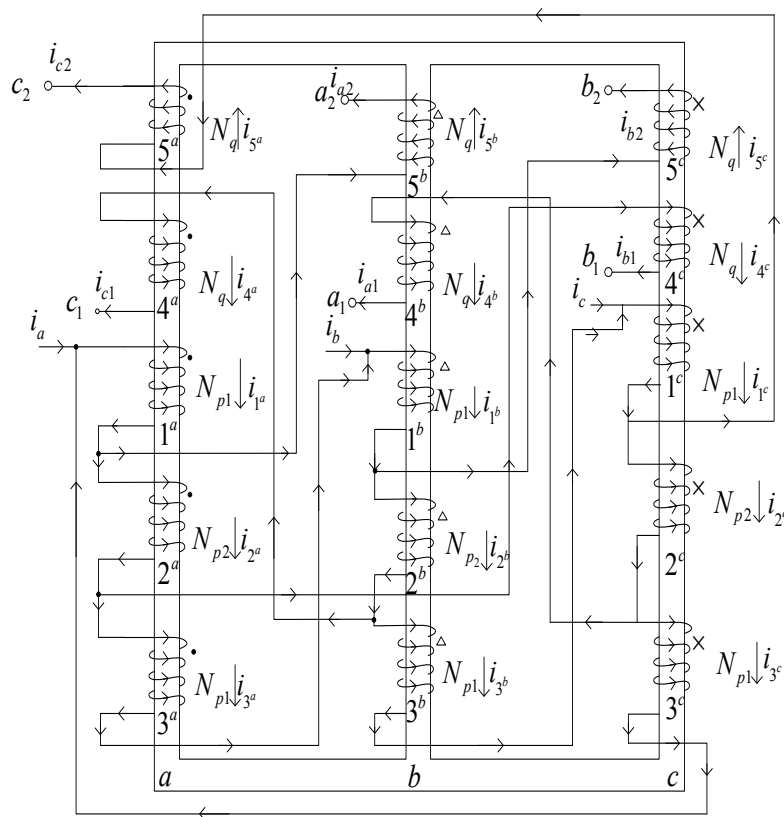


Fig.2 Diagram of the differential delta-connected transformer winding configuration

From figure 2, the MMF equations are expressed as

$$\begin{cases} i_{5a} N_q + i_{4a} N_p = i_{1a} N_{p1} + i_{2a} N_{p2} + i_{3a} N_{p1} \\ i_{5b} N_q + i_{4b} N_p = i_{1b} N_{p1} + i_{2b} N_{p2} + i_{3b} N_{p1} \\ i_{5c} N_q + i_{4c} N_p = i_{1c} N_{p1} + i_{2c} N_{p2} + i_{3c} N_{p1} \end{cases} \quad (5)$$

Where  $N_{p1}$  and  $N_{p2}$  both are primary turns number, while  $N_q$  is the secondary turns number.

KCL equations can be derived from figure 2 as

$$\left\{ \begin{array}{l} i_a + i_{3c} = i_{1a} \\ i_b + i_{3a} = i_{1b} \\ i_c + i_{3b} = i_{1c} \\ i_{1a} = i_{2a} + i_{5b} \\ i_{1b} = i_{2b} + i_{5c} \\ i_{1c} = i_{2c} + i_{5a} \\ i_{2a} = i_{3a} + i_{4c} \\ i_{2b} = i_{3b} + i_{4a} \\ i_{2c} = i_{3c} + i_{4b} \end{array} \right. \quad (6)$$

Figure 3 shows the phase diagram of differential delta-connected transformer.

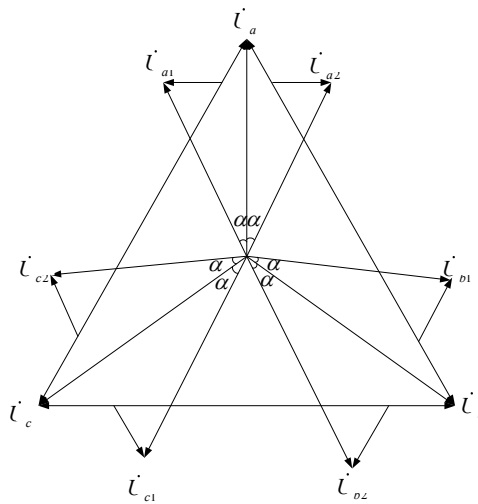


Fig. 3 Phase diagram of differential delta transformer

In figure 1, the input voltages of two diode bridges can be expressed as

$$\left\{ \begin{array}{l} u_{a1} = U_{m1} \sin(\omega t + \alpha) \\ u_{b1} = U_{m1} \sin\left(\omega t - \frac{2\pi}{3} + \alpha\right) \\ u_{c1} = U_{m1} \sin\left(\omega t + \frac{2\pi}{3} + \alpha\right) \\ u_{a2} = U_{m1} \sin(\omega t - \alpha) \\ u_{b2} = U_{m1} \sin\left(\omega t - \frac{2\pi}{3} - \alpha\right) \\ u_{c2} = U_{m1} \sin\left(\omega t + \frac{2\pi}{3} - \alpha\right) \end{array} \right. \quad (7)$$

Where  $U_{m1}$  is the amplitude of the output phase voltage of differential delta-connected autotransformer.

The relation between  $U_{m1}$  and  $U_m$  can be derived from figure 3 as

$$\cos \alpha = 0.9660 \frac{U_m}{U_{m1}} \tag{8}$$

In figure 3, the reasonable ranges of  $\alpha$  is  $0.02 < \alpha < \pi/2$ , therefore, the ranges of phase-shift angle  $\varphi$  is  $0.04 < \varphi < \pi$ .

From figure 2 and figure 3, the secondary and the primary winding turns ratio can be derived as

$$\begin{cases} \frac{N_{p1}}{N_q} = \frac{393}{9660 \tan \alpha - 197} \\ \frac{N_{p2}}{N_q} = \frac{16535}{9660 \tan \alpha - 197} \end{cases} \tag{9}$$

Take  $i_a$  as an example. From (5) and (6), the input line current  $i_a$  can be derived as

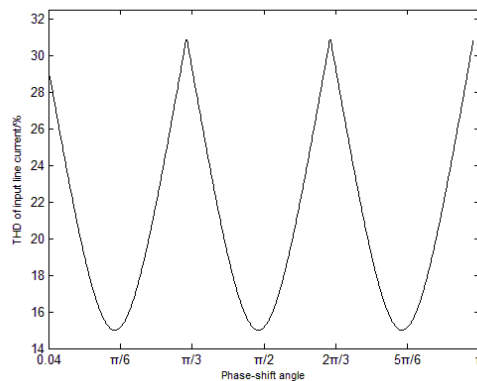
$$i_a = \frac{\frac{N_{p2}}{N_q}}{2 \frac{N_{p1}}{N_q} + \frac{N_{p2}}{N_q}} i_{a1} + \left( 1 - \frac{\frac{N_{p1}}{N_q}}{2 \frac{N_{p1}}{N_q} + \frac{N_{p2}}{N_q}} \right) i_{a2} + \left( 1 + \frac{\frac{N_{p2}}{N_q} - \frac{N_{p1}}{N_q} + 1}{2 \frac{N_{p1}}{N_q} + \frac{N_{p2}}{N_q}} \right) i_{b1} - \frac{1}{2 \frac{N_{p1}}{N_q} + \frac{N_{p2}}{N_q}} i_{b2} - \frac{1}{2 \frac{N_{p1}}{N_q} + \frac{N_{p2}}{N_q}} i_{c1} + \frac{1}{2 \frac{N_{p1}}{N_q} + \frac{N_{p2}}{N_q}} i_{c2} \tag{10}$$

The THD of  $i_a$  is expressed as

$$THD_{ia} = \frac{I_n}{I_1} \times 100\% \tag{11}$$

Where  $I_n$  is the RMS value of the ‘n’ th harmonic current of  $i_a$ ,  $I_1$  is the RMS value of the fundamental current of  $i_a$ .

Setting the highest harmonic current to ‘1000’th and Programming (9)and(10)into (11), we can obtain the relation between phase-shift angle and input current  $i_a$  in figure 4



**Fig. 4 Effect of phase shift angle on input current total harmonic distribution**

In figure 4, the minimum THD value of  $i_a$  is around 15%. When the THD of  $i_a$  reaches to its minimum value, the phase-shift angle  $\varphi$  gets three different values:  $\pi/6$ ,  $\pi/2$  and  $5\pi/6$ . Therefore,  $\pi/6$ ,  $\pi/2$  and  $5\pi/6$  all can be chosen as the phase-shift angle of differential delta-connected transformer to boost the output voltage.

Assume  $k_T = U_{m1}/U_m = 0.9660/\cos \alpha$ , when  $k_T$  is greater than 1, differential delta-connected transformer is operated as a step-up transformer; when  $k_T$  is smaller than 1, differential delta-connected transformer is operated as a step-down transformer. While  $\varphi$  is equal to  $\pi/6$ ,  $\pi/2$  and  $5\pi/6$ ,  $k_T$  is 1.001, 1.3661 and 3.7323 respectively. Therefore, when phase-shift angle is equal to  $\pi/2$  and  $5\pi/6$ , the differential delta-connected transformer is operated as a step-up transformer respectively.

Besides the THD value of the input currents, the equivalent KVA rating of the autotransformer is also important to an autotransformer.

### 3 Effect of Phase-Shift Angle on KVA of Differential Delta-Connected Transformer

Assume the RMS values of the voltage on the turns  $N_{p1}$ , turns  $N_{p2}$  and turns  $N_q$  are  $U_{p1}$ ,  $U_{p2}$  and  $U_q$  respectively, in figure 3, we can calculate  $U_{p1}$ ,  $U_{p2}$  and  $U_q$  as

$$\begin{cases} U_{p1} = \frac{0.0393}{\sqrt{2}} U_m \\ U_{p2} = \frac{1.6535}{\sqrt{2}} U_m \\ U_q = (0.9660 \tan \alpha - 0.0197) U_m \end{cases} \quad (12)$$

From (5) and (6),  $i_{3a}$ ,  $i_{3b}$  and  $i_{3c}$  can be derived as

$$\begin{cases} i_{3a} = \frac{i_{c2} - i_{c1} - \left( \frac{N_{p1}}{N_q} + \frac{N_{p2}}{N_q} \right) i_{b1} - \frac{N_{p1}}{N_q} i_{a2}}{2 \frac{N_{p1}}{N_q} + \frac{N_{p2}}{N_q}} \\ i_{3b} = \frac{i_{a2} - i_{a1} - \left( \frac{N_{p1}}{N_q} + \frac{N_{p2}}{N_q} \right) i_{c1} - \frac{N_{p1}}{N_q} i_{b2}}{2 \frac{N_{p1}}{N_q} + \frac{N_{p2}}{N_q}} \\ i_{3c} = \frac{i_{b2} - i_{b1} - \left( \frac{N_{p1}}{N_q} + \frac{N_{p2}}{N_q} \right) i_{a1} - \frac{N_{p1}}{N_q} i_{c2}}{2 \frac{N_{p1}}{N_q} + \frac{N_{p2}}{N_q}} \end{cases} \quad (13)$$

The differential delta-connected transformer KVA can be expressed as

$$S = \frac{3}{2} \left[ U_q (I_{5^b} + I_{4^c}) + U_{p2} I_{2^a} + U_{p1} (I_{1^a} + I_{3^a}) \right] \quad (14)$$

Where  $I_{5^a}, I_{4^c}, I_{2^a}, I_{1^a}$  and  $I_{3^a}$  is the RMS value of  $i_{5^a}, i_{4^c}, i_{2^a}, i_{1^a}$  and  $i_{3^a}$  respectively.

With the same way,  $i_{5^a}, i_{4^c}, i_{2^a}, i_{1^a}$  and  $i_{3^a}$  can be calculated from (5) and (6) respectively.

The output power  $P_0$  can be expressed as

$$P_0 = U_d I_d \quad (15)$$

Where  $U_d$  is the RMS value of  $u_d$ ,  $I_d$  is the RMS value of  $i_d$ .

$U_d$  can be expressed as[4]

$$\begin{cases} \frac{3U_m}{\sqrt{\pi} \cos \alpha} \sqrt{\left(\frac{\pi}{6} - \alpha\right) \cos^2(\alpha) + \alpha \cos^2\left(\alpha - \frac{\pi}{6}\right) + \frac{1}{4} \cos\left(\frac{\pi}{6} - 2\alpha\right) + \frac{\sqrt{3}}{8}}, & 0 \leq \alpha \leq \frac{\pi}{6} \\ \frac{3U_m}{\sqrt{\pi} \cos \alpha} \sqrt{\left(\alpha - \frac{\pi}{6}\right) \cos^2\left(\alpha - \frac{\pi}{3}\right) + \left(\frac{\pi}{3} - \alpha\right) \cos^2\left(\alpha - \frac{\pi}{6}\right) + \frac{1}{4} \cos\left(\frac{\pi}{2} - 2\alpha\right) + \frac{\sqrt{3}}{8}}, & \frac{\pi}{6} \leq \alpha \leq \frac{\pi}{3} \\ \frac{3U_m}{\sqrt{\pi} \cos \alpha} \sqrt{\left(\frac{\pi}{2} - \alpha\right) \cos^2\left(\alpha - \frac{\pi}{3}\right) + \left(\alpha - \frac{\pi}{3}\right) \cos^2\left(\alpha - \frac{\pi}{2}\right) + \frac{1}{4} \cos\left(\frac{5\pi}{6} - 2\alpha\right) + \frac{\sqrt{3}}{8}}, & \frac{\pi}{3} \leq \alpha \leq \frac{\pi}{2} \end{cases} \quad (16)$$

Define  $S_{eq}$  as the equivalent KVA of the differential delta-connected transformer.

$$S_{eq} = \frac{S}{P_0} \quad (17)$$

Programming (12),(13),(14),(15)and (16)into(17), we can obtain the relationship between phase-shift angle and differential delta-connected transformer equivalent KVA rating in figure 5.

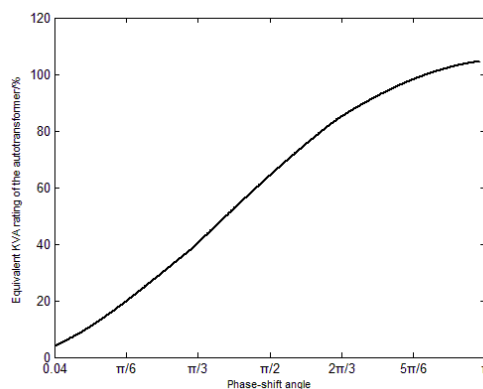


Fig. 5 Effect of the phase-shift angle on equivalent KVA of the autotransformer

In figure 5, when  $\varphi$  are  $\pi/6, \pi/2$  and  $5\pi/6$  the value of  $S_{eq}$  is around 19%, 63% and 98% respectively. While  $\varphi$  is  $5\pi/6$ , the equivalent KVA rating of autotransformer is almost equal to that of isolated transformer. Therefore,  $5\pi/6$  will be not the best choice of multi-pulse rectifier to be a phase-shift angle.

#### 4 Matlab based Simulation

Figure 6 shows the model of 12-pulse rectifier with differential delta-connected transformer.

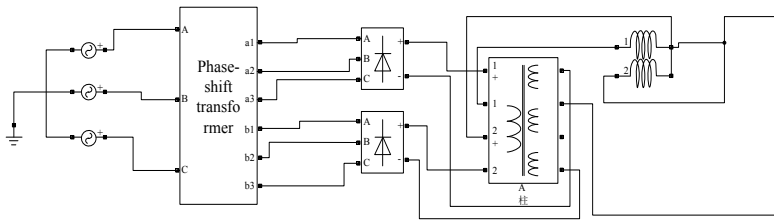


Fig. 6 Diagram of matlab based block of simulation

In figure 6, the basic frequency is 50HZ and the running time is 425 cycles. To verify the correctness of the theory analysis, during this simulation, 47 points are chosen among the range of  $0.04 < \varphi < \pi$  averagely.

Get the experimental data of  $i_a$  THD and the theoretical curve of  $i_a$  THD in figure 7.

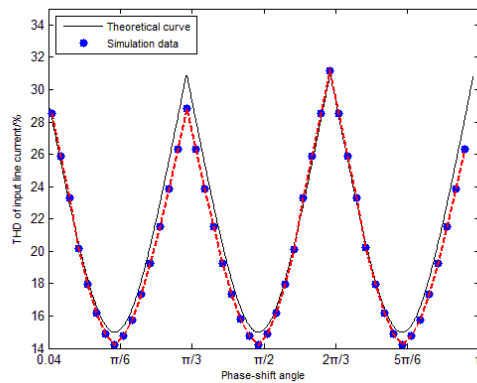


Fig. 7 Diagram of input current total harmonic distribution investigation

In figure 7, the simulation curve and the theoretical curve is almost the same. When  $\varphi$  are  $\pi/6, \pi/2$  and  $5\pi/6$  the input current THD value hits to the minimum value which is around 15%.

Get the experimental data of the autotransformer equivalent KVA and the theoretical curve about its KVA in figure 8.

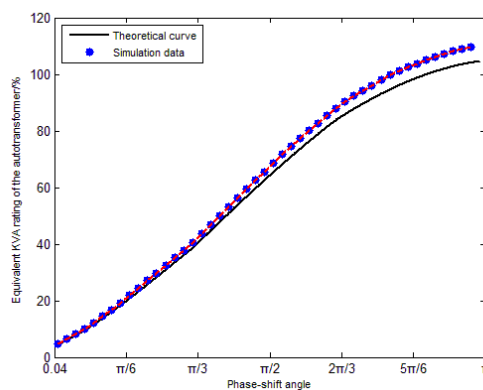


Fig. 8 Diagram of autotransformer equivalent KVA investigation

In figure 8, the simulation curve and the theoretical curve is almost the same. When  $\varphi$  are



$\pi/6$ ,  $\pi/2$  and  $5\pi/6$ , the equivalent KVA of differential delta-connected transformer is about 19%, 65% and 101% respectively.

## 5 Summary

The traditional phase-shift angle of differential delta-connected transformer is not suitable the step-up voltage occasion in the 12-pulse rectifier. However,  $\pi/2$  as a novel phase-shift angle of the differential delta-connected transformer can boost the output voltage to satisfy slightly greater step-up voltage occasion. Meanwhile, with the novel phase-shift angle of  $\pi/2$  the equivalent KVA rating of the autotransformer is about 63%, which is still smaller than the traditional isolated transformer. If the slightly greater step-up voltage situation has no strict requirements on the electric isolation, the differential delta-connected transformer is still a nice choice to provide a suitable step-up voltage and reduce the volume of the rectifier to the isolated transformer at the same time.

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